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*Research Article*

# Optimal Control Channel Selection for Wireless Channel Assignment

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In wireless channel assignment, control channels are often needed for coordination among wireless devices. In this paper, we propose an algorithm for finding the optimal set of control channels given a number of wireless devices and the ranges of channels these devices can access.

## 1. Introduction

Wireless communications are now a crucial part of our everyday life. In wireless communications, there is a fundamental problem called *channel assignment* [1–3]: each wireless device needs to determine which channels it should use for its own communications, before any message can be sent or received. In this process, there need to be one or more *control channels* for coordination among wireless devices. These channels are used by the wireless devices to exchange information regarding their decisions on channel assignment (with the base station). Hence, a natural question is which channel(s) should be used as the control channel(s).

In this paper, we study the following mathematical problem regarding the selection of control channels: given a base station and a number of wireless devices, which control channels should be used as the control channels, so that only the smallest number of control channels is used but each involved device can access at least one control channel? The answer may be trivial if all involved wireless devices can access all channels available. Nevertheless, in reality, many wireless devices actually have limited tunability. Such a device can only access channels in a certain range of frequency. Consequently, in this paper, we assume that for each involved device, the range of channels accessible by this device is also given as input. Our objective is to develop an algorithm that finds the optimal set of control channels when given devices and their ranges of accessible channels.

### *1.1. Related Work and Our Contribution*

Channel assignment has been studied in various settings, under various assumptions. A lot of results have been obtained, for example, [1–6]. Most of these channel assignment schemes require that there is one or more control channels, so that wireless devices can exchange information with each other (and/or with the base station) in the process of channel assignment. However, as far as we know, there is no previous result on optimal selection of control channels in this context.

There are some existing studies (e.g., [7, 8]) of control channel selection in a different context, namely, cognitive radio networks. Since both their models and their objectives differ from ours, their results cannot be applied to our problem.

Our main contribution in this paper is that we design an algorithm to find the optimal set of control channels for channel assignment. Here optimality means under the constraint that each device can access at least one control channel, the total number of control channels is minimized. We provide a formal proof for the correctness of our algorithm. We also analyze its complexity.

We emphasize that the problem studied in this paper is not the channel assignment problem. In fact, it is a closely related, but very different problem, namely, the control channel selection problem. Specifically, our objective is not to assign channels to devices for their data communications, but to choose control channels for devices, so that they can coordinate with each other for their transmissions. One of the key differences here is that in channel assignment, performance metrics such as system throughput are of great importance. In contrast, in control channel selection, such performance metrics are not so important because the number of control packets is typically much smaller than that of data packets.

## 2. Assumptions and Formal Model

To solve the problem of control channel selection, we make the following crucial assumptions.

(1) All involved devices are in a single collision domain.

(2) Devices are equipped with radios that can access arbitrary sets of channels.

(3) Devices coordinate with each other through their common control channels. In particular, we assume it suffices for each pair of devices to have a control channel in common, such that they can use this control channel to coordinate with each other.

(4) Under the constraint that every pair of devices should have a common control channel, it is desirable to minimize the number of control channels, such that more channels can be used for data communications.

Based on these assumptions, we have the following formal model for the problem.

Suppose that there is a base station and $n$ wireless devices, all in a single collision domain. For simplicity, denote the wireless devices by $1, 2, \ldots, n$. Thus the device set $D = \{1, 2, \ldots, n\}$. Let the set of available channels be $\{L, L+1, L+2, \ldots, U\}$ $(L < U)$. For each device $i$, the channels that can be accessed are $\ell_i$ through $u_i$, that is, $\ell_i, \ell_i + 1, \ldots, u_i$. Clearly, we always have $L \le \ell_i \le u_i \le U$.

Our target is to find an algorithm to compute a set $C$ of control channels. This set $C$ must be a subset of available channels $\{L, L+1, L+2, \ldots, U\}$. It should also include at least one accessible channel for each device, that is, for all device $i \in D$, there should exist $c \in C$ such that $\ell_i \le c \le u_i$. We want to minimize the cardinality of $C$ under the above constraint.

```
Step 1:
D′ ← D;
for each i ∈ D′
    for each j ∈ D′ such that j ≠ i
        if ℓ_i ≤ ℓ_j and u_j ≤ u_i
            D′ ← D′ − {i};
            break;
C ← ϕ;
Step 2:
Repeat until D′ = ϕ
    i ← arg min_{i∈D′} ℓ_i;
    C ← C ∪ {u_i};
    for each j ∈ D′ such that j ≠ i
        if ℓ_j ≤ u_i
            D′ ← D′ − {j};
    D′ ← D′ − {i};
```

**Algorithm 1:** Algorithm for finding optimal control channel set.

## 3. The Algorithm

We summarize our algorithm in Algorithm 1.

## 4. Algorithm Analysis

**Theorem 4.1** (correctness). *When the algorithm terminates,* $C$ *is a subset of* $\{L, L+1, L+2, \ldots, U\}$ *with the smallest cardinality such that for all* $i \in D$*, there exists* $c \in C$*,* $\ell_i \leq c \leq u_i$*.*

*Proof.* Let $C^\star$ be the value of $C$ when the algorithm terminates. We divide our proof into two parts: first, we show that for all $i \in D$, there exists $c \in C^\star$ such that $\ell_i \leq c \leq u_i$. Second, we show the cardinality of $C^\star$ is the smallest among all subsets of $\{L, L+1, L+2, \ldots, U\}$ that satisfy the above constraint.

Let the value of $D'$ be $D'_0$ after Step 1 of the algorithm finishes and before Step 2 begins. For any $k \in D'_0$, there exists an iteration in Step 2, in which $k$ is removed from $D'$. There are two possibilities. The first possibility is that $i = k$ in that iteration and thus $k$ is removed from $D'$. The other possibility is that $i \neq k$ in that iteration but $\ell_k \leq u_i$ in that iteration, and thus $k$ is removed from $D'$.

For the first possibility, it is clear that $u_k = u_i$ is added to $C$ in that iteration. Hence, $u_i = u_k \in C^\star$.

For the second possibility, since $u_i$ is added to $C$ in that iteration, we know that $u_i \in C^\star$. We have also known that $\ell_k \leq u_i$. Furthermore, we should have $u_i \leq u_k$ because otherwise we would have $\ell_i \leq \ell_k$ and $u_k < u_i$, which implies that $i$ should have been removed from $D'$ in Step 1. Since $i \in D'_0$, the above is impossible. Consequently, we must have $u_i \leq u_k$.

Putting together the above two possibilities, we know that for any $k \in D'_0$, the value $u_i$ found above always satisfies that $u_i \in C^\star$ and that $\ell_k \leq u_i \leq u_k$. Consequently, we have proved the following Lemma.

**Lemma 1**. *For all $i \in D'_0$, there exists $c \in C^\star$ such that $\ell_i \leq c \leq u_i$.*

Now we replace $D'_0$ with $D$ in Lemma 1. We note that $D'_0 \subseteq D$ and that, due to Step 1 of the algorithm, for any $k \in D - D'_0$, there exists $m$ such that $\ell_k \leq \ell_m$ and $u_m \leq u_k$, and that either $m$ is removed from $D'$ after $k$ has been removed, or $m \in D'_0$. Applying induction to the above statement, we can easily obtain that, for any $k \in D - D'_0$, there exists $m \in D'_0$ such that $\ell_k \leq \ell_m$ and $u_m \leq u_k$. Combining this with Lemma 1, we get that there exists $c \in C^\star$ such that $\ell_k \leq c \leq u_k$.

So far we have finished the first part of our proof. Now let us proceed to the second part.

Consider any subset $C_X$ of $\{L, L+1, L+2, \ldots, U\}$ such that for all $i \in D'_0$, there exists $c \in C_X$ such that $\ell_i \leq c \leq u_i$. Now sort the elements of $C_X$ in increasing order: $e_1, e_2, \ldots, e_{|C_X|}$. Using induction, below we show that $e_i \leq f_i$, where $f_i$ is the element added to $C$ in the $i$th iteration of Step 2 of the algorithm.

Due to the algorithm, $f_1 = u_{i_1}$ where $i_1 = \arg\min_{i \in D'_0} \ell_i$. Assuming for the purpose of contradiction that $e_1 > f_1$, then for all $i$, $e_i > f_1$. Hence, there is no $c \in C_X$ such that $\ell_{i_1} \leq c \leq u_{i_1}$. Contradiction. So $e_1 \leq f_1$.

Next, suppose that $e_k \leq f_k$. Let the value of $D'$ at the end of the $k$th iteration of Step 2 be $D'_k$. Therefore, $f_{k+1} = u_{i_{k+1}}$ where

$$i_{k+1} = \arg\min_{i \in D'_k} \ell_i. \tag{4.1}$$

Now assume for the purpose of contradiction that $e_{k+1} > f_{k+1}$; thus, $e_{k+1} > u_{i_{k+1}}$. Consequently, for all $k' \geq k+1$, $e_{k'} > u_{i_{k+1}}$. On the other hand, for all $k' \leq k$, $e_{k'} \leq e_k \leq f_k = u_{i_k}$. Due to the algorithm, we must have $u_{i_k} < \ell_{i_{k+1}}$ because otherwise $i_{k+1}$ would have been removed from $D'$ in the $k$th iteration. Consequently, for all $k' \leq k$, $e_{k'} < \ell_{i_{k+1}}$. In summary, we have obtained that there is no $e_{k'} \in C_X$ such that $\ell_{i_{k+1}} \leq e_{k'} \leq u_{i_{k+1}}$. Contradiction.

Therefore, if $|C_X| < |C^\star|$, then $e_1 \leq f_1, e_2 \leq f_2, \ldots, e_{|C_X|} \leq f_{|C_X|}$. Since $f_{|C_X|} = u_{i_{|C_X|}} < \ell_{i_{|C_X|+1}}$, this implies that there is no $c \in C_X$ such that $\ell_{i_{|C_X|+1}} \leq c \leq u_{i_{|C_X|+1}}$, which contradicts our definition of $C_X$. Hence, we must have that $|C_X| \geq |C^\star|$.

The above has actually proved the following lemma.

**Lemma 2**. *$C^\star$ is a subset of $\{L, L+1, L+2, \ldots, U\}$ with the smallest cardinality such that for all $i \in D'_0$, there exists $c \in C^\star$, $\ell_i \leq c \leq u_i$.*

Finally, it suffices to observe that any subset of $\{L, L+1, L+2, \ldots, U\}$ such that for all $i \in D$, there exists $c \in C^\star$, $\ell_i \leq c \leq u_i$ is also a subset of $\{L, L+1, L+2, \ldots, U\}$ such that for all $i \in D'_0$, there exists $c \in C^\star$, $\ell_i \leq c \leq u_i$. Consequently, combining Lemmas 1 and 2, we get that $C^\star$ is a subset of $\{L, L+1, L+2, \ldots, U\}$ with the smallest cardinality such that for all $i \in D$, there exists $c \in C^\star$, $\ell_i \leq c \leq u_i$. □

**Theorem 4.2.** *The time complexity of the algorithm is $O(n^2)$.*

*Proof.* In step 1, at the beginning of the loop, $|D'| = |D| = n$. Hence, the loop clearly has $n(n-1)$ iterations. The time complexity of this step is thus $O(n^2)$.

Note that throughout step 1, $|D'|$ can only decrease but not increase. Hence, in step 2, at the beginning of the loop, $|D'| \leq n$. In each iteration of this loop, $|D'|$ must decrease by at least 1 because of the last line of the algorithm. Therefore, the number of iterations in the external loop must be no more than $n$. On the other hand, the number of iternations in each internal loop is no more than the current value of $|D'| - 1$ and so no more than $n - 1$. Consequently, the time complexity of this step is $O(n^2)$.

Summarizing the above analysis, the total time complexity of the algorithm is $O(n^2)$. □

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