Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 532610, 6 pages
doi:10.1155/2012/532610

Research Article

Stable One-Dimensional Periodic Wave in Kerr-Type and Quadratic Nonlinear Media

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Received 6 December 2011; Revised 9 February 2012; Accepted 13 February 2012

Academic Editor: Cristian Toma

We present the propagation of optical beams and the properties of one-dimensional (1D) spatial solitons ("bright" and "dark") in saturated Kerr-type and quadratic nonlinear media. Special attention is paid to the recent advances of the theory of soliton stability. We show that the stabilization of bright periodic waves occurs above a certain threshold power level and the dark periodic waves can be destabilized by the saturation of the nonlinear response, while the dark quadratic waves turn out to be metastable in the broad range of material parameters. The propagation of (1+1) a dimension-optical field on saturated Kerr media using nonlinear Schrödinger equations is described. A model for the envelope one-dimensional evolution equation is built up using the Laplace transform.

1. Introduction

The discrete spatial optical solitons have been introduced and studied theoretically as spatially localized modes of periodic optical structures [1]. A standard theoretical approach in the study of the discrete spatial optical solitons is based on the derivation of an effective discrete nonlinear Schrödinger equation and the analysis of its stationary localized solitons-discrete localized modes [1, 2].

The spatial solitons may exist in a broad branch of nonlinear materials, such as cubic Kerr, saturable, thermal, reorientation, photorefractive, and quadratic media, and periodic systems. Furthermore, the existence of solitons varies in topologies and dimensions [3].

The theory of spatial optical solitons has been based on the nonlinear Schrödinger (NLS) equation with a cubic nonlinearity, which is exactly integrable by means of the inverse scattering (IST) technique. From the physical point of view, the integrable NLS equation describes the (1+1)-dimensional beams in a Kerr (cubic) nonlinear medium in the framework of the so-called paraxial approximation [4].

Bright solitons are formed due to the diffraction or dispersion compensated by self-focusing nonlinearity and appear as an intensity hump in a zero background. Solitons, which appear as intensity dips with a CW background, are called *dark soliton* [3].

Kerr solitons rely primarily on a physical effect, which produces an intensity-dependent change in refractive index [3].

The periodic wave structures play an important role in the nonlinear wave domain so that they are core of instability modulation development and optics chaos on continuous nonlinear media, modes of quasidiscrete systems or discrete system on mechanic and electric domain. Thus, periodic wave structures are unstable in the propagation process. For example, photorefractive crystals accept relatively high nonlinearity of saturated character at an already known intensity for He-Ne laser in continuous regime.

2. Methodology

The propagation of the optical radiation in $(1+1)$ dimensions in saturable Kerr-type medium is described by the nonlinear Schrödinger equation for the varying field amplitude $\Phi(\varsigma,\rho)$ [5]:

$$2i\frac{\partial\Phi(\varsigma,\rho)}{\partial\varsigma}+\frac{\partial^2\Phi(\varsigma,\rho)}{\partial\rho^2}-2\frac{\Phi(\varsigma,\rho)\left|\Phi(\varsigma,\rho)\right|^2}{1+S\left|\Phi(\varsigma,\rho)\right|^2}=0. \tag{2.1}$$

The transverse ς and the longitudinal ρ coordinates are scaled in terms of the characteristic pulse (beam) width and dispersion (diffraction) length, respectively; S is the saturation parameter; $\sigma=-1$ (+1) stands for focusing (defocusing) media [5]

$$\begin{gathered}\varsigma=\sigma KZ,\\ \rho=\sqrt{\sigma}K\sqrt{X^2+Y^2},\\ \varphi=\operatorname{arctg}\left(\frac{Y}{X}\right),\\ \eta=\rho\sin\varphi,\\ \xi=\rho\cos\varphi.\end{gathered} \tag{2.2}$$

The simplest periodic stationary solutions of (2.1) have the following form:

$$\Phi(\varsigma,\rho)=U(\rho)e^{+2ih\varsigma}, \tag{2.3}$$

where h is the propagation constant.

By replacing the field in such a form into (2.1), one gets

$$\frac{\partial^2U(\rho)}{\partial\rho^2}-2hU(\rho)-\frac{2U^3(\rho)}{1+SU^2(\rho)}=0. \tag{2.4}$$

To perform the linear stability analysis of periodic waves in the saturable medium, we use the mathematical formalism initially developed for periodic waves in cubic nonlinear media [5].

We consider an analytic model, which used the Laplace transform of (2.4):

$$\left(\widehat{\alpha}(U(\rho)) = \int_0^{+\infty} U(\rho)e^{-p\rho}d\rho = \widetilde{U}(p)\right)$$
$$p = u_1 + iv_1 \tag{2.5}$$
$$-\left(\frac{p^2}{2} - 2h\right)\widetilde{U}(p) + \left[\frac{p}{2}U(0) + \frac{1}{2}\left(\frac{\partial U(\rho)}{\partial \rho}\right)_{\rho=0}\right] + \int_0^{\infty} \frac{U^3(\rho)}{1+SU^2(\rho)}e^{-p\rho}d\rho = 0.$$

With the boundary conditions,

$$U(\rho)\big|_{\rho=0} = U(0) = U_0,$$
$$\left.\frac{\partial U(\rho)}{\partial \rho}\right|_{\rho=0} = 0. \tag{2.6}$$

From (2.5) we get the Laplace transform of the field:

(i) direct form:

$$\widetilde{U}(p) = \frac{pU_o + 2\int_0^{+\infty}((U^3(\rho))/(1+SU^2(\rho)))e^{-p\rho}d\rho}{((p^2/2) - 2h)} \tag{2.7}$$

(ii) inverse transformation form:

$$U(\rho) = \frac{1}{2\pi i}\int_{u-i\infty}^{u+i\infty} \frac{pU_0 + 2\int_0^{+\infty}(U^3(\rho)/1+SU^2(\rho))e^{-p\rho}d\rho}{(p^2-4h)}e^{+p\rho}dp, \tag{2.8}$$

where u is a finite number.

For the integration on real ($h > 0$) and imaginary ($h < 0$) poles, we calculated the complex amplitude of nonlinear equation such as

$$U(\rho) = U_0\operatorname{ch}\left(2\sqrt{h}\rho\right) - 4\int_0^{+\infty} d\left(\frac{\operatorname{sh}2\sqrt{h}(\rho-\rho')}{\left(2\sqrt{h}\right)^2}\right)\left(\frac{U^3(\rho')}{1+SU^2(\rho')}\right),$$
$$U(\rho) = U_0\cos\left(2\sqrt{h}\rho\right) + 4\int_0^{+\infty} d\rho'\left(\frac{U^3(\rho')}{1+SU^2(\rho')}\right)\left(\frac{\cos 2\sqrt{h}(\rho-\rho')}{2\sqrt{h}}\right). \tag{2.9}$$

For the harmonic case ($h < 0$) integration form of the complex amplitude is

$$U(\rho) = U_0 \cos\left(2\sqrt{h}\rho\right) + \frac{1}{h}\cos\left(2\sqrt{h}\rho\right)\int_0^{+\infty}\frac{U^3(\rho')}{1+SU^2(\rho')}d\left(\sin 2\sqrt{h}\rho'\right)$$
$$-\frac{1}{h}\sin\left(2\sqrt{h}\rho\right)\int_0^{+\infty}\frac{U^3(\rho')}{1+SU^2(\rho')}d\left(\cos 2\sqrt{h}\rho'\right). \tag{2.10}$$

By using the integration, we get

$$U(\rho) = U_0 \cos\left(2\sqrt{h}\rho\right) + \frac{1}{h}\frac{U^3(\rho)}{1+SU^2(\rho)}\sin\left(2\sqrt{h}\rho\right) \tag{2.11}$$

or

$$U(\rho) = \sqrt{U_0^2 + \left(\frac{1}{h}\frac{U^3(\rho)}{1+SU^2(\rho)}\right)^2}\sin\left(2\sqrt{h}\rho + \varphi_1\right),$$
$$\varphi_1 = \operatorname{arctg}\left(\frac{U_0}{(1/h)(U^3(\rho)/1+SU^2(\rho))}\right). \tag{2.12}$$

The total phase of the optical field envelope is as follows:

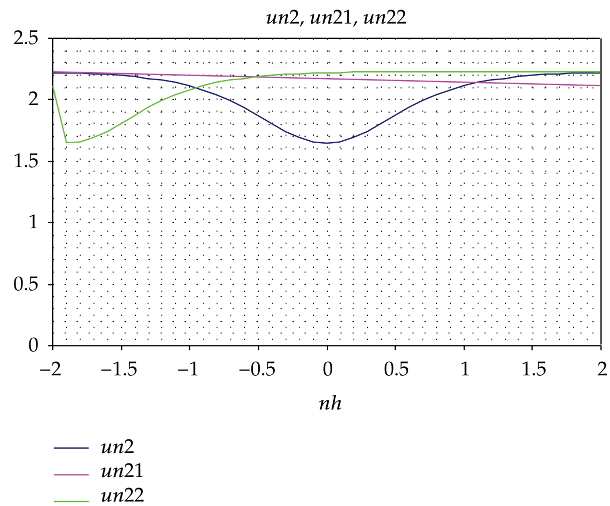
$$\varphi_T = 2\sqrt{h}\rho + \operatorname{arctg}\left(\frac{U_0}{(1/h)(U^3(\rho)/1+SU^2(\rho))}\right). \tag{2.13}$$

We assume a frequency (ω) as a speed variation of total phase such as

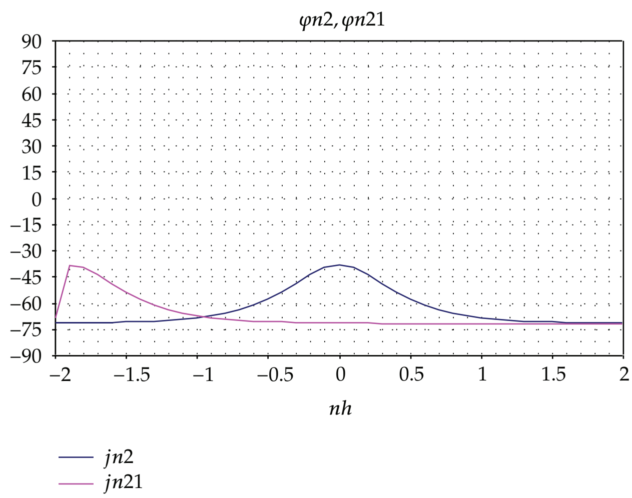
$$\omega \overset{\text{Def}}{=} \frac{d\varphi_T}{d\rho} = \left(2\sqrt{h}\right) + \frac{d}{d\rho}\left\{\operatorname{arctg}\left(hU_0\frac{1+SU^2(\rho)}{U^3(\rho)}\right)\right\}. \tag{2.14}$$

We have the complex amplitude of envelope field with the following form:

$$U(\rho) = A(\omega,\rho)\cos\left(2\sqrt{h}\rho\right) + B(\omega,\rho)\sin\left(2\sqrt{h}\rho\right). \tag{2.15}$$



(a)



(b)

Figure 1: Numerical simulations of complex amplitude and phase.

The hyperbolic secant plays this equation resulting in a conservative effect. The longitudinal component is

$$A(\omega,\rho) = U_0 - \frac{5\omega}{2(\mathrm{Sh})^{3/2}(1 + (\omega^3/4h))\mathrm{ch}(\omega\rho)},$$
$$B(\omega,\rho) = \left(\frac{1}{h}\frac{U_0^3}{1 + SU^2(0)}\right) - \frac{\left(5\omega\left(\omega^2/2\sqrt{h}\right)\right) + h(\omega\rho)}{2(\mathrm{Sh})^{3/2}(1 + (\omega^3/4h))\mathrm{ch}(\omega\rho)}. \tag{2.16}$$

Some numerical simulations of the complex amplitude of the nonlinear equation and the total phase of the optical field depending on the propagation constant and an integer number n are illustrated in Figure 1.

Figure 1 represents the model amplitude and the phase functions of the complex total number, which explained the theoretical model presented. Thanks to the complex model, the initial solution includes the hyperbolic secant and the conjugate complex part

$$\begin{aligned}
\Phi(\xi,\rho) &= \left\{ \begin{array}{l} \left[U_0 - \dfrac{5\omega}{2(\mathrm{Sh})^{3/2}(1+(\omega^3/4h))\mathrm{ch}(\omega\rho)}\right]\cos\left(2\sqrt{h}\rho\right) \\ +\left[\left(\dfrac{1}{h}\dfrac{U_0^3}{1+SU_0^2}\right) - \dfrac{5\omega\left(\omega^2/2\sqrt{h}\right)+h(\omega\rho)}{2(\mathrm{Sh})^{3/2}(1+(\omega^3/4h))\mathrm{ch}(\omega\rho)}\right]\sin\left(2\sqrt{h}\right)\rho \end{array} \right\} e^{2ih\xi}, \\
\omega_0 &= 2\sqrt{h}, \\
\omega &= \frac{d\varphi_T}{d\rho}, \\
\omega(\rho) &= \omega_0 + \frac{e^{-4\omega_0\rho}}{4\left((\omega_0^2/4)U_0 S^{3/2}\right)^2}\frac{1}{\mathrm{ch}^2(\omega_0\rho)}, \\
\omega(0) - \omega_0 &= \frac{16}{U_0^2 S^3}\frac{1}{\omega_0^4}.
\end{aligned} \tag{2.17}$$

3. Conclusions

We have described the propagation in quadratic nonlinear media of the periodic waves in saturated Kerr type. The analytic solution for one-dimensional, bright and dark spatial solitons was found. To describe the spatial optical solitons in saturated Kerr type and the quadratic nonlinear media, we propose an analytical model based on Laplace transform. The theoretical model consists in solving analytically the Schrödinger equation with photonic network using Laplace transform. The propagation properties were found by using different forms of saturable nonlinearity. However, an exact analytic solution of the propagation problem presented herein creates possibilities for further theoretical investigation. As a result, it is a useful structure, which obtains one-dimensional "bright" and "dark" solitons with transversal structure and transversal one-dimensional periodic waves.

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