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Research Article

Semi-Slant Warped Product Submanifolds of a Kenmotsu Manifold

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We study semi-slant warped product submanifolds of a Kenmotsu manifold. We obtain a characterization for warped product submanifolds in terms of warping function and shape operator and finally proved an inequality for squared norm of second fundamental form.

1. Introduction

In [1] Tanno classified the connected almost contact metric manifold whose automorphism group has maximum dimension, there are three classes:

(a) homogeneous normal contact Riemannian manifolds with constant ϕ holomorphic sectional curvature if the sectional curvature of the plane section contains ξ, say $C(X, \xi) > 0$;

(b) global Riemannian product of a line or a circle and Kaehlerian manifold with constant holomorphic sectional curvature, $C(X, \xi) = 0$;

(c) a warped product space $R \times_f C^n$, if $C(X, \xi) < 0$.

Manifolds of class (a) are characterized by some tensorial equations, it has a Sasakian structure and manifolds of class (b) are characterized by some tensor equations called Cosymplectic manifolds. Kenmotsu [2] obtained some tensorial equations to Characterize manifolds of class (c), these manifolds are called Kenmotsu manifolds.

The notion of semi-slant submanifolds of almost Hermitian manifolds was introduced by Papaghiuc [3] after that cabrerizo et al. [4] defined and studied semi-slant submanifolds in the setting of almost contact manifolds.

Bishop and O'Neill [5] introduced the notion of warped product manifolds. These manifolds are generalization of Riemannian product manifolds and occur naturally. Recently, many important physical applications of warped product manifolds have been discovered, giving motivation to study of these spaces with differential geometric point of view. For instance, it has been accomplished that warped product manifolds provide an excellent setting to model space time near black hole or bodies with large gravitational fields (c.f., [5–7]). Due to wide applications of these manifolds in physics as well as engineering this becomes a fascinating and interesting topic for research, and many articles are available in literature (c.f., [3, 8, 9]).

Recently, Atçeken [10] proved that the warped product submanifolds of type $N_\theta \times_f N_T$ and $N_\theta \times_f N_\perp$ of a Kenmotsu manifold $\overline{M}$ do not exist where the manifolds N_θ and N_T (resp. $N_\perp$) are proper slant and invariant (resp., anti-invariant) submanifolds of Kenmotsu manifold $\overline{M}$, respectively. After that Siraj-Uddin et al. [11] investigated warped product of the types $N_T \times_f N_\theta$ and $N_\perp \times_f N_\theta$ and obtained some interesting results. In this continuation we obtain a characterization and an inequality for squared norm of second fundamental form.

2. Preliminaries

A $2n+1$ dimensional C^∞ manifold $\overline{M}$ is said to have an almost contact structure if there exist on $\overline{M}$ a tensor field ϕ of type $(1,1)$, a vector field ξ, and 1-form η satisfying the following properties:

$$\phi^2 = -I + \eta \otimes \xi, \qquad \phi(\xi) = 0, \qquad \eta \circ \phi = 0, \qquad \eta(\xi) = 1. \tag{2.1}$$

There always exists a Riemannian metric g on an almost contact manifold $\overline{M}$ satisfying the following conditions:

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi), \tag{2.2}$$

where X, Y are vector fields on $\overline{M}$.

An almost contact metric structure (ϕ, ξ, η, g) is said to be Kenmotsu manifold, if it satisfies the following tensorial equation [2]:

$$\left(\overline{\nabla}_X \phi\right) Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.3}$$

for any $X, Y \in T\overline{M}$, where $T\overline{M}$ is the tangent bundle of $\overline{M}$ and $\overline{\nabla}$ denotes the Riemannian connection of the metric g. Moreover, for a Kenmotsu manifold

$$\overline{\nabla}_X \xi = X - \eta(X)\xi. \tag{2.4}$$

Let M be a submanifold of an almost contact metric manifold $\overline{M}$ with induced metric g and if ∇ and $\nabla^{\perp}$ are the induced connection on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M, respectively, then Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y),$$
$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{2.5}$$

for each $X, Y \in TM$ and $N \in T^{\perp}M$, where h and A_N are the second fundamental form and the shape operator, respectively, for the immersion of M into $\overline{M}$ and they are related as

$$g(h(X,Y),N) = g(A_N X, Y), \tag{2.6}$$

where g denotes the Riemannian metric on $\overline{M}$ as well as on M.

For any $X \in TM$, we write

$$\phi X = PX + FX, \tag{2.7}$$

where PX is the tangential component and FX is the normal component of ϕX.

Similarly, for any $N \in T^{\perp}M$, we write

$$\phi N = tN + fN, \tag{2.8}$$

where tN is the tangential component and fN is the normal component of ϕN. The covariant derivatives of the tensor field P and F are defined as

$$\left(\overline{\nabla}_X P\right)Y = \nabla_X PY - P\nabla_X Y,$$
$$\left(\overline{\nabla}_X F\right)Y = \nabla^{\perp}_X \mathrm{F}Y - F\nabla_X Y. \tag{2.9}$$

From (2.3), (2.5), (2.7) and (2.8) we have

$$\left(\overline{\nabla}_X P\right)Y = A_{FY}X + th(X,Y) - g(X,PY)\xi - \eta(Y)PX, \tag{2.10}$$

$$\left(\overline{\nabla}_X F\right)Y = fh(X,Y) - h(X,PY) - \eta(Y)FX. \tag{2.11}$$

Definition 2.1 (see [12]). A submanifold M of an almost contact metric manifold $\overline{M}$ is said to be slant submanifold if for any $x \in M$ and $X \in T_x M - \langle \xi \rangle$ is constant. The constant angle $\theta \in [0, \pi/2]$ is then called slant angle of M in $\overline{M}$. If $\theta = 0$ the submanifold is invariant submanifold, if $\theta = \pi/2$ then it is anti-invariant submanifold, if $\theta \neq 0, \pi/2$ then it is proper slant submanifold.

For slant submanifolds of contact manifolds Cabrerizo et al. [13] proved the following lemma.

Lemma 2.2. *Let M be a submanifold of an almost contact manifold $\overline{M}$, such that $\xi \in TM$ then M is slant submanifold if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$P^2 = \lambda(I - \eta \otimes \xi), \tag{2.12}$$

where $\lambda = -\cos^2\theta$.

Thus, one has the following consequences of above formulae:

$$g(PX, PY) = \cos^2\theta[g(X,Y) - \eta(X)\eta(Y)], \tag{2.13}$$

$$g(FX, FY) = \sin^2\theta[g(X,Y) - \eta(X)\eta(Y)]. \tag{2.14}$$

A submanifold M of $\overline{M}$ is said to be semi-slant submanifold of an almost contact manifold $\overline{M}$ if there exist two orthogonal complementary distributions D_T and D_θ on M such that

(i) $TM = D_T \oplus D_\theta \oplus \langle\xi\rangle$,

(ii) the distribution D_T is invariant that is, $\phi D_T \subseteq D_T$,

(iii) the distribution D_θ is slant with slant angle $\theta \neq 0$.

It is straight forward to see that semi-invariant submanifolds and slant submanifolds are semi-slant submanifolds with $\theta = \pi/2$ and $D_T = \{0\}$, respectively.

If μ is invariant subspace under ϕ of the normal bundle $T^\perp M$, then in the case of semi-slant submanifold, the normal bundle $T^\perp M$ can be decomposed as

$$T^\perp M = \mu \oplus FD_\theta. \tag{2.15}$$

A semi-slant submanifold M is called a semi-slant product if the distributions D_T and D_θ are parallel on M. In this case M is foliated by the leaves of these distributions.

As a generalization of the product manifolds and in particular of a semi-slant product submanifold, one can consider warped product of manifolds which are defined as

Definition 2.3. Let (B, g_B) and (F, g_F) be two Riemannian manifolds with Riemannian metric g_B and g_F, respectively, and f be a positive differentiable function on B. The warped product of B and F is the Riemannian manifold $(B \times F, g)$, where

$$g = g_B + f^2 g_F. \tag{2.16}$$

For a warped product manifold $N_1 \times_f N_2$, we denote by D_1 and D_2 the distributions defined by the vectors tangent to the leaves and fibers, respectively. In other words, D_1 is obtained by the tangent vectors of N_1 via the horizontal lift, and D_2 is obtained by the tangent vectors of N_2 via vertical lift. In case of semi-slant warped product submanifolds D_1 and D_2 are replaced by D_T and D_θ, respectively.

The warped product manifold $(B \times F, g)$ is denoted by $B \times_f F$. If X is the tangent vector field to $M = B \times_f F$ at (p, q) then

$$\|X\|^2 = \|d\pi_1 X\|^2 + f^2(p)\|d\pi_2 X\|^2. \tag{2.17}$$

Bishop and O'Neill [5] proved the following.

Theorem 2.4. *Let $M = B \times_f F$ be warped product manifolds. If $X, Y \in TB$ and $V, W \in TF$ then*

(i) $\nabla_X Y \in TB$,

(ii) $\nabla_X V = \nabla_V X = (Xf/f)V$,

(iii) $\nabla_V W = (-g(V, W)/f)\nabla f$.

∇f is the gradient of f and is defined as

$$g(\nabla f, X) = Xf, \tag{2.18}$$

for all $X \in TM$.

Corollary 2.5. *On a warped product manifold $M = N_1 \times_f N_2$, the following statements hold:*

(i) *N_1 is totally geodesic in M,*

(ii) *N_2 is totally umbilical in M.*

Throughout, one denotes by N_T and N_θ an invariant and a slant submanifold, respectively, of an almost contact metric manifold $\overline{M}$.

Khan et al. [14] proved the following corollary.

Corollary 2.6. *Let $\overline{M}$ be a Kenmotsu manifold and N_1 and N_2 be any Riemannian submanifolds of $\overline{M}$, then there do not exist a warped product submanifold $M = N_1 \times_f N_2$ of $\overline{M}$ such that ξ is tangential to N_2.*

Thus, one assumes that the structure vector field ξ is tangential to N_1 of a warped product submanifold $N_1 \times_f N_2$ of $\overline{M}$.

In this paper we will consider the warped product of the type $N_\theta \times_f N_T$ and $N_T \times_f N_\theta$. The warped product of the type $N_\theta \times_f N_T$ is called warped product semi-slant submanifolds; this type of warped product is studied by Atçeken [10], they proved that the warped product $N_\theta \times_f N_T$ does not exist. The warped product of the type $N_T \times_f N_\theta$ is called semi-slant warped product; these submanifolds were studied by Siraj-Uddin et al. [11] and they proved the following Lemma

Lemma 2.7. *Let $M = N_T \times_f N_\theta$ be warped product semi-slant submanifold of a Kenmotsu manifold $\overline{M}$ such that ξ is tangent to N_T, where N_T and N_θ are invariant and proper slant submanifolds of $\overline{M}$. then*

(i) $g(h(X, Z), FPZ) = g(h(X, PZ), FZ) = \{X \ln f - \eta(X)\}\cos^2\theta\|Z\|^2$,

(ii) $g(h(X, Z), FZ) = -PX \ln f\|Z\|^2$,

for any $X \in TN_T$ and $Z \in TN_\theta$.

Replacing X by PX in part (ii) of above lemma one has

$$g(h(PX,Z),FZ) = X\ln f\|Z\|^2. \tag{2.19}$$

3. Semi-Slant Warped Product Submanifolds

Throughout this section we will study the warped product of the type $N_T \times_f N_\theta$, for these submanifolds by Theorem 2.4 we have

$$\nabla_X Z = \nabla_Z X = X\ln f Z, \tag{3.1}$$

for any $X \in TN_T$ and $Z \in TN_\theta$.

Lemma 3.1. *Let $M = N_T \times_f N_\theta$ be a semi-slant warped product submanifolds of a Kenmotsu manifold $\overline{M}$, then*

$$g(h(X,Y),FZ) = 0, \tag{3.2}$$

for any $X, Y \in TN_T$ and $Z \in TN_\theta$.

Proof. As N_T is totally geodesic in M then $(\overline{\nabla}_X P)Y \in TN_T$ and therefore by formula (2.10):

$$\left(\overline{\nabla}_X P\right)Y = th(X,Y) - g(X,PY)\xi - \eta(Y)PX, \tag{3.3}$$

taking inner product with $Z \in TN_\theta$ we get (3.2). □

Now we have the following Characterization.

Theorem 3.2. *A semi-slant submanifold M of a Kenmotsu manifold $\overline{M}$ with integrable invariant distribution $D_T \oplus \langle\xi\rangle$ and integrable slant distribution D_θ is locally a semi-slant warped product if and only if $\nabla_Z PZ \in D_\theta$ and there exists a C^∞- function α on M with $Z\alpha = 0$,*

$$A_{FZ}X = \left(X\alpha - \eta(X)\right)PZ - PX\alpha Z, \tag{3.4}$$

for all $X \in D_T \oplus \langle\xi\rangle$ and $Z \in D_\theta$.

Proof. From (2.10) and (3.1) we have

$$A_{FZ}X + th(X,Z) = 0. \tag{3.5}$$

Similarly,

$$PX\ln f Z - X\ln f PZ = th(X,Z) - \eta(X)PZ, \tag{3.6}$$

from (3.5) and (3.6), we get

$$A_{FZ}X = X \ln f PZ - PX \ln f Z - \eta(X)PZ, \tag{3.7}$$

taking inner product with $W \in TN_\theta$, we have

$$g(A_{FZ}X, W) = \big(X \ln f - \eta(X)\big) g(PZ, W) - PX \ln f g(Z, W). \tag{3.8}$$

From Lemma 3.1 and (3.8) we get the desired result.

Conversely, let M be a semi-slant submanifold of $\overline{M}$ satisfying the hypothesis of the theorem, then for any $X, Y \in D_T \oplus \langle \xi \rangle$ and $Z \in D_\theta$

$$g(h(X, Y), FZ) = 0, \tag{3.9}$$

that means $h(X, Y) \in \mu$. Then from (2.11)

$$-F\nabla_X Y = fh(X, Y) - h(X, PY). \tag{3.10}$$

Since $h(X, Y) \in \mu$, then we have $F\nabla_X Y = 0$, that is, $\nabla_X Y \in D_T \oplus \langle \xi \rangle$. Hence, each leaf of $D_T \oplus \langle \xi \rangle$ is totally geodesic in M.

Further, suppose N_θ be a leaf of D_θ and h_θ be second fundamental form of the immersion of N_θ in M, then for any $X \in D_T \oplus \langle \xi \rangle$ and $Z \in D_\theta$, we have

$$g\big(h_\theta(Z, Z), \phi X\big) = g(\nabla_Z Z, \phi X), \tag{3.11}$$

using (2.7) and (2.5), the above equation yields

$$g\big(h_\theta(Z, Z), \phi X\big) = g(\nabla_Z PZ, X) + g(A_{FZ}Z, X), \tag{3.12}$$

applying (3.4), we get

$$g\big(h_\theta(Z, Z), \phi X\big) = -PX \ln f g(Z, Z). \tag{3.13}$$

Replacing X by PX, the above equation gives

$$h_\theta(Z, Z) = \nabla \alpha g(Z, Z). \tag{3.14}$$

From above equation it is easy to derive

$$h_\theta(Z, W) = \nabla \alpha g(Z, W), \tag{3.15}$$

that is, N_θ is totally umbilical and as $Z\alpha = 0$, for all $Z \in D_\theta$, $\nabla \mu$ is defined on N_T, this mean that mean curvature vector of N_θ is parallel, that is, the leaves of D_θ are extrinsic spheres in

M. Hence by virtue of result of [15] which says that if the tangent bundle of a Riemannian manifold M splits into an orthogonal sum $TM = E_0 \oplus E_1$ of nontrivial vector subbundles such that E_1 is spherical and its orthogonal complement E_0 is autoparallel, then the manifold M is locally isometric to a warped product $M_0 \times_f M_1$, we can say M is locally semi-slant warped product submanifold $N_T \times_f N_\theta$, where the warping function $f = e^\alpha$.

Let us denote by D_T and D_θ the tangent bundles on N_T and N_θ, respectively, and let $\{X_0 = \xi, X_1, \dots, X_p, X_{p+1} = \phi X_1, \dots, X_{2p} = \phi X_p\}$ and $\{Z_1, \dots, Z_q, Z_{q+1} = PZ_1, \dots, Z_{2q} = PZ_q\}$ be local orthonormal frames of vector fields on N_T and N_θ, respectively, with $2p$ and $2q$ being real dimension. Since $h(X, \xi) = 0$ for all $X \in TM$, then the second fundamental form can be written as

$$\begin{aligned} \|h\|^2 = {} & \sum_{i,j=1}^{2p} g\big(h(X_i, X_j), h(X_i, X_j)\big) + \sum_{i=1}^{2p}\sum_{r=1}^{2q} g(h(X_i, Z_r), h(X_i, Z_r)) \\ & + \sum_{r,s=1}^{2q} g(h(Z_r, Z_s), h(Z_r, Z_s)). \end{aligned} \tag{3.16}$$

Now, on a semi-slant warped product submanifold of a Kenmotsu manifold, we prove the following. □

Theorem 3.3. *Let $M = N_T \times_f N_\theta$ be a semi-slant warped product submanifold of a Kenmotsu manifold $\overline{M}$ with N_T and N_θ invariant and slant submanifolds, respectively, of $\overline{M}$. If $\eta(X) \geq 2X \ln f$ for all $X \in TN_T$, then the squared norm of the second fundamental form h satisfies*

$$\|h\|^2 \geq 4q \csc^2\theta \left\{1 + \cos^4\theta\right\} \left\|\nabla \ln f\right\|^2, \tag{3.17}$$

where $\nabla \ln f$ is the gradient of $\ln f$ and $2q$ is the dimension N_θ.

Proof. In view of the decomposition (2.15), we may write

$$h(U, V) = h_{FD_\theta}(U, V) + h_\mu(U, V), \tag{3.18}$$

for each $U, V \in TM$, where $h_{FD_\theta}(U, V) \in FD_\theta$ and $h_\mu(U, V) \in \mu$ with

$$h_{FD_\theta}(U, V) = \sum_{r=1}^{2q} h^r(U, V) FZ_r, \tag{3.19}$$

$$h^r(U, V) = \csc^2\theta\, g(h(U, V), FZ_r), \tag{3.20}$$

for each $U, V \in TM$. In view of above formulae we have

$$\begin{aligned} g(h_{FD_\theta}(PX_i, Z_r), h_{FD_\theta}(PX_i, Z_r)) = {} & g(h^r(PX_i, Z_r)FZ_r, h^r(PX_i, Z_r)FZ_r) \\ & + g(h^s(PX_i, Z_r)FZ_r, h^s(PX_i, Z_r)FZ_r). \end{aligned} \tag{3.21}$$

Now using (2.14) and (2.19)

$$g(h_{FD_\theta}(PX_i, Z_r), h_{FD_\theta}(PX_i, Z_r)) = h^r(PX_i, Z_r)X_i \ln f + \sin^2\theta \sum_{s\neq r}(h^s(PX_i, Z_r))^2. \tag{3.22}$$

In view of (3.20) and (2.19), we get

$$g(h_{FD_\theta}(PX_i, Z_r), h_{FD_\theta}(PX_i, Z_r)) = \csc^2\theta(X_i \ln f)^2 + \sin^2\theta \sum_{s\neq r}(h^s(PX_i, Z_r))^2. \tag{3.23}$$

Summing over $i = 1, \dots, 2p$ and $r = 1, \dots, 2q$ the above equation yields

$$\begin{aligned}\sum_{i=1}^{2p}\sum_{r=1}^{2q} g(h_{FD_\theta}(PX_i, Z_r), h_{FD_\theta}(PX_i, Z_r)) &= 4q\csc^2\theta\|\nabla \ln f\|^2 \\ &\quad + \sin^2\theta \sum_{i=1}^{2p}\sum_{r,s=1,r\neq s}^{2q}(h^s(PX_i, Z_r))^2.\end{aligned} \tag{3.24}$$

Since we have choose the orthonormal frame of vector fields on D_θ as $\{Z_1, \dots, Z_q, Z_{q+1} = PZ_1, \dots, Z_{2q} = PZ_q\}$, then the second term in the right-hand side of (3.24) is written as

$$\begin{aligned}\csc^2\theta \sum_{i=1}^{2p}\Bigg[&\sum_{r=1}^{q}\left\{(g(h(PX_i, Z_r), FPZ_r))^2 + (g(h(PX_i, PZ_r), FZ_r))^2\right\} \\ &+ \sum_{r=1}^{q}\sum_{s=1,s\neq r}^{q}\left\{(g(h(PX_i, Z_r), FPZ_s))^2 + (g(h(PX_i, PZ_r), FZ_s))^2\right\}\Bigg].\end{aligned} \tag{3.25}$$

From part (i) of Lemma 2.7, the first two terms of above equation can be written as

$$\csc^2\theta \sum_{i=1}^{2p}\left[2q(X_i \ln f - \eta(X_i))^2 \cos^4\theta\right]. \tag{3.26}$$

In account of to hypothesis $\eta(X_i) \geq 2X_i \ln f$ the above expression is greater than equal to the following term:

$$4q\csc^2\theta\|\nabla \ln f\|^2\cos^4\theta. \tag{3.27}$$

Using above inequality into (3.24), we have

$$g(h_{FD_\theta}(PX_i, Z_r), h_{FD_\theta}(PX_i, Z_r)) \geq 4q\csc^2\theta\left\{1 + \cos^4\theta\right\}\|\nabla \ln f\|^2. \tag{3.28}$$

The inequality (3.17) follows from (3.16) and (3.28).

The equality holds if $h(D_T, D_T) = 0$, $h(D_\theta, D_\theta) = 0$, $h(PX, Z)$ is orthogonal to FZ and FPZ for all $X \in D_T$ and $Z \in D_\theta$ and $\eta(X) = 2X \ln f$. □

4. Conclusion

In this paper we study nontrivial warped product submanifolds of a Kenmotsu manifold and in this study there emerge natural problems of finding the estimates of the squared norm of second fundamental form and to find the relation between shape operator and warping function. This study predict the geometric behavior of underlying warped product submanifolds. Further, as it is known that the warping function of a warped product manifold is a solution of some partial differential equations (c.f., [8]) and most of physical phenomenon is described by partial differential equations. We hope that our study may find applications in physics as well as in engineering.

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