

Research Article

Modeling of Signal Plans for Transit Signal Priority at Isolated Intersections under Stochastic Condition

Lv Bin

School of Traffic and Transportation, Lanzhou Jiaotong University, Gansu, Lanzhou 730070, China

Correspondence should be addressed to Lv Bin, tylbxx@163.com

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Transit signal priority (TSP) is recognized as having the potential to improve transit service reliability at small cost to general traffic. The popular preference for TSP encounters the challenges of various and challenging test scenarios. According to the stochastic characteristics of traffic flow, the signal timing model was established for TSP at an isolated signal intersection, where the passenger average delay was used as the optimization objective, and the weights of all phases were considered. The priority logic that is considered in the study provides cycle length and green time within a fixed-time traffic signal control environment. Using the Gauss elimination, the quantitative relationships were determined between phase clearance reliability (PCR), cycle length, and green time. Simulation experiments conducted by the particle swarm optimization (PSO) algorithm indicated that (1) the random variation of arrival rate has an obvious effect on traffic signal settings; (2) the proposed TSP model can reduce passenger delays, especially under stochastic traffic flow.

1. Introduction

Transit signal priority (TSP) strategies will have the potential improvements in transit schedule reliability, which have been recognized to speed up transit service in common. The TSP strategies can also be categorized into two groups: optimization based and logic rule based [1]. The existed TSP strategies, which are used at isolated signalized intersections, focus on intersections primarily without exclusive bus lanes and operate with an acyclic manner often [2, 3].

Yagar and Han [4, 5] analyzed the influences that passengers' getting on and off transits at transit stations have on the traffic stream of intersections. In order to optimize transit and street cars delay, a signal timing optimization model was built by taking

the influences of passengers' getting on and off transits into consideration. According to the changes of such parameters as passenger amount, on time performance of transits, transit departure frequency, and so forth, Mirchandani [6] established a transit priority weight model in order to optimize signal timing parameters.

Aimed at passenger average delay, Yang et al. [7] founded an optimal liner planning model of transit signal priority control system at isolated intersections, thus realizing preferential control passive transit signals at isolated intersections. Sunkari et al. [8] proposed another technique that used the delay equation from the 1985 highway capacity manual (HCM). The HCM delay equation was applied to the no-TSP signal timings and to signal timing plans associated with four TSP cases, namely, maximum and minimum green extension and maximum and minimum early green.

With the purpose of optimizing phase sequence and time of green light, Ma et al. [9] have designed priority control point number model of transit signals with different departure frequencies and consider to optimize signal timing with the aim of minimizing average delay of transits when the departure frequency is of integral multiple of the length of the cycle. Bie et al. [10] analyzed the relations of the length of the cycle, average passenger, and transit delay indicators at intersections, determined the method to calculate the cycle length, and taking the minimization of average passenger delay as a goal for optimization, brought forward green signal ratio optimization method according to surplus green light time allocated to passenger amount of each phase under the precondition of no traffic jam. Abdy and Hellenga [11] proposed TSP model that is developed on the basis of deterministic queuing theory. In developing the proposed model, analytical expressions are developed assuming that the intersection modeled as a D/D/1 queuing system.

In Polus' study [12], the observed travel time of bus on links was found to follow Beta distribution, whose parameters were estimated and found linearly related with each other and with the link length in the off peak hours. Because of the influence of bus dwell time, number of stops, and passenger activities (i.e., arrival, boarding, and alighting), the dispatching headway of bus was assumed gamma distributed [13]. The fact that a design of timing plan of test intersections, typically regardless of the stochastic characteristics of traffic flow and the demands of passenger, necessarily makes test scenarios less challenging may be the reason why timing parameters was found having an insignificant impact on the performance of TSP strategies [14]. Moreover, the traditional measures of effectiveness, such as delay, stops, travel time, transit schedule reliability, and fuel assumption [14], are fully vehicle oriented and the interests of passenger are neglected to a great extent.

This paper attempts to analyze the impact of timing parameters on the development and performance of an optimization-based transit signal priority, which is for use at isolated signalized intersections with two approaches busway. The study differs from previous studies in two ways. First, the attribute for passenger average delay is introduced to analyze the principles of timing parameters. Second, the stochastic characteristics of traffic flow were analyzed, and a model was established for TSP at an isolated signal intersection.

2. Model Building

The essence of transit priority is to demonstrate care for people. In optimizing the signal timing parameters of transits, we should consider the changing tendency of both delays of street cars and average passenger delays at intersections as well [15]. Therefore, we assume that the total delays at the approach of intersection are equal to the product of average

vehicle delay and the number of passengers in all vehicles at this approach, and that the total delays of pedestrians at the intersection are equal to the sum of delays of pedestrians at all approaches.

Suppose that the average number of passengers carried by transits is P_b and that of street cars is P_s , then the total delays of people in one signal cycle D_p will be

$$D_p = \sum_{i=1}^n \sum_{j=1}^{m_i} D_{ij}^p = C \sum_{i=1}^n \sum_{j=1}^{m_i} (d_{ij}^s Q_{ij}^s P_s + d_{ij}^b Q_{ij}^b P_b), \quad (2.1)$$

where D_{ij}^p is the total passengers delay of j approach of i phase in one cycle (s); C is cycle length at a signal intersection (s); d_{ij}^s , and d_{ij}^b are the delays of street cars and transits at the j approach of i phase (s). Q_{ij}^s is the arrival rate of street cars at the j approach of i phase (pcu/s); Q_{ij}^b means the arrival rate of transits at the j approach of i phase (veh/s); m_i signifies the amount at the approach of i phase.

Define d_p^i as the passenger average delay at the i phase. This value is related to the arrival rate of street cars Q_{ij}^s , that of transits Q_{ij}^b at each approach related to the i phase and their respective passenger carrying numbers P_b and P_s :

$$d_p^i = \frac{D_p}{C \sum_{j=1}^{m_i} (Q_{ij}^s P_s + Q_{ij}^b P_b)}. \quad (2.2)$$

According to the definition above, a signal timing optimization model can be built with regard to the optimization of transit priority signal timing at isolated intersection under stochastic conditions:

$$\min \sum_{i=1}^n u_p^i d_p^i, \quad (2.3)$$

$$\text{s.t. } P\{v_i C \leq g_i s_i\} = \alpha_i, \quad \forall i, \quad (2.4)$$

$$\sum_{i=1}^n g_i + L = C, \quad (2.5)$$

$$L = \sum_{i=1}^n (L_s + I_i - A), \quad (2.6)$$

$$0 \leq g_{\min}^i \leq g_i, \quad (2.7)$$

where n is the number of phase at signal intersections; u_p^i is the weight factor of passenger average delay of i phase and can be determined according to the grade of road or the arrival rate of transits; v_i , s_i , and g_i are the arrival rate (pcu/s), the saturated flow rate (pcu/s) of critical lane, and the green time (s) of i phase, respectively. C is the cycle length (s); α_i is the phase clearance reliability (PCR) at i phase; PCR is the probability that traffic could be entirely discharged within the available green time of the i phase; L is the total lost time of each cycle (s); A and L_s are the yellow time (s), the startup lost time (s) of each phase. I_i is

the intergreen interval time (s) of i phase. g_{\min}^i is the minimum green time of i phase which should meet pedestrians crossing the street and traffic safety (s).

In this model, the goal of the objective function (2.3) is to minimize the summation of passenger average delay of each phase, and the value of passenger average delay of each phase is correlative to PCR (α_i).

The constraint condition (2.4) is a PCR constraint that should be satisfied by the i phase [16]. The PCR could reflect the actual traffic running of each phase, and it could also be used to evaluate whether signalized timing parameters of each phase could meet traffic demand. Based on this, PCR would be used to calculate and analyse the reliability of signalized intersection comprehensively. PCR associates with v_i and s_i in a given timing plan at an intersection. It is stated that the arrival has no relation with saturated flow rate in previous researches, and the arrival rate and the capacity are independent from each other. Therefore, the saturated flow rate is assumed constant. If the arrival rate is constant, then PCR is constant. When the arrival rate is random variable, the PCR varies with arrival rate within the specified period. To make the state more stable, or get the higher value of the PCR at an intersection, it is necessary to consider the optimization of signal timing parameters, and all of them are affected by the arrival rate.

Equation (2.5) means that the sum of green time of each phase and time loss of each cycle equals the cycle length; (2.6) stands for the total time loss of cycle signals constituted by the time loss of each phase; (2.7) refers to the minimum green light time constraint that should be satisfied by phase green time for the safe intersection of pedestrians and motor vehicles.

The research team conducted a traffic investigation at a signal intersection in Lanzhou City. The results and data analysis indicated that the traffic flow arrival rate was assumed to be normal distribution (normal distribution can achieve good effects when fitting the traffic data of the various directions, the various vehicle models, and the various approaches of heavy traffic congested roads and intersections. Therefore, if the valid model testing results cannot be affected, let the random variable v_i (road vehicle arrival rate of critical approach) of i phase be subject to normal distribution, and its distribution parameter is (μ_i, σ_i^2) . Using the Gauss elimination, the model can be transformed into

$$\begin{aligned} & \min \sum_{i=1}^n \mu_p^i d_p^i \\ & \text{s.t.} \begin{cases} g_1 = \frac{L \prod_{i=2}^n B_i}{\prod_{i=1}^n B_i - \prod_{i=2}^n B_i - \sum_{k=2}^n \left(\prod_{i=1, i \neq k}^n B_i \right)}, & g_i = \frac{g_1 B_1}{B_i}, \\ C = \frac{g_i s_i}{\mu_i + \sigma_i \cdot \Phi^{-1}(\alpha_i)}, \\ L = \sum_{i=1}^n (L_s + I_i - A), \\ 0 \leq g_{\min}^i \leq g_i, \end{cases} \end{aligned} \quad (2.8)$$

where $B_i = s_i / (\mu_i + \sigma_i \cdot \Phi^{-1}(\alpha_i))$.

According to different specific control targets, for example, different grades of roads at each direction at the intersections and the amount of transit flows, we should consider using

road directions with relatively higher grade and maximize the traffic capacity at phases with large transit flow. We can reflect the above requirement by reasonably setting the value of u_p^i in the model.

3. Solution Algorithm

Particle swarm optimization (PSO) algorithm is an evolutionary computation technique. It optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solutions and moving these particles around in the search space according to simple mathematical formulae over the particle's position and velocity. Each particle's movement is influenced by its local best known position and is also guided toward the best known positions in the search space, which are updated as better positions are found by other particles. This is expected to move the swarm toward the best solutions.

PSO is originally attributed to Kennedy [17] and was first intended for simulating social behavior, as a stylized representation of the movement of organisms in a bird flock or fish school. The algorithm was simplified and it was observed to be performing optimization. PSO is a metaheuristic as it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions. More specifically, PSO does not use the gradient of the problem being optimized, which means PSO does not require that the optimization problem be differentiable as is required by classic optimization methods, such as gradient descent and quasi-Newton methods. PSO can therefore also be used on optimization problems that are partially irregular, noisy, change over time, and so forth. PSO algorithm displays apparent advantages in solving stochastic expected value model. Hence, this paper will throughout use it in model solution.

3.1. PSO Algorithm

Every particle in the particle swarm will fly in the searched space at certain speed $V_i = (V_{i1}, V_{i2}, \dots, V_{id})$ in the n dimension space. First, those particles will randomly generate a group of initial solutions x_1, x_2, \dots, x_N , where N represents the number of particles. Adjust the dynamic location and speed according to the flying experience of particles in solution space and the group, and use the adaptive value to evaluate the merits of solutions. Choose current individual extremum P_{best} , global extremum G_{best} , and record their locations. Update the speed and location of the next generation of particles according to formula (3.1) to (3.3) and continue the next iteration. The equation of updating is

$$V_{id} = \beta \times V_{id} + a_1 \times \text{rand}() \times (P_{id} - X_{id}) + a_2 \times \text{rand}() \times (P_{gd} - X_{id}), \quad (3.1)$$

$$V_{id} = V_{\max}, \quad \text{if } V_{id} > V_{\max}, \quad (3.2)$$

$$V_{id} = -V_{\max}, \quad \text{if } V_{id} < -V_{\max},$$

$$X_{id} = X_{id} + V_{id}, \quad (3.3)$$

where β is inertia weight which can maintain the kinetic inertia of particles and enable them to explore new areas. a_1 and a_2 stand for positive acceleration constant which is usually

made 2, and they can make every particle move towards P_{best} and G_{best} in accelerated motion. $\text{rand}()$ is the random number distributed equally on $[0, 1]$ and can be used to imitate random disturbances of group behaviors in nature. P_{id} and P_{gd} are the d dimension component of individual and overall extremum value, respectively.

Equation (3.2) constraints the largest speed of particles: if the accelerated speed of the particle at present will make one of its dimension speed component V_{id} exceed this dimension's largest speed V_{max} , then the speed of this dimension will be limited as V_{max} which determines the searching precision of a particle in the solution space. If its V_{max} is too large, a particle is prone to fly past the optimal solution; otherwise, it will probably be trapped in partial searching space, unable to realize overall searching. If the searching space of the issue is limited within $[-X_{\text{max}}, X_{\text{max}}]$, then we can assume that $V_{\text{max}} = kX_{\text{max}}, 0 \leq k \leq 1$.

3.2. Procedures of Algorithm

By combining stochastic simulation and PSO algorithm, we can arrive at the solving algorithm.

- (1) Initialize the particle group in n dimension problem space: suppose the size of the group is popsize , a random number will appear in the feasible region of green light time g_i at the i phase; use the expected value estimation method of stochastic stimulation to compute and test the feasibility of this random number, namely, judging whether g_i can satisfy $B_1 - B_1 B_n + B_n \neq 0$ and $0 \leq g_{\text{min}}^i \leq g_i$; repeat this process for popsize times and popsize initial feasible particles will be obtained: $\mathbf{g}_j = (g_{j1}, g_{j2}, \dots, g_{jm}), j = 1, 2, \dots, \text{popsize}$; initialize the speed parameter.
- (2) Compute the adaptive value, namely, $u_p^i d_p^i$, of every particle with the expected value estimation method of stochastic stimulation.
- (3) Compare the adaptive value of each particle with that at the best position experienced by it; if the former is better, then take it as the best position at present.
- (4) Compare the adaptive value of each particle with the best adaptive value experienced by the particle throughout the whole situation; if the former is better, then take it as the best position of the whole situation at present.
- (5) Evolve according to the updating (3.1), (3.2), and (3.3).
- (6) Compute $u_p^i d_p^i$ of particles updated with the expected value estimation method of random stimulation and test the feasibility of particles; if the feasibility is acceptable, then keep their original positions unchanged.
- (7) Repeat (2) to (6) until the largest iteration presupposed or a good enough adaptive value is obtained.
- (8) Export the best particles and corresponding adaptive values as optimal solutions.

4. Simulation Calculation

To verify the model and the formulas, let the intersection be four approaches in main direction of road (including one busway) and two approaches in minor direction of road, as shown in Figure 1. Three-phase signal control plan is adopted at the intersection. Saturated flow rate of each approach equals to $s_0 \times f_b$, where saturated flow rate of straight approach equals to

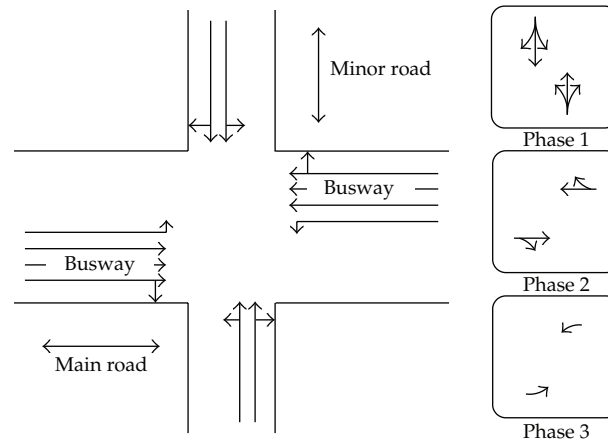


Figure 1: The graph of intersection channelization and phase scheme.

1800 pcu/h, saturated flow rate of left-turn approach equals to 1600 pcu/h, $g_{\min}^1 = g_{\min}^2 = 20$ s, $g_{\min}^3 = 10$ s, and $L = 9$ s. Each of phase clearance reliability is equal, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$. The number of passenger street cars is 2.5, and the number of passenger buses is 40. Simulation results are presented in the following sections.

Let the vehicle arrival rates of all phases be subject to the normal distribution, where arrival rate expectation of street cars is 1100 pcu/h, arrival rate expectation of buses is 240 veh/h, and the buses are running in the main road. The traffic flow ratio of left-turn, straight, and right-turn is 5%, 85%, and 10%, respectively. If street cars and buses arrival rate is constant, that is, the arrival rate of street cars is 1100 pcu/h, while that of the buses is 240 veh/h, then use the solving model to acquire the followings: cycle length = 76 s, $g_1 = 26$ s, $g_2 = 31$ s, and $g_3 = 10$ s. Using the PSO algorithm, the results are shown from Tables 1, 2 and Figure 2 when arrival rate of social vehicles and buses is random variable.

As shown in Tables 1, 2 and Figure 2:

- (1) to ensure the phase clearance reliability without reducing and to meet the random features of the arrival rate, the cycle length and the green time need to be set larger at signalized intersection;
- (2) to get the larger phase clearance reliability, a larger cycle length is needed. And if the vehicle arrival rate is increasing, this trend would be more obvious. As shown in Table 1, if the variance of street cars arrival rate is 220 pcu/h, and the buses arrival rate is constant, the phase clearance reliability increased from 0.75 to 0.95 and the cycle length from 89 s to 126 s, and the phase which has the larger variance of arrival rate needs much green time;
- (3) if all phases have the equal phase clearance reliability, then the phase with higher arrival rate should be allocated with much green time;
- (4) to obtain the higher phase clearance reliability with the larger variance of arrival rate, the cycle length of an intersection would exceed the maximum value. On this circumstance, using fixed-time control mode would not meet the control requirements. Therefore, actuated control or adaptive control mode may be considered.

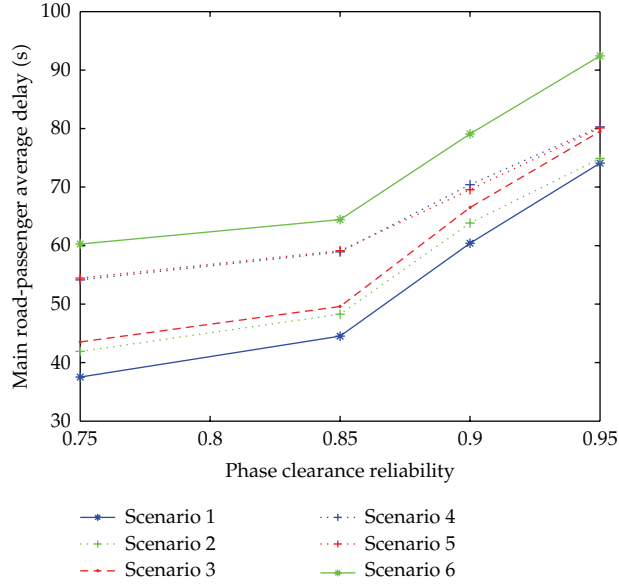


Figure 2: Average delay per passenger of main road.

Table 1: Calculation results when arrival rate of social vehicles is random variable.

α	Scenario 1 variance of street cars = 110 pcu/h number of bus is constant				Scenario 2 variance of street cars = 220 pcu/h number of bus is constant				Scenario 3 variance of street cars = 330 pcu/h number of bus is constant			
	C (s)	g_1 (s)	g_2 (s)	g_3 (s)	C (s)	g_1 (s)	g_2 (s)	g_3 (s)	C (s)	g_1 (s)	g_2 (s)	g_3 (s)
0.75	82	28	32	13	89	31	33	16	95	34	36	16
0.85	92	34	36	13	101	37	38	17	116	44	45	18
0.90	99	37	39	14	112	43	43	17	126	51	48	18
0.95	108	42	43	14	126	48	50	19	135	53	53	20

5. Conclusion

This paper analyzes the limitations of existing transit signal priority control method and considers the stochastic characteristics of arrival rate of street cars and transits; aimed at signal plans for transit signal priority at isolated intersections under the condition of TSP, it establishes a signal timing model with the target of optimizing passenger average delay, designs a corresponding PSO algorithm, and carries out stimulation.

The signal plans for transit signal timing priority model at isolated intersections can reflect the control strategy of transit priority, with the indicator of passenger average delay being a key factor. The optimizing method of phase sequence is an important research content of prior transit signal control method. This research assumes that transit stream travels on the main roads, and there is no turning transit which does not conform to the actual conditions; therefore, the issue of signal plans for transit signal priority at joint phase should be further considered.

Table 2: Calculation results when arrival rate of social vehicles and buses are random variables.

α	Scenario 4				Scenario 5				Scenario 6			
	variance of street cars = 110 pcu/h variance of buses = 24 veh/h				variance of street cars = 220 pcu/h variance of buses = 48 veh/h				variance of street cars = 330 pcu/h variance of buses = 72 veh/h			
	C (s)	g_1 (s)	g_2 (s)	g_3 (s)	C (s)	g_1 (s)	g_2 (s)	g_3 (s)	C (s)	g_1 (s)	g_2 (s)	g_3 (s)
0.75	85	29	33	14	98	35	38	16	103	38	40	16
0.85	99	37	39	14	109	39	42	19	123	48	45	21
0.90	107	41	43	14	119	45	46	19	135	49	54	23
0.95	118	45	49	15	132	49	54	20	144	54	57	24

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