

Research Article

Multiobjective Interaction Programming Problem with Interaction Constraint for Two Players

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This paper extends an existing cooperative multi-objective interaction programming problem with interaction constraint for two players (or two agents). First, we define an *s*-optimal joint solution with weight vector to multi-objective interaction programming problem with interaction constraint for two players and get some properties of it. It is proved that the *s*-optimal joint solution with weight vector to the multi-objective interaction programming problem can be obtained by solving a corresponding mathematical programming problem. Then, we define another *s*-optimal joint solution with weight value to multi-objective interaction programming problem with interaction constraint for two players and get some of its properties. It is proved that the *s*-optimal joint solution with weight vector to multi-objective interaction programming problem can be obtained by solving a corresponding mathematical programming problem. Finally, we build a pricing multi-objective interaction programming model for a bi-level supply chain. Numerical results show that the interaction programming pricing model is better than Stackelberg pricing model and the joint pricing model.

1. Introduction

There exists a kind of interactional and complex decision-making problem characterized with conflicts, incompatibility and complexity among multiagent systems, which has received much attention from researchers. Ever since the 20th century, researchers have studied two-player and multiplayer interaction problems and developed a new field called Game Theory [1], which has been widely applied in economics, engineering, military affairs, computers, and so forth [2, 3]. The game model contains the following main factors: players, strategies,

and payoffs. In the game, people focus on finding the optimal strategies that benefit all the players, which are the equilibrium solutions or cooperative solutions to the interactional problems. In recent years, some researchers studied the cooperative games and negotiation games [4], which laid emphasis on the cooperative rules, for example, multiagent model [5]. However, there are still some interactional problems that cannot be solved by the existing game models. In 1999, Meng and Li introduced a definition of interaction decision-making problems [6], which mainly considers the multiagent decision-making problems that involve two persons and more and concerns how decision is made if the decision-making process of every agent is influenced by the other agents. Therefore, the interaction problems turn out to be the interaction decision-making problems. In some cases, the interaction decision-making problems may contain conflicts, so the interaction decision-making problems can be seen as an extension of the game models. The interaction decision-making model is complex and mainly contains the following five factors: decision makers (persons or agents), sets of constraints, decision variables, objective function, and interactional constraints.

Some interaction decision-making problems can be described by nonlinear programming models with parameters, called interaction programming problem (hereinafter called IPP) and studied in [6]. Generally speaking, the game problems can always be described as interaction programming models. However, problems with conflicts and under complex constraints sometimes cannot be described by normal game models, for example, multiagent problems, and cannot be solved. After 1999, researchers made in-depth researches as to the existence and equivalence of the solution to and the method of solving the IPP [7–10]. Ma and Ding studied the relation between interaction programming and multiobjective programming by adopting the converse problem of parametric programming [8]. Meng et al. discussed two new types of IPP that are used to solve the problems with or without conflicts and introduced the definition of its joint optimal solution and the method of solving this model [9, 10]. Jiang et al. discussed the multiobjective interaction programming for two persons [11].

In this paper, first, we introduce a definition of an s -optimal joint solution with weight vector to a multiobjective interaction programming problem with two players (or two agents). In fact, Meng et al. have proved the s -optimal joint solution is a better solution to interaction programming problems than Nash equilibrium and can be obtained by solving an equivalent mathematical programming problem [7]. Moreover, the s -optimal joint solution is obtained under the assumption that all the decision makers make the same concession. For some interaction decision-making problems, there are always multiobjective decisions for decision-makers to make. Therefore, we are to extend the s -optimal joint solution of interaction programming problem discussed in [7, 11] to multiobjectives interaction programming problem with two players and study its properties. Then, we introduce a definition of s -optimal joint solution with weight value to a multiobjective interaction programming problem with two players (or two agents), which differs from the definition of an s -optimal joint solution. Finally, we build an interaction programming pricing model for bilevel supply chain. Numerical results show that the pricing interaction programming model is better than a Stackelberg pricing model or a joint pricing model.

2. s -Optimal Joint Solution

Let $f : R^m \times R^n \rightarrow R^k$, $g : R^m \times R^n \rightarrow R^k$ be the multiobjective functions, and let $X, Y, H \subset R^m \times R^n$ be nonempty sets where n , m , and k are positive integers. There exist the following

multiobjectives interaction programming problem with interaction constraint $(x, y) \in H$ for Player 1 (or Agent 1) and Player 2 (or Agent 2):

$$\begin{aligned} \min_x \quad & f(x, y) \text{ s.t. } (x, y) \in X \\ \min_y \quad & g(x, y) \text{ s.t. } (x, y) \in Y \\ \text{s.t.} \quad & (x, y) \in H. \end{aligned} \quad (\text{FG})$$

(x, y) satisfies the following constraint $(x, y) \in H$. Then, such multiobjectives interaction programming problem with interaction constraint is defined as two-player (or two-agent) multiobjective interaction programming problem (FG), and H is called the interaction constraint. Let $Z = \{(x, y) \mid (x, y) \in X, (x, y) \in Y, (x, y) \in H\}$ be a feasible set to problem (FG).

Definition 2.1. For $(x^*, y^*) \in Z$,

(i) if it satisfies

$$f(x^*, y^*) \leq f(x, y), \quad g(x^*, y^*) \leq g(x, y), \quad \forall (x, y) \in Z, \quad (2.1)$$

then (x^*, y^*) is called an optimal joint solution for Player 1 (or Agent 1) and Player 2 (or Agent 2) or to problem (FG);

(ii) if there does not exist $(x, y^*) \in Z, (x^*, y) \in Z$ which satisfies

$$f(x, y^*) \leq f(x, y), \quad g(x^*, y) \leq g(x, y), \quad (2.2)$$

then (x^*, y^*) is called a Nash-equilibrium solution for Player 1 (or Agent 1) and Player 2 (or Agent 2) or to problem (FG).

Obviously, the optimal joint solution is a Nash-equilibrium solution.

Let a given weight $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k) > 0$.

Definition 2.2. For $(x^*, y^*) \in Z$ and $\mathbf{s} = (s_1, s_2, \dots, s_k) > 0$, if it satisfies

$$f_i(x^*, y^*) - \lambda_i s_i \leq f_i(x, y), \quad g_i(x^*, y^*) - \lambda_i s_i \leq g_i(x, y), \quad i = 1, 2, \dots, k, \quad \forall (x, y) \in Z, \quad (2.3)$$

then (x^*, y^*) is called an \mathbf{s} -joint solution with weight vector λ for Player 1 (or Agent 1) and Player 2 (or Agent 2) or to problem (FG). \mathbf{s} is called a joint value of problem (FG), and the set of all joint values is denoted by S .

When $\lambda = (1, 1, \dots, 1)$, an \mathbf{s} -joint solution with weight vector λ is an \mathbf{s} -joint solution in [11].

Theorem 2.3. Consider the following single objective programming problems (F_i) and (G_i) ($i = 1, 2, \dots, k$):

$$\begin{aligned} \min \quad & f_i(x, y) \\ \text{s.t.} \quad & (x, y) \in Z, \end{aligned} \tag{F_i}$$

$$\begin{aligned} \min \quad & g_i(x, y) \\ \text{s.t.} \quad & (x, y) \in Z. \end{aligned} \tag{G_i}$$

Let $(x_{f_i}^*, y_{f_i}^*)$ be the optimal solution to (F_i) , and let $(x_{g_i}^*, y_{g_i}^*)$ be the optimal solution to (G_i) . For any given $(x', y') \in Z$, let $s'_i = (1/\lambda_i) \max\{f_i(x', y') - f_i(x_{f_i}^*, y_{f_i}^*), g_i(x', y') - g_i(x_{g_i}^*, y_{g_i}^*)\}$ ($i = 1, 2, \dots, k$) and $\mathbf{s}' = (s'_1, s'_2, \dots, s'_k)$. Then (x', y') is an \mathbf{s}' -joint solution with weight vector λ to problem (FG) .

Proof. For any $(x, y) \in Z$ and $i = 1, 2, \dots, k$, it concludes from the assumption that

$$f_i(x_{f_i}^*, y_{f_i}^*) \leq f_i(x, y), \quad g_i(x_{g_i}^*, y_{g_i}^*) \leq g_i(x, y), \tag{2.4}$$

which implies

$$\begin{aligned} f_i(x', y') - [f_i(x', y') - f_i(x_{f_i}^*, y_{f_i}^*)] &\leq f_i(x, y), \\ g_i(x', y') - [g_i(x', y') - g_i(x_{g_i}^*, y_{g_i}^*)] &\leq g_i(x, y). \end{aligned} \tag{2.5}$$

It follows with the assumption that

$$\begin{aligned} f_i(x', y') - \lambda_i s'_i &\leq f_i(x, y), \\ g_i(x', y') - \lambda_i s'_i &\leq g_i(x, y). \end{aligned} \tag{2.6}$$

Then, by Definition 2.2, the proof completes. \square

By Theorem 2.3, for any $(x, y) \in Z$, there exists a joint value \mathbf{s} such that (x, y) is an \mathbf{s} -joint solution with weight vector λ to problem (FG) . The \mathbf{s} -joint solution illustrates the same concession \mathbf{s} the agents make. Furthermore, we define a joint value $|\mathbf{s}| = s_1 + s_2 + \dots + s_k$, expecting to get a minimum of all the joint values as an optimal solution with weight vector λ to problem (FG) .

Definition 2.4. Let (x^*, y^*) be an \mathbf{s}^* -joint solution with weight vector λ to problem (FG) with the corresponding joint value $|\mathbf{s}^*|$, that is, the minimum of all the joint values. Then, (x^*, y^*) is called an \mathbf{s}^* -optimal joint solution with weight vector λ for Player 1 (or Agent 1) and Player 2 (or Agent 2) or to problem (FG) .

Obviously, if $|\mathbf{s}^*| = 0$, then the \mathbf{s}^* -optimal joint solution with weight vector λ is the optimal joint solution as per Definition 2.1. In fact, an \mathbf{s}^* -optimal joint solution with weight vector λ is an \mathbf{s}^* -joint solution too, but an \mathbf{s}^* -joint solution with weight vector λ is not always an \mathbf{s}^* -optimal joint solution with weight vector λ .

Theorem 2.5. For $i = 1, 2, \dots, k$, let $(x_{f_i}^*, y_{f_i}^*)$ be an optimal solution to (F_i) , and let $(x_{g_i}^*, y_{g_i}^*)$ be the optimal solution to (G_i) . Then, (x^*, y^*, s^*) is an optimal solution to the following problem:

$$\begin{aligned} \min \quad & |s| = s_1 + s_2 + \dots + s_k \\ \text{s.t.} \quad & f_i(x, y) - \lambda_i s_i \leq f_i(x_{f_i}^*, y_{f_i}^*), \quad i = 1, 2, \dots, k, \\ & g_i(x, y) - \lambda_i s_i \leq g_i(x_{g_i}^*, y_{g_i}^*), \quad i = 1, 2, \dots, k, \\ & (x, y) \in Z, s_i \geq 0, i = 1, 2, \dots, k \end{aligned} \quad (S)$$

if and only if (x^*, y^*) is an s^* -optimal joint solution with weight vector λ to problem (FG).

Proof. For any $(x, y) \in Z$, by the assumption, (x^*, y^*, s^*) is an optimal solution to problem (S) such that

$$\begin{aligned} f_i(x^*, y^*) - \lambda_i s_i &\leq f_i(x_{f_i}^*, y_{f_i}^*) \leq f(x, y), \quad i = 1, 2, \dots, k, \\ g_i(x^*, y^*) - \lambda_i s_i &\leq g_i(x_{g_i}^*, y_{g_i}^*) < g_i(x, y), \quad i = 1, 2, \dots, k. \end{aligned} \quad (2.7)$$

Then, it concludes from Definition 2.2 that (x^*, y^*) is an s^* -joint solution to problem (FG). Let (x', y') be an s' -optimal joint solution with weight vector λ to problem (FG). By Definitions 2.2 and 2.4, we have $|s^*| \geq |s'|$ and, for all $(x, y) \in Z$,

$$\begin{aligned} f_i(x', y') - \lambda_i s'_i &\leq f_i(x, y), \\ g_i(x', y') - \lambda_i s'_i &\leq g_i(x, y). \end{aligned} \quad (2.8)$$

Then, (x', y') is feasible solution to problem (S), and with $|s^*| \leq |s'|$, we conclude $|s^*| = |s'|$. That is to say, (x^*, y^*) is an s^* -optimal joint solution with weight vector λ to problem (FG).

The converse statement is also true. If (x^*, y^*) is an s^* -optimal joint solution with weight vector λ to problem (FG), then it implies (x^*, y^*, s^*) is feasible to problem (S) by Definition 2.2. Suppose (x', y', s') is an optimal solution to (S), from the previous proof, it concludes that (x', y') is an s' -optimal joint solution with weight vector λ to problem (FG) and $|s^*| = |s'|$. Therefore, (x^*, y^*, s^*) is an optimal solution to problem (S), and the proof completes. \square

Corollary 2.6. Let Z be compact, and let f and g be continuous functions. Then there exists an s^* -optimal joint solution to problem (FG).

Theorem 2.7. For $i = 1, 2, \dots, k$, let $(x_{f_i}^*, y_{f_i}^*)$ be the optimal solution to (F_i) , and let $(x_{g_i}^*, y_{g_i}^*)$ be the optimal solution to (G_i) . If $[(x', y', s'), (x''_1, y''_1), (x''_2, y''_2)]$ ($i = 1, 2, \dots, k$) is an optimal solution to the following problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^k [s_i + f(x'_i, y'_i) + g(x''_i, y''_i)] \\ \text{s.t.} \quad & f_i(x, y) - \lambda_i s_i \leq f_i(x'_{i1}, y'_{i1}), \quad i = 1, 2, \dots, k, \\ & g_i(x, y) - \lambda_i s_i \leq g_i(x''_i, y''_i), \quad i = 1, 2, \dots, k, \\ & (x, y), (x'_i, y'_i), (x''_i, y''_i) \in Z, \quad s_i \geq 0, \quad i = 1, 2, \dots, k, \end{aligned} \quad (S')$$

then (x', y') is an s^* -optimal joint solution with weight vector λ to problem (FG), where

$$s_i^* = s'_i + \frac{1}{\lambda_i} \max \left\{ \left(f_i(x'_{i1}, y'_{i1}) - f_i(x^*_{fi}, y^*_{fi}) \right), \left(g_i(x''_{i2}, y''_{i2}) - g_i(x^*_{gi}, y^*_{gi}) \right) \right\}, \quad i = 1, 2, \dots, k. \quad (2.9)$$

Proof. Supposing (x^*, y^*) is an s^* -optimal joint solution with weight vector λ to problem (FG), from Theorem 2.5, we have that (x^*, y^*, s^*) is an optimal solution to problem (S). Then, $[(x^*, y^*, s^*), (x^*_{fi}, y^*_{fi}), (x^*_{gi}, y^*_{gi})]$ ($i = 1, 2, \dots, k$) is feasible to problem (S'), which implies

$$\sum_{i=1}^k [s'_i + f(x'_{i1}, y'_{i1}) + g(x''_{i2}, y''_{i2})] \leq \sum_{i=1}^k [s_i^* + f(x^*_{fi}, y^*_{fi}) + g(x^*_{gi}, y^*_{gi})], \quad (2.10)$$

that is:

$$\sum_{i=1}^k \left[s'_i + \left(f(x'_{i1}, y'_{i1}) - f(x^*_{fi}, y^*_{fi}) \right) + \left(g(x''_{i2}, y''_{i2}) - g(x^*_{gi}, y^*_{gi}) \right) \right] \leq \sum_{i=1}^k s_i^*. \quad (2.11)$$

Letting

$$s_i^\circ = s'_i + \frac{1}{\lambda_i} \max \left\{ \left(f(x'_{i1}, y'_{i1}) - f(x^*_{fi}, y^*_{fi}) \right), \left(g(x''_{i2}, y''_{i2}) - g(x^*_{gi}, y^*_{gi}) \right) \right\}, \quad i = 1, 2, \dots, k, \quad (2.12)$$

then it follows $\sum_{i=1}^k s_i^\circ \leq \sum_{i=1}^k s_i^*$. On the other hand, from the assumption we have that

$$\begin{aligned} f_i(x', y') - \lambda_i s'_i &\leq f_i(x'_{i1}, y'_{i1}), \quad i = 1, 2, \dots, k, \\ g_i(x', y') - \lambda_i s'_i &\leq g_i(x''_{i2}, y''_{i2}), \quad i = 1, 2, \dots, k, \end{aligned} \quad (2.13)$$

which implies

$$\begin{aligned} f_i(x', y') &\leq \lambda_i s'_i + f_i(x'_{i1}, y'_{i1}) - f_i(x^*_{fi}, y^*_{fi}) + f_i(x^*_{fi}, y^*_{fi}), \quad i = 1, 2, \dots, k, \\ g_i(x', y') &\leq \lambda_i s'_i + g_i(x''_{i2}, y''_{i2}) - g_i(x^*_{gi}, y^*_{gi}) + g_i(x^*_{gi}, y^*_{gi}), \quad i = 1, 2, \dots, k, \end{aligned} \quad (2.14)$$

that is:

$$\begin{aligned} f_i(x', y') &\leq s_i^\circ + f_i(x^*_{fi}, y^*_{fi}), \quad i = 1, 2, \dots, k, \\ g_i(x', y') &\leq s_i^\circ + g_i(x^*_{gi}, y^*_{gi}), \quad i = 1, 2, \dots, k. \end{aligned} \quad (2.15)$$

Then, $(x', y', \mathbf{s}^\circ)$ is a feasible solution to problem (S). Thus, $\sum_{i=1}^k s_i^\circ \geq \sum_{i=1}^k s_i^*$ holds and (x', y') becomes an \mathbf{s}° -optimal joint solution with weight vector λ to problem (FG) with $\sum_{i=1}^k s_i^\circ = \sum_{i=1}^k s_i^*$. This completes the proof. \square

Remark 2.8. By Theorem 2.7, we can get an \mathbf{s}^* -optimal joint solution with weight vector λ to problem (FG) if the optimal solution to problem (S') is obtained. However, we cannot get the optimal joint value \mathbf{s}^* unless the optimal solutions to problems (F_i) and (G_i) ($i = 1, 2, \dots, k$) are obtained.

3. \mathbf{s} -Optimal Joint Solution with Weight Value

In this section, we discuss another optimal joint solution to (FG), where $f : R^m \times R^n \rightarrow R^{k_1}$ and $g : R^m \times R^n \rightarrow R^{k_2}$. When $k_1 \neq k_2$, the definition of \mathbf{s} -optimal joint solution to problem (FG) is not appropriate. Consider the following multiobjectives interaction programming problem with interaction constraint $(x, y) \in H$ for Player 1 (or Agent 1) and Player 2 (or Agent 2):

$$\begin{aligned} \min_x \quad & f(x, y) \text{ s.t. } (x, y) \in X, \\ \min_y \quad & g(x, y) \text{ s.t. } (x, y) \in Y \\ \text{s.t.} \quad & (x, y) \in H. \end{aligned} \quad (\text{FG})$$

Hence, we need to define a new \mathbf{s} -optimal joint solution to problem (FG). Suppose the weight of f_i is $p_i > 0$ and the weight of g_j is $q_j > 0$, $i = 1, 2, \dots, k_1$, $j = 1, 2, \dots, k_2$. Let

$$p = (p_1, p_2, \dots, p_{k_1}), \quad q = (q_1, q_2, \dots, q_{k_2}). \quad (3.1)$$

Definition 3.1. For $(x^*, y^*) \in Z$, $\mathbf{s} \geq 0$, if it satisfies

$$f_i(x^*, y^*) - p_i \mathbf{s} \leq f_i(x, y), \quad g_j(x^*, y^*) - q_j \mathbf{s} \leq g_j(x, y) \quad (3.2)$$

for $(x, y) \in Z$ ($i = 1, 2, \dots, k_1$, $j = 1, 2, \dots, k_2$), then (x^*, y^*) is called an \mathbf{s} -joint solution with weight vector (p, q) for Player 1 (or Agent 1) and Player 2 (or Agent 2) or to problem (FG). \mathbf{s} is called a joint value with weight vector (p, q) . Here, \mathbf{s} is real value.

Theorem 3.2. Suppose $(x_{f_i}^*, y_{f_i}^*)$ is the optimal solution to (F_i) and $(x_{g_j}^*, y_{g_j}^*)$ is the optimal solution to (G_j) . For any $(\bar{x}, \bar{y}) \in Z$, letting

$$\bar{\mathbf{s}} = \max \left\{ \frac{1}{p_i} \left(f_i(\bar{x}, \bar{y}) - f_i(x_{f_i}^*, y_{f_i}^*) \right), \frac{1}{q_j} \left(g_j(\bar{x}, \bar{y}) - g_j(x_{g_j}^*, y_{g_j}^*) \right) \mid i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2 \right\}, \quad (3.3)$$

then (\bar{x}, \bar{y}) is the $\bar{\mathbf{s}}$ -joint solution with weight vector (p, q) to (FG).

Proof. For all $(x, y) \in Z, i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2$, it follows from the assumption that

$$f_i(x_{fi}^*, y_{fi}^*) \leq f_i(x, y), \quad g_j(x_{gj}^*, y_{gj}^*) \leq g_j(x, y), \quad (3.4)$$

which implies

$$\begin{aligned} f_i(\bar{x}, \bar{y}) - [f_i(\bar{x}, \bar{y}) - f_i(x_{fi}^*, y_{fi}^*)] &\leq f_i(x, y), \\ g_j(\bar{x}, \bar{y}) - [g_j(\bar{x}, \bar{y}) - g_j(x_{gj}^*, y_{gj}^*)] &\leq g_j(x, y), \end{aligned} \quad (3.5)$$

so

$$f_i(\bar{x}, \bar{y}) - p_i \bar{s} \leq f_i(x, y), \quad g_j(\bar{x}, \bar{y}) - q_j \bar{s} \leq g_j(x, y). \quad (3.6)$$

This completes the proof with Definition 3.1. \square

By Theorem 2.3, for all $(x, y) \in X$, and for all $(x, y) \in Y$, there exists a value \mathbf{s} such that (x, y) becomes an \mathbf{s} -joint solution with weight vector (p, q) to (FG). \mathbf{s} -joint solution implies the decision makers give the same concession \mathbf{s} with weight vector (p, q) , which is fair for all the decision makers. Thus, it is useful for us to find the minimum of all the joint values.

Definition 3.3. Suppose (x^*, y^*) is an \mathbf{s}^* -joint solution with weight vector (p, q) to (FG) with \mathbf{s}^* the minimum of all the joint values. Then, (x^*, y^*) is called an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG).

Obviously, if $\mathbf{s}^* = 0$, (x^*, y^*) becomes the optimal joint solution for Player 1 (or Agent 1) and Player 2 (or Agent 2).

Theorem 3.4. For $i = 1, \dots, k_1, j = 1, \dots, k_2$, let (x_{fi}^*, y_{fi}^*) be the optimal solution to (F_i) , and let (x_{gj}^*, y_{gj}^*) be the optimal solution to (G_j) . Then, (x^*, y^*, \mathbf{s}^*) is an optimal solution to the following problem (SW):

$$\begin{aligned} \min \quad & \mathbf{s} \\ \text{s.t.} \quad & f_i(x, y) - p_i \mathbf{s} \leq f_i(x_{fi}^*, y_{fi}^*), \quad i = 1, 2, \dots, k_1, \\ & g_j(x, y) - q_j \mathbf{s} \leq g_j(x_{gj}^*, y_{gj}^*), \quad j = 1, 2, \dots, k_2, \\ & \mathbf{s} \geq 0, \quad (x, y) \in Z, \end{aligned} \quad (\text{SW})$$

if and only if (x^*, y^*) is an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG).

Proof. Suppose (x^*, y^*, \mathbf{s}^*) is the optimal solution to the problem (SW). For any $(x, y) \in Z, i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2$, it follows with the constraints of (SW) that

$$f_i(x^*, y^*) - p_i \mathbf{s}^* \leq f_i(x_{fi}^*, y_{fi}^*) \leq f_i(x, y), \quad g_j(x^*, y^*) - q_j \mathbf{s}^* \leq g_j(x_{gj}^*, y_{gj}^*) \leq g_j(x, y), \quad (3.7)$$

which implies (x^*, y^*) is an \mathbf{s}^* -joint solution with weight vector (p, q) to (FG). Assume (\bar{x}, \bar{y}) is the $\bar{\mathbf{s}}$ -optimal joint solution with weight vector (p, q) to (FG). Then, with Definitions 3.1 and 3.3, it gets $\mathbf{s}^* \geq \bar{\mathbf{s}}$ and

$$f_i(\bar{x}, \bar{y}) - p_i \bar{\mathbf{s}} \leq f_i(x, y), \quad g_j(\bar{x}, \bar{y}) - q_j \bar{\mathbf{s}} \leq g_j(x, y), \quad \forall (x, y) \in Z, \quad (3.8)$$

which implies (\bar{x}, \bar{y}) is feasible to (S) and $\bar{\mathbf{s}} \geq \mathbf{s}^*$. Then, we get $\bar{\mathbf{s}} = \mathbf{s}^*$, and (x^*, y^*) becomes the \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG).

On the contrary, if (x^*, y^*) is an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG), by Definition 3.1, it gets (x^*, y^*, \mathbf{s}^*) is feasible to (SW). Supposing $(\bar{x}, \bar{y}, \bar{\mathbf{s}})$ is the optimal solution to (SW), it is easily obtained that (\bar{x}, \bar{y}) is the $\bar{\mathbf{s}}$ -optimal joint solution with weight vector (p, q) to (FG), which implies $\bar{\mathbf{s}} = \mathbf{s}^*$. Thus, (x^*, y^*, \mathbf{s}^*) is the optimal solution to (SW), and this completes the proof. \square

Remark 3.5. If the optimal solutions to (F_i) and (G_i) exist, the \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG) exists, too. By Theorem 3.4, we can obtain a method of solving the problem IPP as follows. First, get the optimal solutions to (F_i) and (G_i) . Then, we can get an optimal solution (x^*, y^*, \mathbf{s}^*) to (SW), and (x^*, y^*) is an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG).

Theorem 3.6. Suppose (x^*, y^*) is an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG). Then, there does not exist $(x, y) \in Z$ such that

$$f_i(x, y) < f_i(x^*, y^*), \quad g_j(x, y) < g_j(x^*, y^*), \quad i = 1, 2, \dots, k_1, \quad j = 1, 2, \dots, k_2. \quad (3.9)$$

Proof. It is obviously correct if $\mathbf{s}^* = 0$. For $\mathbf{s}^* > 0$, suppose there exists $(x', y') \in Z$ such that

$$f_i(x', y') < f_i(x^*, y^*), \quad g_j(x', y') < g_j(x^*, y^*), \quad i = 1, 2, \dots, k_1, \quad j = 1, 2, \dots, k_2. \quad (3.10)$$

Let $\delta = \min\{f_i(x^*, y^*) - f_i(x', y'), g_j(x^*, y^*) - g_j(x', y') \mid i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2\}$ with $\delta > 0$. Then, there exists sufficiently small $\alpha > 0$ such that $\mathbf{s}^* - \alpha\delta > 0$, $p_i\alpha < 1$, and $q_j\alpha < 1$ for $i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2$. Letting $\mathbf{s}' = \mathbf{s}^* - \alpha\delta$, then, we get $\mathbf{s}' < \mathbf{s}^*$. It follows with Definition 3.1 that

$$\begin{aligned} f_i(x', y') - p_i \mathbf{s}' &= f_i(x', y') - p_i \mathbf{s}^* + p_i \alpha \delta < f_i(x', y') - p_i \mathbf{s}^* + f_i(x^*, y^*) - f_i(x', y') \\ &\leq f_i(x, y), \\ g_j(x', y') - q_j \mathbf{s}' &= g_j(x', y') - q_j \mathbf{s}^* + q_j \alpha \delta \\ &< g_j(x', y') - q_j \mathbf{s}^* + g_j(x^*, y^*) - g_j(x', y') \leq g_j(x, y), \end{aligned} \quad (3.11)$$

for any $(x, y) \in Z$. Thus, (x', y') is \mathbf{s}' -optimal joint solution with weight vector (p, q) to (FG) and $\mathbf{s}' > \mathbf{s}^*$, which contradicts the assumption of $\mathbf{s}' < \mathbf{s}^*$; this completes the proof. \square

Theorem 3.7. For $i = 1, \dots, k_1$, $j = 1, \dots, k_2$, let (x_{fi}^*, y_{fi}^*) be an optimal solution to (F_i) , and let (x_{gj}^*, y_{gj}^*) be an optimal solution to (G_j) . If $[(\bar{x}, \bar{y}, \bar{s}), (\bar{x}'_i, \bar{y}'_i), (\bar{x}''_j, \bar{y}''_j)]$ ($i = 1, 2, \dots, k_1$, $j = 1, 2, \dots, k_2$) is the optimal solution of the following problem (\bar{S}) :

$$\begin{aligned} \min \quad & \mathbf{s} + \sum_{i=1}^{k_1} \frac{1}{p_i} f_i(x'_i, y'_i) + \sum_{j=1}^{k_2} \frac{1}{q_j} g_j(x''_j, y''_j) \\ \text{s.t.} \quad & f_i(x, y) - p_i \mathbf{s} \leq f_i(x'_i, y'_i), \quad i = 1, 2, \dots, k_1, \\ & g_j(x, y) - q_j \mathbf{s} \leq g_j(x''_j, y''_j), \quad j = 1, 2, \dots, k_2, \\ & \mathbf{s} \geq 0, (x, y), (x'_i, y'_i), (x''_j, y''_j) \in Z, \quad i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2, \end{aligned} \quad (\bar{S})$$

then (\bar{x}, \bar{y}) is an \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG) , where

$$\begin{aligned} \mathbf{s}^* = \bar{s} + \max \left\{ \frac{1}{p_i} (f_i(\bar{x}'_i, \bar{y}'_i) - f_i(x_{fi}^*, y_{fi}^*)), \right. \\ \left. \frac{1}{q_j} (g_j(\bar{x}''_j, \bar{y}''_j) - g_j(x_{gj}^*, y_{gj}^*)) \mid i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2 \right\}. \end{aligned} \quad (3.12)$$

Proof. Supposing (x^*, y^*) is the \mathbf{s}^* -optimal joint solution with weight vector (p, q) to (FG) , it concludes from Theorem 3.4 that (x^*, y^*, \mathbf{s}^*) is the optimal solution to (SW) . Then, $[(x^*, y^*, \mathbf{s}^*), (x_{fi}^*, y_{fi}^*), (x_{gj}^*, y_{gj}^*)]$ ($i = 1, 2, \dots, k_1$, $j = 1, 2, \dots, k_2$) is feasible to (\bar{S}) . Thus, we have

$$\bar{s} + \sum_{i=1}^{k_1} \frac{1}{p_i} f_i(\bar{x}'_i, \bar{y}'_i) + \sum_{j=1}^{k_2} \frac{1}{q_j} g_j(\bar{x}''_j, \bar{y}''_j) \leq \mathbf{s}^* + \sum_{i=1}^{k_1} \frac{1}{p_i} f_i(x_{fi}^*, y_{fi}^*) + \sum_{j=1}^{k_2} \frac{1}{q_j} g_j(x_{gj}^*, y_{gj}^*). \quad (3.13)$$

Let $\tilde{\mathbf{s}} = \bar{s} + \max\{(1/p_i)(f_i(\bar{x}'_i, \bar{y}'_i) - f_i(x_{fi}^*, y_{fi}^*)), (1/q_j)(g_j(\bar{x}''_j, \bar{y}''_j) - g_j(x_{gj}^*, y_{gj}^*)) \mid i = 1, 2, \dots, k_1, j = 1, 2, \dots, k_2\}$.

It concludes from (3.13) that $\tilde{\mathbf{s}} \leq \mathbf{s}^*$. Further, with the assumption, we get

$$\begin{aligned} f_i(\bar{x}, \bar{y}) - p_i \tilde{\mathbf{s}} &\leq f_i(\bar{x}'_i, \bar{y}'_i), \quad i = 1, 2, \dots, k_1, \\ g_j(\bar{x}, \bar{y}) - q_j \tilde{\mathbf{s}} &\leq g_j(\bar{x}''_j, \bar{y}''_j), \quad j = 1, 2, \dots, k_2, \end{aligned} \quad (3.14)$$

which implies

$$\begin{aligned} f_i(\bar{x}, \bar{y}) &\leq p_i \tilde{\mathbf{s}} + f_i(\bar{x}'_i, \bar{y}'_i) - f_i(x_{fi}^*, y_{fi}^*) + f_i(x_{fi}^*, y_{fi}^*), \quad i = 1, 2, \dots, k_1, \\ g_j(\bar{x}, \bar{y}) &\leq q_j \tilde{\mathbf{s}} + g_j(\bar{x}''_j, \bar{y}''_j) - g_j(x_{gj}^*, y_{gj}^*) + g_j(x_{gj}^*, y_{gj}^*), \quad j = 1, 2, \dots, k_2. \end{aligned} \quad (3.15)$$

Thus,

$$\begin{aligned} f_i(\bar{x}, \bar{y}) &\leq p_i \tilde{s} + f_i(x_{f_i}^*, y_{f_i}^*), \quad i = 1, 2, \dots, k_1, \\ g_j(\bar{x}, \bar{y}) &\leq q_j \tilde{s} + g_j(x_{g_j}^*, y_{g_j}^*), \quad j = 1, 2, \dots, k_2. \end{aligned} \quad (3.16)$$

Therefore, $(\bar{x}, \bar{y}, \tilde{s})$ is feasible to (S), which implies $\tilde{s} \geq s^*$, and (\bar{x}, \bar{y}) is the \tilde{s} -joint solution with weight vector (p, q) to (FG). Thus, $\tilde{s} = s^*$, and this completes the proof. \square

In Theorem 3.4, as soon as the optimal solutions to (F_i) and (G_i) are found, it is possible to find out an s^* -optimal joint solution with weight vector to (FG) by solving the problem (SW). In Theorem 3.7, we can obtain an s^* -optimal joint solution with weight vector to (FG) by solving the problem (\bar{S}) . However, if we do not solve the optimal solutions to (F_i) and (G_i) , it is unluckily that we cannot get an s^* .

Example 3.8. Considering the following IPP:

$$\begin{aligned} \min \quad & f(x, y) = \left((x-2)^2 + y^2, x^2 + y^2 \right), & (F(y)) \\ \min \quad & g(x, y) = \left(x^2 + (y-2)^2, x^2 + y^2 \right) & (G(x)) \\ \text{s.t.} \quad & x, y \in R, \end{aligned}$$

where $H = R \times R$. For $p_1 = p_2 = q_1 = q_2 = 2$, by Theorem 3.4, it is easily obtained $(1, 1)$ that is the 1-optimal joint solution with weight vector $(2, 2, 2, 2)$ to this IPP.

4. A Bilevel Supply Chain with Two Players

Now, we show an example of a bilevel supply chain with two players, which is solved with the method given in Section 2. The bilevel supply chain can be seen as a two-player system where a manufacturer is one agent while a retailer is the other, with the manufacturer providing goods for the retailer to sell. Then, the problem is to decide the prices of the goods at a level such that both the manufacturer and retailer can gain the most. Clearly, this is an IPP with two players. Suppose there is a manufacturer which manufactures products P_i ($i = 1, 2, \dots, n$). Let c_i denote the production cost for P_i ($i = 1, 2, \dots, n$), c_{it} the transportation cost for P_i ($i = 1, 2, \dots, n$), and w_i the price of P_i ($i = 1, 2, \dots, n$). The manufacturer provides products to the retailer at $w_i^{\min} \leq w_i \leq w_i^{\max}$, where w_i^{\min} is the minimum of w_i and w_i^{\max} is the maximum of w_i . Let q_i denote the quantity ordered for P_i ($i = 1, 2, \dots, n$) with $q_i^{\min} \leq q_i \leq q_i^{\max}$, where q_i^{\min} is the minimum of q_i and q_i^{\max} is the maximum of q_i . Supposing the retailer sells the product P_i ($i = 1, 2, \dots, n$) at the price of p_i , then it is clear that $w_i \leq p_i$. Let $D_i = D_i(p_i)$, which denotes the market demand for P_i ($i = 1, 2, \dots, n$). Then we get the following pricing model for the bi-level supply chain:

$$\begin{aligned} \max_w \quad & f(w, p) = \left((w_1 - c_1 - c_{t1})D_1(p_1), (w_2 - c_2 - c_{t2})D_2(p_2), \dots, (w_n - c_n - c_{tn})D_n(p_n) \right) \\ \text{s.t.} \quad & w_i^{\min} \leq w_i \leq w_i^{\max}, \quad i = 1, 2, \dots, n, \\ & q_i^{\min} \leq D_i(p_i) \leq q_i^{\max}, \quad i = 1, 2, \dots, n, \end{aligned} \quad (F_p)$$

$$\begin{aligned}
\max_p \quad & g(w, p) = ((p_1 - w_1)D_1(p_1), (p_2 - w_2)D_2(p_2), \dots, ((p_n - w_n)D_n(p_n))) \\
\text{s.t.} \quad & w_i^{\min} \leq w_i \leq w_i^{\max}, \quad i = 1, 2, \dots, n, \\
& q_i^{\min} \leq D_i(p_i) \leq q_i^{\max}, \quad i = 1, 2, \dots, n, \\
& w_i \leq p_i, \quad i = 1, 2, \dots, n,
\end{aligned} \tag{G_w}$$

where $p = (p_1, p_2, \dots, p_n)$ is the parameter for (F_p) , $w = (w_1, w_2, \dots, w_n)$ is the parameter for (G_w) , and $w_i \leq p_i$ ($i = 1, 2, \dots, n$) is the interaction constraint.

As is known to all, Stackelberg pricing model (which is a pricing model constructed as per Stackelberg model) and joint pricing model are widely used in the supply chain pricing decision. Stackelberg pricing model tends to benefit the manufacturer since the price is decided by the manufacturer, while the price is always decided by the retailer in joint pricing model.

Then we compare the results of our model with those of Stackelberg pricing model and joint pricing model. For the previous problem, let $i = 1, 2, 3$, and let all the P_i be independent to each other; the transportation cost $c_{it} = 5$ for all P_i ; the production costs: $c_1 = 10$, $c_2 = 12$ and $c_3 = 13$; the production capacity $40 \leq q_1 \leq 100$, $30 \leq q_2 \leq 100$ and $30 \leq q_3 \leq 100$; the price w_i ($i = 1, 2, 3$) restricted by $20 \leq w_1 \leq 30$, $20 \leq w_2 \leq 35$ and $20 \leq w_3 \leq 35$; the market demand for P_i : $D_1(p_1) = 200 - 4p_1$, $D_2(p_2) = 250 - 5p_2$ and $D_3(p_3) = 300 - 6p_3$.

We have the pricing model of (FG):

$$\begin{aligned}
\max_w \quad & f(w, p) = ((w_1 - 15)(200 - 4p_1), (w_2 - 17)(250 - 5p_2), (w_3 - 18)(300 - 6p_3)) \\
\text{s.t.} \quad & 20 \leq w_1 \leq 30, \quad 20 \leq w_2 \leq 35, \quad 20 \leq w_3 \leq 35, \\
& 40 \leq 200 - 4p_1 \leq 100, \quad 30 \leq 250 - 5p_2 \leq 100, \quad 30 \leq 300 - 6p_3 \leq 100,
\end{aligned} \tag{F_p}$$

$$\begin{aligned}
\max_p \quad & g(w, p) = ((p_1 - w_1)(200 - 4p_1), (p_2 - w_2)(250 - 5p_2), (p_3 - w_3)(300 - 6p_3)) \\
\text{s.t.} \quad & 20 \leq w_1 \leq 30, \quad 20 \leq w_2 \leq 35, \quad 20 \leq w_3 \leq 35, \\
& 40 \leq 200 - 4p_1 \leq 100, \quad 30 \leq 250 - 5p_2 \leq 100, \quad 30 \leq 300 - 6p_3 \leq 100, \\
& p_1 \geq w_1, \quad p_2 \geq w_2, \quad p_3 \geq w_3.
\end{aligned} \tag{G_w}$$

We give the Stackelberg pricing model:

$$\begin{aligned}
\max_{w,p} \quad & f(w, p) = ((w_1 - 15)(200 - 4p_1), (w_2 - 17)(250 - 5p_2), (w_3 - 18)(300 - 6p_3)) \\
\text{s.t.} \quad & 20 \leq w_1 \leq 30, \quad 20 \leq w_2 \leq 35, \quad 20 \leq w_3 \leq 35, \\
& 40 \leq 200 - 4p_1 \leq 100, \quad 30 \leq 250 - 5p_2 \leq 100, \quad 30 \leq 300 - 6p_3 \leq 100.
\end{aligned} \tag{4.1}$$

p solves the lower level problem:

$$\begin{aligned} \max_p \quad & g(w, p) = ((p_1 - w_1)(200 - 4p_1), (p_2 - w_2)(250 - 5p_2), (p_3 - w_3)(300 - 6p_3)) \\ \text{s.t.} \quad & 20 \leq w_1 \leq 30, 20 \leq w_2 \leq 35, 20 \leq w_3 \leq 35, \\ & 40 \leq 200 - 4p_1 \leq 100, 30 \leq 250 - 5p_2 \leq 100, 30 \leq 300 - 6p_3 \leq 100, \\ & p_1 \geq w_1, p_2 \geq w_2, p_3 \geq w_3. \end{aligned} \tag{4.2}$$

Let $f_1(w, p) = (w_1 - 15)(200 - 4p_1) + (w_2 - 17)(250 - 5p_2) + (w_3 - 18)(300 - 6p_3)$ and let $g_1(w, p) = (p_1 - w_1)(200 - 4p_1) + (p_2 - w_2)(250 - 5p_2) + (p_3 - w_3)(300 - 6p_3)$. We give the joint pricing model:

$$\begin{aligned} \max_p \quad & f_1(w, p) + g_1(w, p) \\ \text{s.t.} \quad & 20 \leq w_1 \leq 30, 20 \leq w_2 \leq 35, 20 \leq w_3 \leq 35, \\ & 40 \leq 200 - 4p_1 \leq 100, 30 \leq 250 - 5p_2 \leq 100, 30 \leq 300 - 6p_3 \leq 100, \\ & p_1 \geq w_1, p_2 \geq w_2, p_3 \geq w_3. \end{aligned} \tag{4.3}$$

Then, we solve this pricing problem of supply chain with the previously mentioned three pricing models, and the numerical results are given in Table 1.

From Table 1, it is found that the profit π_m of the manufacturer is about 4 times less than the profit π_r of the retailer in the solution of joint pricing model which cannot be accepted by the manufacturer. The profit π_m of the manufacturer is about 2 times more than the profit π_r of the retailer in the solution of Stackelberg model which may not be accepted by the retailer. However, in s-optimal joint solution to the IPP, the difference between the profit π_m of the manufacturer and the profit π_r of the retailer is much closer, around 40%. Therefore, the s-optimal joint solution to (FG) provides a better equilibrium solution that can provide maximum profit for both the manufacturer and the retailer. Thus, the s-optimal joint solution is an acceptable solution for the manufacturer and the retailer.

5. Conclusion

In this paper, s-optimal joint solution and s-optimal joint solution with weight vector to the multiobjective interaction programming problem with two players are discussed, and they are obtained by solving some equivalent mathematical programming problems. Furthermore, the proposed model in this paper can be extended to that of multiagents. The numerical results illustrate that the solution of the multiobjective interaction programming model to the bilevel supply chain is better than those of Stackelberg model and joint pricing model. Moreover, the multiobjective interaction programming may be applied in other fields, such as allocation of multi-jobs in computer networks and allocation of resources in market.

Table 1: Solutions of the three models.

		Profit of s-joint solution	Profit of Stackelberg solution	Profit of pricing solution
Manufacturer	π_{m1}	563.22	612.5	350
	π_{m2}	657.3	680.625	247.5
	π_{m3}	755.44	765	192
	π_m	1976.36	2058.125	789.5
Retailer	π_{r1}	273.22	288.75	875
	π_{r2}	569.1	344.4375	1113.75
	π_{r3}	571.9	433.5	1344
	π_r	1414.22	1066.6875	3332.75
System	π	3390.58	3124.8125	4122.25

π_{mi} ($i = 1, 2, 3$) denotes the profit of P_i for the manufacturer.

$\pi_m = \sum_{i=1}^3 \pi_{mi}$ denotes the total profit of P_i for the manufacturer.

π_{ri} ($i = 1, 2, 3$) denotes the profit of P_i for the retailer.

$\pi_r = \sum_{i=1}^3 \pi_{ri}$ denotes the total profit of P_i for the retailer.

$\pi = \pi_m + \pi_r$ denotes the total profit of P_i for the system, the manufacturer, and the retailer as a whole.

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