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Research Article

Traveling Wave Solutions for the Generalized Zakharov Equations

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We use the bifurcation method of dynamical systems to study the traveling wave solutions for the generalized Zakharov equations. A number of traveling wave solutions are obtained. Those solutions contain explicit periodic wave solutions, periodic blow-up wave solutions, unbounded wave solutions, kink profile solitary wave solutions, and solitary wave solutions. Relations of the traveling wave solutions are given. Some previous results are extended.

1. Introduction

The Zakharov equations

$$\begin{aligned} iu_t + u_{xx} - uv &= 0, \\ v_{tt} - v_{xx} + \left(|u|^2\right)_{xx} &= 0, \end{aligned} \tag{1.1}$$

which is one of the fundamental models governing dynamics of nonlinear waves in one-dimensional systems. It describes the interaction between high-frequency and low-frequency waves. The physically most important example involves the interaction between the Langmuir and ion-acoustic waves in plasmas [1]. The equations can be derived from a hydrodynamic description of the plasma [2, 3]. However, some important effects such as transit-time damping and ion nonlinearities, which are also implied by the fact that the values used for the ion damping have been anomalously large from the point of view of linear ion-acoustic wave dynamics, have been ignored in (1.1). This is equivalent to saying that (1.1) is a simplified model of strong Langmuir turbulence. Thus we have to generalize (1.1) by

taking more elements into account. Starting from the dynamical plasma equations with the help of relaxed Zakharov simplification assumptions, and through making use of the time-averaged two-time-scale two-fluid plasma description, (1.1) are generalized to contain the self-generated magnetic field [4, 5], and the first related study on magnetized plasmas in [6, 7]. The generalized Zakharov equations are a set of coupled equations and may be written as [8]

$$\begin{aligned} iu_t + u_{xx} - 2\lambda|u|^2u + 2uv = 0,\\ v_{tt} - v_{xx} + \left(|u|^2\right)_{xx} = 0. \end{aligned} \tag{1.2}$$

Malomed et al. [8] analyzed internal vibrations of a solitary wave in (1.2) by means of a variational approach. Wang and Li [9] obtained a number of periodic wave solutions of (1.2) by using extended F-expansion method. Javidi and Golbabai [10] used the He's variational iteration method to obtain solitary wave solutions of (1.2). Zhang [11] obtained the exact traveling wave solutions of (1.2) by using the direct algebraic method. Zhang [12] used He's semi-inverse method to search for solitary wave solutions of (1.2). Javidi and Golbabai [13] obtained the exact and numerical solutions of (1.2) by using the variational iteration method. Li et al. [14] used the Exp-function method to seek exact solutions of (1.2). Borhanifar et al. [15] obtained the generalized solitary solutions and periodic solutions of (1.2) by using the Exp-function method. Khan et al. [16] used He's variational approach to obtain new soliton solutions of (1.2).

The aim of this paper is to study the traveling wave solutions and their limits for (1.2) by using the bifurcation method and qualitative theory of dynamical systems [17–24]. Through some special phase orbits, we obtain many smooth periodic wave solutions and periodic blow-up solutions. Their limits contain kink-profile solitary wave solutions, unbounded wave solutions, periodic blow-up solutions, and solitary wave solutions.

The remainder of this paper is organized as follows. In Section 2, by using the bifurcation theory of planar dynamical systems, two-phase portraits for the corresponding traveling wave system of (1.2) are given under different parameter conditions. The relations between the traveling wave solutions and the Hamiltonian h are presented. In Section 3, we obtain a number of traveling wave solutions of (1.2) and give the relations of the traveling wave solutions. A short conclusion will be given in Section 4.

2. Phase Portraits and Qualitative Analysis

We assume that the traveling wave solutions of (1.2) is of the form

$$u(x,t) = e^{i\eta}\varphi(\xi), \qquad v(x,t) = \psi(\xi), \qquad \eta = px + qt, \qquad \xi = k(x - 2pt), \tag{2.1}$$

where $\varphi(\xi)$ and $\psi(\xi)$ are real functions; p, q, and k are real constants.

Substituting (2.1) into (1.2), we have

$$\begin{aligned} k^2\varphi'' + 2\varphi\psi - \left(p^2 + q\right)\varphi - 2\lambda\varphi^3 = 0,\\ k^2\left(4p^2 - 1\right)\psi'' + k^2\left(\varphi^2\right)'' = 0. \end{aligned} \tag{2.2}$$

Integrating the second equation of (2.2) twice, and letting the first integral constant be zero, we have

$$\psi = \frac{\varphi^2}{1-4p^2} + g, \quad p \neq \frac{1}{2}, \tag{2.3}$$

where g is integral constant.

Substituting (2.3) into the first equation of (2.2), we have

$$k^2\varphi'' + \left(2g - p^2 - q\right)\varphi + 2\left(\frac{1}{1-4p^2} - \lambda\right)\varphi^3 = 0. \tag{2.4}$$

Letting $\varphi' = y$, $\alpha = (2/k^2)(\lambda - 1/(1-4p^2))$, and $\beta = (2g - p^2 - q)/k^2$, then we get the following planar system

$$\begin{aligned} \frac{d\varphi}{d\xi} &= y, \\ \frac{dy}{d\xi} &= \alpha\varphi^3 - \beta\varphi. \end{aligned} \tag{2.5}$$

Obviously, the above system (2.5) is a Hamiltonian system with Hamiltonian function

$$H(\varphi, y) = y^2 - \frac{1}{2}\alpha\varphi^4 + \beta\varphi^2. \tag{2.6}$$

In order to investigate the phase portrait of (2.5), set

$$f(\varphi) = \alpha\varphi^3 - \beta\varphi. \tag{2.7}$$

Obviously, $f(\varphi)$ has three zero points, φ_-, φ_0, and φ_+, which are given as follows:

$$\varphi_- = -\sqrt{\frac{\beta}{\alpha}}, \qquad \varphi_0 = 0, \qquad \varphi_+ = \sqrt{\frac{\beta}{\alpha}}. \tag{2.8}$$

Letting $(\varphi_i, 0)$ be one of the singular points of system (2.5), then the characteristic values of the linearized system of system (2.5) at the singular points $(\varphi_i, 0)$ are

$$\lambda_\pm = \pm\sqrt{f'(\varphi_i)}. \tag{2.9}$$

From the qualitative theory of dynamical systems, we know that:

(1) if $f'(\varphi_i) > 0$, $(\varphi_i, 0)$ is a saddle point;

(2) if $f'(\varphi_i) < 0$, $(\varphi_i, 0)$ is a center point;

(3) if $f'(\varphi_i) = 0$, $(\varphi_i, 0)$ is a degenerate saddle point;

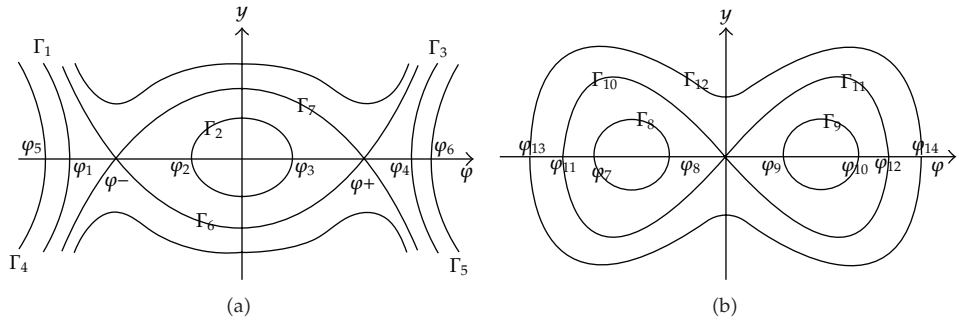

Figure 1: The phase portraits of system (2.5). (a) $\alpha > 0,\ \beta > 0$, (b) $\alpha < 0,\ \beta < 0$.

Therefore, we obtain the phase portraits of system (2.5) in Figure 1.
Let

$$H(\varphi, y) = h, \tag{2.10}$$

where h is Hamiltonian.

Next, we consider the relations between the orbits of (2.5) and the Hamiltonian h.
Set

$$h^* = \left|H(\varphi_+, 0)\right| = \left|H(\varphi_-, 0)\right| = \frac{\beta^2}{2|\alpha|}. \tag{2.11}$$

According to Figure 1, we get the following propositions.

Proposition 2.1. *Suppose that $\alpha > 0$ and $\beta > 0$, we have the following.*

(1) *When $h < 0$ or $h > h^*$, system (2.5) does not have any closed orbit.*

(2) *When $0 < h < h^*$, system (2.5) has three periodic orbits Γ_1, Γ_2, and Γ_3.*

(3) *When $h = 0$, system (2.5) has two periodic orbits Γ_4 and Γ_5.*

(4) *When $h = h^*$, system (2.5) has two heteroclinic orbits Γ_6 and Γ_7.*

Proposition 2.2. *Suppose that $\alpha < 0$ and $\beta < 0$, we have the following.*

(1) *When $h \leqslant -h^*$, system (2.5) does not have any closed orbit.*

(2) *When $-h^* < h < 0$, system (2.5) has two periodic orbits Γ_8 and Γ_9.*

(3) *When $h = 0$, system (2.5) has two homoclinic orbits Γ_{10} and Γ_{11}.*

(4) *When $h > 0$, system (2.5) has a periodic orbit Γ_{12}.*

From the qualitative theory of dynamical systems, we know that a smooth solitary wave solution of a partial differential system corresponds to a smooth homoclinic orbit of a traveling wave equation. A smooth kink wave solution or an unbounded wave solution corresponds to a smooth heteroclinic orbit of a traveling wave equation. Similarly, a periodic orbit of a traveling wave equation corresponds to a periodic traveling wave solution of a

partial differential system. According to the above analysis, we have the following propositions.

Proposition 2.3. *If $\alpha > 0$ and $\beta > 0$, we have the following.*

(1) *When $0 < h < h^*$, (1.2) has two periodic wave solutions (corresponding to the periodic orbit Γ_2 in Figure 1) and two periodic blow-up wave solutions (corresponding to the periodic orbits Γ_1 and Γ_3 in Figure 1).*

(2) *When $h = 0$, (1.2) has two periodic blow-up wave solutions (corresponding to the periodic orbits Γ_4 and Γ_5 in Figure 1).*

(3) *When $h = h^*$, (1.2) has two kink-profile solitary wave solutions and two unbounded wave solutions (corresponding to the heteroclinic orbits Γ_6 and Γ_7 in Figure 1).*

Proposition 2.4. *If $\alpha < 0$ and $\beta < 0$, we have the following.*

(1) *When $-h^* < h < 0$, (1.2) has two periodic wave solutions (corresponding to the periodic orbits Γ_8 and Γ_9 in Figure 1).*

(2) *When $h = 0$, (1.2) has two solitary wave solutions (corresponding to the homoclinic orbits Γ_{10} and Γ_{11} in Figure 1).*

(3) *When $h > 0$, (1.2) has two periodic wave solutions (corresponding to the periodic orbit Γ_{12} in Figure 1).*

3. Traveling Wave Solutions and Their Relations

Firstly, we will obtain the explicit expressions of traveling wave solutions for the (1.2) when $\alpha > 0$ and $\beta > 0$.

(1) From the phase portrait, we note that there are three periodic orbits Γ_1, Γ_2, and Γ_3 passing the points $(\varphi_1, 0)$, $(\varphi_2, 0)$, $(\varphi_3, 0)$, and $(\varphi_4, 0)$. In (φ, y) plane the expressions of the orbits are given as

$$y = \pm\sqrt{\frac{\alpha}{2}}\sqrt{(\varphi-\varphi_1)(\varphi-\varphi_2)(\varphi-\varphi_3)(\varphi-\varphi_4)}, \tag{3.1}$$

where $\varphi_1 = -\sqrt{(\beta+\sqrt{\beta^2-2\alpha h})/\alpha}$, $\varphi_2 = -\sqrt{(\beta-\sqrt{\beta^2-2\alpha h})/\alpha}$, $\varphi_3 = \sqrt{(\beta-\sqrt{\beta^2-2\alpha h})/\alpha}$, $\varphi_4 = \sqrt{(\beta+\sqrt{\beta^2-2\alpha h})/\alpha}$, and $0 < h < h^*$.

Substituting (3.1) into $d\varphi/d\xi = y$ and integrating them along Γ_1, Γ_2, and Γ_3, we have

$$\begin{aligned}
\pm\int_{\varphi}^{\infty} \frac{1}{\sqrt{(s-\varphi_1)(s-\varphi_2)(s-\varphi_3)(s-\varphi_4)}}\,ds &= \sqrt{\frac{\alpha}{2}}\int_0^{\xi} ds,\\
\pm\int_{0}^{\varphi} \frac{1}{\sqrt{(s-\varphi_1)(s-\varphi_2)(s-\varphi_3)(s-\varphi_4)}}\,ds &= \sqrt{\frac{\alpha}{2}}\int_0^{\xi} ds.
\end{aligned} \tag{3.2}$$

Completing above integrals we obtain

$$\varphi = \pm \frac{\varphi_4}{\mathrm{sn}\left(\varphi_4\sqrt{\alpha/2}\,\xi, \varphi_3/\varphi_4\right)},$$
$$\varphi = \pm\varphi_3 \mathrm{sn}\left(\varphi_4\sqrt{\frac{\alpha}{2}}\,\xi, \frac{\varphi_3}{\varphi_4}\right). \tag{3.3}$$

Noting that (2.1) and (2.3), we get the following periodic wave solutions:

$$u_1(x,t) = \pm \frac{e^{i\eta}\varphi_4}{\mathrm{sn}\left(\varphi_4\sqrt{\alpha/2}\,\xi, \varphi_3/\varphi_4\right)},$$
$$v_1(x,t) = \frac{\varphi_4^2}{(1-4p^2)\left(\mathrm{sn}\left(\varphi_4\sqrt{\alpha/2}\,\xi, \varphi_3/\varphi_4\right)\right)^2} + g,$$
$$u_2(x,t) = \pm e^{i\eta}\varphi_3 \mathrm{sn}\left(\varphi_4\sqrt{\frac{\alpha}{2}}\,\xi, \frac{\varphi_3}{\varphi_4}\right), \tag{3.4}$$
$$v_2(x,t) = \frac{\left(\varphi_3 \mathrm{sn}\left(\varphi_4\sqrt{\alpha/2}\,\xi, \varphi_3/\varphi_4\right)\right)^2}{1-4p^2} + g,$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

(2) From the phase portrait, we note that there are two special orbits Γ_4 and Γ_5, which have the same hamiltonian as that of the center point $(0,0)$. In (φ, y) plane the expressions of the orbits are given as

$$y = \pm\sqrt{\frac{\alpha}{2}}\varphi\sqrt{(\varphi-\varphi_5)(\varphi-\varphi_6)}, \tag{3.5}$$

where $\varphi_5 = -\sqrt{2\beta/\alpha}$ and $\varphi_6 = \sqrt{2\beta/\alpha}$.

Substituting (3.5) into $\mathrm{d}\varphi/\mathrm{d}\xi = y$, and integrating them along the two orbits Γ_4 and Γ_5, it follows that

$$\pm\int_{\varphi}^{+\infty} \frac{1}{s\sqrt{(s-\varphi_5)(s-\varphi_6)}}\mathrm{d}s = \sqrt{\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s. \tag{3.6}$$

Completing above integrals we obtain

$$\varphi = \pm\sqrt{\frac{2\beta}{\alpha}}\csc\sqrt{\beta}\xi. \tag{3.7}$$

Noting (2.1) and (2.3), we get the following periodic blow-up wave solutions:

$$u_3(x,t) = \pm e^{i\eta}\sqrt{\frac{2\beta}{\alpha}}\csc\sqrt{\beta}\xi,$$
$$v_3(x,t) = \frac{2\beta\left(\csc\sqrt{\beta}\xi\right)^2}{\alpha\left(1-4p^2\right)} + g, \tag{3.8}$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

(3) From the phase portrait, we see that there are two heteroclinic orbits Γ_6 and Γ_7 connected at saddle points $(\varphi_-, 0)$ and $(\varphi_+, 0)$. In (φ, y) plane the expressions of the heteroclinic orbits are given as

$$y = \pm\sqrt{\frac{\alpha}{2}}\sqrt{(\varphi-\varphi_-)^2(\varphi-\varphi_+)^2}. \tag{3.9}$$

Substituting (3.9) into $\mathrm{d}\varphi/\mathrm{d}\xi = y$, and integrating them along the heteroclinic orbits Γ_6 and Γ_7, it follows that

$$\pm\int_0^{\varphi}\frac{1}{(s-\varphi_-)(\varphi_+-s)}\mathrm{d}s = \sqrt{\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s,$$
$$\pm\int_{\varphi}^{+\infty}\frac{1}{(s-\varphi_-)(s-\varphi_+)}\mathrm{d}s = \sqrt{\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s. \tag{3.10}$$

Completing above integrals we obtain

$$\varphi = \pm\sqrt{\frac{\beta}{\alpha}}\tanh\sqrt{\frac{\beta}{2}}\xi,$$
$$\varphi = \pm\sqrt{\frac{\beta}{\alpha}}\coth\sqrt{\frac{\beta}{2}}\xi. \tag{3.11}$$

Noting (2.1) and (2.3), we get the following kink profile solitary wave solutions:

$$u_4(x,t) = \pm e^{i\eta}\sqrt{\frac{\beta}{\alpha}}\tanh\sqrt{\frac{\beta}{2}}\xi,$$
$$v_4(x,t) = \frac{\beta\left(\tanh\sqrt{\beta}\xi\right)^2}{\alpha\left(1-4p^2\right)} + g, \tag{3.12}$$

and unbounded wave solutions

$$u_5(x,t) = \pm e^{i\eta}\sqrt{\frac{\beta}{\alpha}}\coth\sqrt{\frac{\beta}{2}}\xi,$$
$$v_5(x,t) = \frac{\beta\left(\coth\sqrt{\beta}\xi\right)^2}{\alpha\left(1-4p^2\right)} + g, \tag{3.13}$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

Secondly, we will obtain the explicit expressions of traveling wave solutions for (1.2) when $\alpha < 0$ and $\beta < 0$.

(1) From the phase portrait, we see that there are two closed orbits Γ_8 and Γ_9 passing the points $(\varphi_7, 0)$, $(\varphi_8, 0)$, $(\varphi_9, 0)$, and $(\varphi_{10}, 0)$. In (φ, y) plane the expressions of the closed orbits are given as

$$y = \pm\sqrt{-\frac{\alpha}{2}}\sqrt{(\varphi-\varphi_7)(\varphi-\varphi_8)(\varphi-\varphi_9)(\varphi_{10}-\varphi)}, \tag{3.14}$$

where $\varphi_7 = -\sqrt{(\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $\varphi_8 = -\sqrt{(\beta + \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $\varphi_9 = \sqrt{(\beta + \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $\varphi_{10} = \sqrt{(\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, and $-h^* < h < 0$.

Substituting (3.14) into $d\varphi/d\xi = y$, and integrating them along Γ_8 and Γ_9, we have

$$\pm\int_{\varphi_7}^{\varphi}\frac{1}{\sqrt{(\varphi_{10}-s)(\varphi_9-s)(\varphi_8-s)(s-\varphi_7)}}ds = \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}ds,$$
$$\pm\int_{\varphi_{10}}^{\varphi}\frac{1}{\sqrt{(s-\varphi_7)(s-\varphi_8)(s-\varphi_9)(\varphi_{10}-s)}}ds = \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}ds. \tag{3.15}$$

Completing above integrals we obtain

$$\varphi = \frac{(\varphi_{10}-\varphi_8)\varphi_7 + (\varphi_8-\varphi_7)\varphi_{10}\left(\mathrm{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2}{\varphi_{10}-\varphi_8+(\varphi_8-\varphi_7)\left(\mathrm{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2},$$
$$\varphi = \sqrt{\varphi_{10}^2 - (\varphi_{10}^2-\varphi_9^2)\left(\mathrm{sn}\left(\varphi_{10}\sqrt{-\frac{\alpha}{2}}\,\xi, \frac{\sqrt{\varphi_{10}^2-\varphi_9^2}}{\varphi_{10}}\right)\right)^2}, \tag{3.16}$$

where $\omega = \sqrt{(\varphi_{10}-\varphi_8)(\varphi_9-\varphi_7)}/2$ and $\kappa = \sqrt{(\varphi_{10}-\varphi_9)(\varphi_8-\varphi_7)/(\varphi_{10}-\varphi_8)(\varphi_9-\varphi_7)}$.

Noting (2.1) and (2.3), we get the following periodic wave solutions:

$$
\begin{aligned}
u_6(x,t) &= \frac{e^{i\eta}\left((\varphi_{10}-\varphi_8)\varphi_7+(\varphi_8-\varphi_7)\varphi_{10}\left(\operatorname{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2\right)}{\varphi_{10}-\varphi_8+(\varphi_8-\varphi_7)\left(\operatorname{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2},\\
v_6(x,t) &= \frac{\left((\varphi_{10}-\varphi_8)\varphi_7+(\varphi_8-\varphi_7)\varphi_{10}\left(\operatorname{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2\right)^2}{(1-4p^2)\left(\varphi_{10}-\varphi_8+(\varphi_8-\varphi_7)\left(\operatorname{sn}\left(\omega\sqrt{-(\alpha/2)}\,\xi,\kappa\right)\right)^2\right)^2}+g,\\
u_7(x,t) &= e^{i\eta}\sqrt{\varphi_{10}^2-(\varphi_{10}^2-\varphi_9^2)\left(\operatorname{sn}\left(\varphi_{10}\sqrt{-(\alpha/2)}\,\xi,\frac{\sqrt{\varphi_{10}^2-\varphi_9^2}}{\varphi_{10}}\right)\right)^2},\\
v_7(x,t) &= \frac{\varphi_{10}^2-(\varphi_{10}^2-\varphi_9^2)\left(\operatorname{sn}\left(\varphi_{10}\sqrt{-(\alpha/2)}\,\xi,\sqrt{\varphi_{10}^2-\varphi_9^2}/\varphi_{10}\right)\right)^2}{1-4p^2}+g,
\end{aligned}
\tag{3.17}
$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

(2) From the phase portrait, we see that there are two symmetric homoclinic orbits Γ_{10} and Γ_{11} connected at the saddle point $(0,0)$. In (φ, y) plane the expressions of the homoclinic orbits are given as

$$
y = \pm\sqrt{-\frac{\alpha}{2}}\,\varphi\sqrt{(\varphi-\varphi_{11})(\varphi_{12}-\varphi)},
\tag{3.18}
$$

where $\varphi_{11} = -\sqrt{2\beta/\alpha}$ and $\varphi_{12} = \sqrt{2\beta/\alpha}$.

Substituting (3.18) into $\mathrm{d}\varphi/\mathrm{d}\xi = y$, and integrating them along the orbits Γ_{10} and Γ_{11}, we have

$$
\begin{aligned}
\pm\int_{\varphi_{11}}^{\varphi}\frac{1}{s\sqrt{(s-\varphi_{11})(\varphi_{12}-s)}}\mathrm{d}s &= \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s,\\
\pm\int_{\varphi_{12}}^{\varphi}\frac{1}{s\sqrt{(s-\varphi_{11})(\varphi_{12}-s)}}\mathrm{d}s &= \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s.
\end{aligned}
\tag{3.19}
$$

Completing above integrals we obtain

$$\varphi = \sqrt{\frac{2\beta}{\alpha}}\operatorname{sech}\sqrt{-\beta}\xi,$$
$$\varphi = -\sqrt{\frac{2\beta}{\alpha}}\operatorname{sech}\sqrt{-\beta}\xi. \tag{3.20}$$

Noting (2.1) and (2.3), we get the following solitary wave solutions:

$$u_8(x,t) = e^{i\eta}\sqrt{\frac{2\beta}{\alpha}}\operatorname{sech}\sqrt{-\beta}\xi,$$
$$v_8(x,t) = \frac{2\beta\left(\operatorname{sech}\sqrt{-\beta}\xi\right)^2}{\alpha\left(1-4p^2\right)} + g,$$
$$u_9(x,t) = -e^{i\eta}\sqrt{\frac{2\beta}{\alpha}}\operatorname{sech}\sqrt{-\beta}\xi, \tag{3.21}$$
$$v_9(x,t) = \frac{2\beta\left(\operatorname{sech}\sqrt{-\beta}\xi\right)^2}{\alpha\left(1-4p^2\right)} + g,$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

(3) From the phase portrait, we see that there is a closed orbit Γ_{12} passing the points $(\varphi_{13}, 0)$ and $(\varphi_{14}, 0)$. In (φ, y) plane the expressions of the closed orbits are given as

$$y = \pm\sqrt{-\frac{\alpha}{2}}\sqrt{(\varphi_{14}-\varphi)(\varphi-\varphi_{13})(\varphi-c_1)(\varphi-\bar{c}_1)}, \tag{3.22}$$

where $\varphi_{14} = \sqrt{(\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $\varphi_{13} = -\sqrt{(\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $c_1 = i\sqrt{(-\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, $\bar{c}_1 = -i\sqrt{(-\beta - \sqrt{\beta^2 - 2\alpha h})/\alpha}$, and $h > 0$.

Substituting (3.22) into $\mathrm{d}\varphi/\mathrm{d}\xi = y$, and integrating them along the orbit Γ_{12}, we have

$$\pm\int_{\varphi_{13}}^{\varphi} \frac{1}{\sqrt{(\varphi_{14}-s)(s-\varphi_{13})(s-c_1)(s-\bar{c}_1)}}\mathrm{d}s = \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s,$$
$$\pm\int_{\varphi}^{\varphi_{14}} \frac{1}{\sqrt{(\varphi_{14}-s)(s-\varphi_{13})(s-c_1)(s-\bar{c}_1)}}\mathrm{d}s = \sqrt{-\frac{\alpha}{2}}\int_0^{\xi}\mathrm{d}s. \tag{3.23}$$

Completing above integrals we obtain

$$\varphi = \varphi_{13}\mathrm{cn}\left(\sqrt{-\beta}\xi, -\varphi_{13}\sqrt{\frac{\alpha}{2\beta}}\right),$$
$$\varphi = \varphi_{14}\mathrm{cn}\left(\sqrt{-\beta}\xi, \varphi_{14}\sqrt{\frac{\alpha}{2\beta}}\right). \tag{3.24}$$

Noting (2.1) and (2.3), we get the following periodic wave solutions:

$$u_{10}(x,t) = e^{i\eta}\varphi_{13}\mathrm{cn}\left(\sqrt{-\beta}\xi, -\varphi_{13}\sqrt{\frac{\alpha}{2\beta}}\right),$$
$$v_{10}(x,t) = \frac{\left(\varphi_{13}\mathrm{cn}\left(\sqrt{-\beta}\xi, -\varphi_{13}\sqrt{\alpha/2\beta}\right)\right)^2}{1-4p^2} + g,$$
$$u_{11}(x,t) = e^{i\eta}\varphi_{14}\mathrm{cn}\left(\sqrt{-\beta}\xi, \varphi_{14}\sqrt{\alpha/2\beta}\right), \tag{3.25}$$
$$v_{11}(x,t) = \frac{\left(\varphi_{14}\mathrm{cn}\left(\sqrt{-\beta}\xi, \varphi_{14}\sqrt{\alpha/2\beta}\right)\right)^2}{1-4p^2} + g,$$

where $\eta = px + qt$ and $\xi = k(x - 2pt)$.

Thirdly, we will give the relations of the traveling wave solutions.

(1) Letting $h \to h^*-$, it follows that $\varphi_4 \to \sqrt{\beta/\alpha}$, $\varphi_3 \to \sqrt{\beta/\alpha}$, $\varphi_3/\varphi_4 \to 1$ and $\mathrm{sn}(\sqrt{\beta}\xi, 1) = \tanh\sqrt{\beta}\xi$. Therefore, we obtain $u_1(x,t) \to u_5(x,t)$, $v_1(x,t) \to v_5(x,t)$, $u_2(x,t) \to u_4(x,t)$ and $v_2(x,t) \to v_4(x,t)$.

(2) Letting $h \to 0+$, it follows that $\varphi_4 \to \sqrt{2\beta/\alpha}$, $\varphi_3 \to 0$, $\varphi_3/\varphi_4 \to 0$ and $\mathrm{sn}(\sqrt{\beta}\xi, 0) = \sin\sqrt{\beta}\xi$. Therefore, we obtain $u_1(x,t) \to u_3(x,t)$ and $v_1(x,t) \to v_3(x,t)$.

(3) Letting $h \to 0-$, it follows that $\varphi_{10} \to \sqrt{2\beta/\alpha}$, $\varphi_9 \to 0$, $\varphi_8 \to 0$, $\varphi_7 \to -\sqrt{2\beta/\alpha}$, $\omega \to \sqrt{\beta/2\alpha}$, $k \to 1$ and $\mathrm{sn}((\sqrt{-\beta}/2)\xi, 1) = \tanh(\sqrt{-\beta}/2)\xi$. Therefore, we obtain $u_6(x,t) \to u_8(x,t)$ and $v_6(x,t) \to v_8(x,t)$.

(4) Letting $h \to 0-$, it follows that $\varphi_{10} \to \sqrt{2\beta/\alpha}$, $\varphi_9 \to 0$, $\varphi_8 \to 0$, $\varphi_7 \to -\sqrt{2\beta/\alpha}$, $\sqrt{\varphi_{10}^2 - \varphi_9^2}/\varphi_{10} \to 1$ and $\mathrm{sn}(\sqrt{-\beta}\xi, 1) = \tanh\sqrt{-\beta}\xi$. Therefore, we obtain $u_7(x,t) \to u_9(x,t)$ and $v_7(x,t) \to v_9(x,t)$.

(5) Letting $h \to 0+$, it follows that $\varphi_{14} \to \sqrt{2\beta/\alpha}$, $\varphi_{13} \to -\sqrt{2\beta/\alpha}$, $-\varphi_{13}\sqrt{\alpha/2\beta} \to 1$, $\varphi_{14}\sqrt{\alpha/2\beta} \to 1$ and $\mathrm{cn}(\sqrt{-\beta}\xi, 1) = \mathrm{sech}\sqrt{-\beta}\xi$. Therefore, we obtain $u_{10}(x,t) \to u_9(x,t)$, $v_{10}(x,t) \to v_9(x,t)$, $u_{11}(x,t) \to u_8(x,t)$ and $v_{11}(x,t) \to v_8(x,t)$.

Finally, we will show that the periodic wave solutions $u_2(x,t)$ evolute into the kink-profile solitary wave solutions $u_4(x,t)$ when the Hamiltonian $h \to h^*-$ (corresponding to

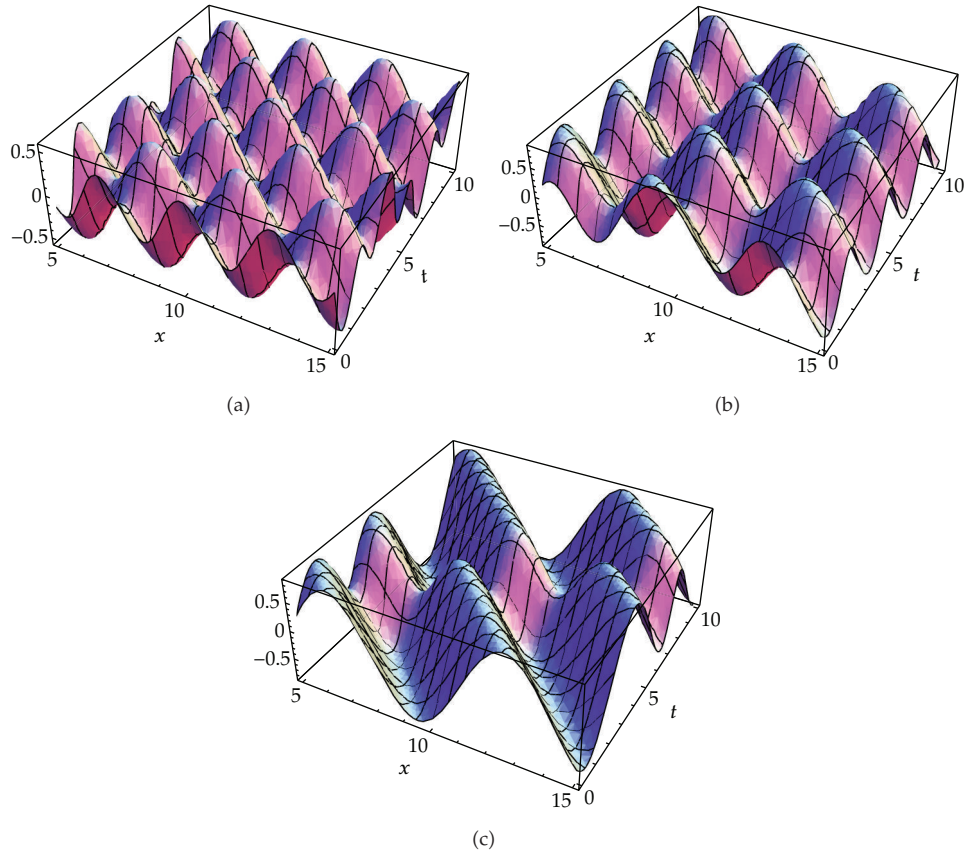

Figure 2: The real part of the periodic wave solution $u_2(x,t)$ evolutes into the kink-profile solitary wave solutions $u_4(x,t)$ with the conditions (3.26). (a) $h = 0.5$; (b) $h = 0.749$; (c) $h = 0.75$.

the changes of phase orbits of Figure 1 as h varies). We take some suitable choices of the parameters, such as

$$\lambda = 1, \qquad k = 1, \qquad p = 1, \qquad q = 1, \qquad g = 2, \tag{3.26}$$

as an illustrative sample and draw their plots (see Figures 2 and 3).

4. Conclusion

In this paper, we obtain phase portraits for the corresponding traveling wave system of (1.2) by using the bifurcation theory of planar dynamical systems. Furthermore, a number of exact traveling wave solutions are also obtained, and their relations are given. The method can be applied to many other nonlinear evolution equations, and we believe that many new results wait for further discovery by this method.

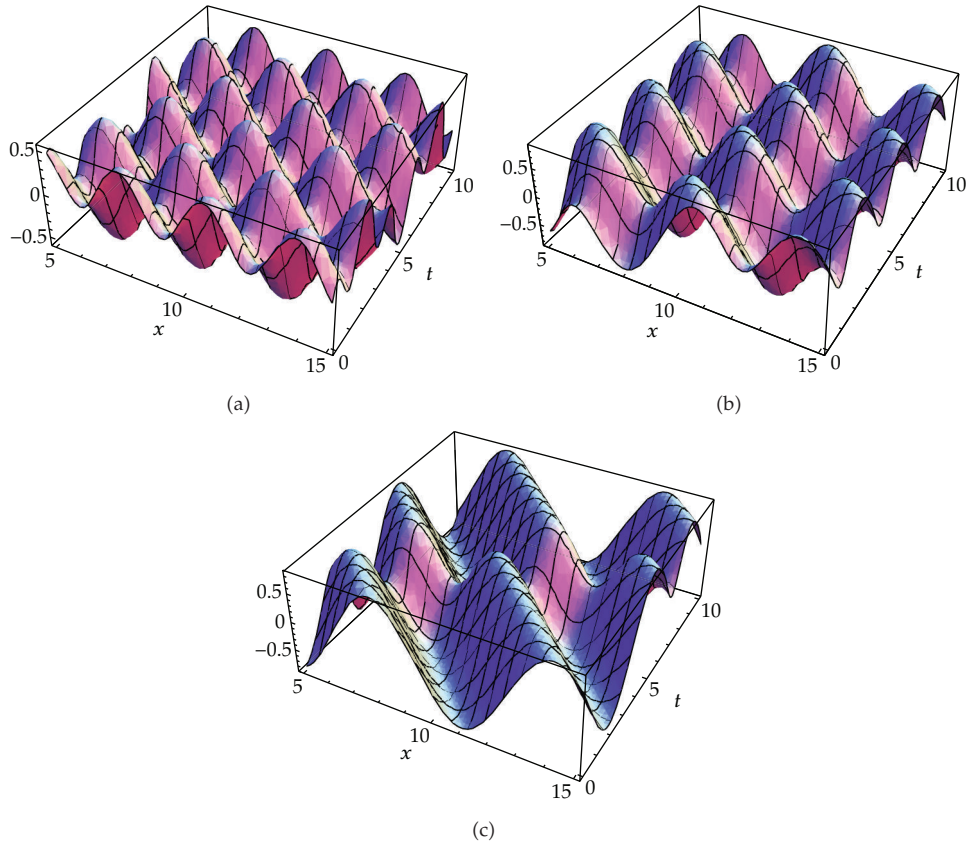

Figure 3: The imaginary part of the periodic wave solution $u_2(x,t)$ evolutes into the kink-profile solitary wave solutions $u_4(x,t)$ with the conditions (3.26). (a) $h = 0.5$; (b) $h = 0.749$; (c) $h = 0.75$.

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