Review Article

A Review of Deterministic Optimization Methods in Engineering and Management

Ming-Hua Lin,[1] Jung-Fa Tsai,[2] and Chian-Son Yu[1]

[1] *Department of Information Technology and Management, Shih Chien University, No. 70, Ta-Chih Street, Taipei City 10462, Taiwan*

[2] *Department of Business Management, National Taipei University of Technology, No. 1, Section 3, Chung-Hsiao E. Road, Taipei City 10608, Taiwan*

Correspondence should be addressed to Jung-Fa Tsai, jftsai@ntut.edu.tw

With the increasing reliance on modeling optimization problems in practical applications, a number of theoretical and algorithmic contributions of optimization have been proposed. The approaches developed for treating optimization problems can be classified into deterministic and heuristic. This paper aims to introduce recent advances in deterministic methods for solving signomial programming problems and mixed-integer nonlinear programming problems. A number of important applications in engineering and management are also reviewed to reveal the usefulness of the optimization methods.

1. Introduction

The field of optimization has grown rapidly during the past few decades. Many new theoretical, algorithmic, and computational contributions of optimization have been proposed to solve various problems in engineering and management. Recent developments of optimization methods can be mainly divided into deterministic and heuristic approaches. Deterministic approaches take advantage of the analytical properties of the problem to generate a sequence of points that converge to a global optimal solution. Heuristic approaches have been found to be more flexible and efficient than deterministic approaches; however, the quality of the obtained solution cannot be guaranteed. Moreover, the probability of finding the global solution decreases when the problem size increases. Deterministic approaches (e.g., linear programming, nonlinear programming, and mixed-integer nonlinear programming, etc.) can provide general tools for solving optimization problems to obtain a global or an approximately global optimum. With the increasing reliance on

modeling optimization problems in real applications, a number of deterministic methods for optimization problems have been presented. This paper focuses on discussing and reviewing the recent advances in deterministic optimization approaches.

Optimization methods have been applied in different fields, such as finance [1–3], allocation and location problems [4–6], engineering design [7–12], system and database design [13–17], chemical engineering design and control [18–22], and molecular biology [23]. For additional literature on real-world applications or developments of optimization methods, readers may refer to the following works. Mockus et al. [24] treated network problems, knapsack, travelling salesman, flow-shop problems, and batch process scheduling problems by Bayesian heuristic approaches. Grossmann [25] discussed the global optimization algorithms and their applications in chemical engineering design. Bomze et al. [26] investigated decision support systems and techniques for solving molecular structures, queuing systems, image reconstruction, location analysis, and process network synthesis problems. Migdalas et al. [27] presented multilevel optimization algorithms and their applications. Mistakidis and Stavroulakis [28] studied engineering applications of the finite element method. De Leone et al. [29] proposed various interesting applications of high-performance software for nonlinear optimization. Hendrix [30] utilized global optimization techniques on environmental management, geometric design, robust product design, and parameter estimation. Corliss and Kearfott [31] presented a rigorous global search method on industrial applications. Floudas and Pardalos [32] studied optimization approaches in the fields of computational chemistry and molecular biology. Laguna and González-Velarde [33] discussed advanced computing tools for tackling various challenging optimization problems. Papalambros and Wilde [34] investigated the principles and practice of optimal engineering design. Atkinson et al. [35] gave a detailed discussion on optimum experimental design. Edgar et al. [36] explored the optimization methods of chemical processes. Pardalos and Romeijn [37] provided a more complete and broad spectrum of approaches including deterministic and heuristic techniques for dealing with global optimization problems. Tawarmalani and Sahinidis [38] provided an insightful and comprehensive treatment of convexification and global optimization of continuous and mixed-integer nonlinear programs. Hadjisavvas et al. [39] investigated generalized convexity and generalized monotonicity and offered an advanced and broad overview of the state of the field. Moreover, Floudas et al. [40] presented an overview of the research progress in optimization during 1998–2003, including the deterministic global optimization advances in mixed-integer nonlinear programming and related applications. Pintér [41] illustrated the applicability of global optimization modeling techniques and solution strategies to real-world problems such as agroecosystem management, assembly line design, bioinformatics, biophysics, cellular mobile network design, chemical product design, composite structure design, controller design for induction motors, electrical engineering design, feeding strategies in animal husbandry, the inverse position problem in kinematics, laser design, radiotherapy planning, robot design, and satellite data analysis. Mishra and Giorgi [42] presented results on invex function, and their properties in smooth and nonsmooth cases, pseudolinearity, and eta-pseudolinearity. Mishra et al. [43] discussed the Kuhn-Tucker optimality, Karush-Kuhn-Tucker necessary and sufficient optimality conditions in presence of various types of generalized convexity assumptions. Floudas and Gounaris [44] also presented an overview of the research progress in deterministic global optimization during the last decade (1998–2008).

Figure 1 gives an overview of the problem types related to optimization problems. Each type of problems has received substantial attention from the practitioners and the
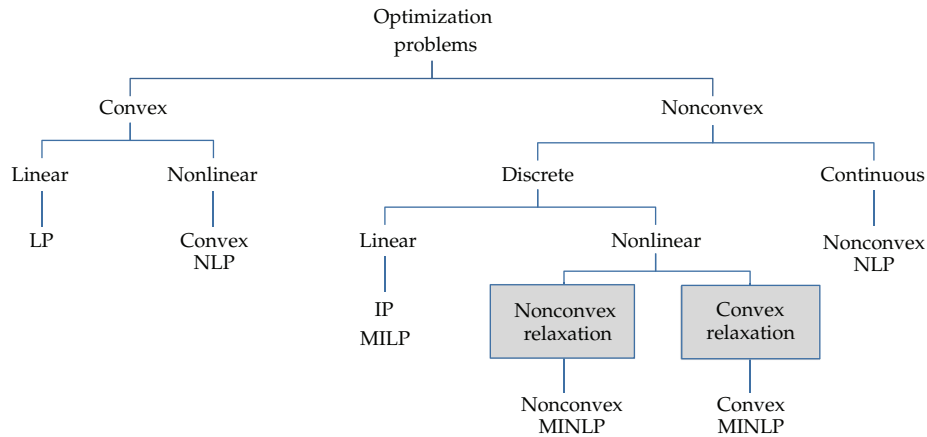
Optimization
problems

Convex

Nonconvex

Linear

Nonlinear

Discrete

Continuous

LP

Convex
NLP

Linear

Nonlinear

Nonconvex
NLP

IP
MILP

Nonconvex
relaxation

Convex
relaxation

Nonconvex
MINLP

Convex
MINLP

Figure 1: Overview of problem types related to optimization problems.

researchers in the last few decades. In this paper, we investigate the advances in deterministic global optimization of nonconvex nonlinear programming (NLP) problems and nonconvex mixed-integer nonlinear programming (MINLP) problems. For NLP problems, we focus on signomial programming problems that are an important class of NLP problems and have played a crucial role in applications.

The rest of this paper is organized as follows. Section 2 discusses the deterministic methods for signomial programming problems. Section 3 reviews the theoretical and algorithmic developments of mixed-integer nonlinear programming problems. Conclusions are made in Section 4.

2. Signomial Programming

Signomial programming (SP) is an optimization technique for solving a class of nonconvex nonlinear programming problems. Although SP problems occur frequently in engineering and management science, SP problems with nonconvex functions are still difficult to be solved to obtain a global optimum. The term geometric program (GP) was introduced by Duffin et al. [7] since the analysis of geometric programs relied heavily upon geometric-arithmetic mean inequality. The early work by Duffin and Peterson [45] solved the posynomial geometric program analytically via the dual problem. Then, Duffin [46] developed a numerical method for GPs based on solving a sequence of linear programs [47]. For solving SP problems, Duffin and Peterson [48] reformulated an SP problem as a geometric program with reversed constraints. The reversed constraints give rise to a nonconvex feasible region that the local minima for SP problems are not guaranteed to be global minima [49]. The developed methods for SP can be divided into two approaches. The first class of SP approaches includes various heuristic techniques such as genetic algorithms, simulated annealing, tabu search, ant colony optimization, and particle swarm optimization. Although the heuristic methods have the advantage of easy implementation and offer a better potential for complex problems, the obtained solution is not guaranteed to be a globally optimal solution. The second class of SP approaches is the deterministic method. For example, Maranas and Floudas [50], Floudas and Pardalos [51], Maranas and

Floudas [22], and Floudas [19–21] developed global optimization algorithms for solving SP problems based on the exponential variable transformation, the convex relaxation, and the branch and bound type algorithm. These methods transform the original nonconvex problem into a convex problem and then solve it to obtain the global optimum. The use of the logarithmic/exponential transformation in global optimization algorithms on SP problems restricts these exponential-based methods to handle the problems with strictly positive variables. Although positive variables are employed frequently to represent engineering and scientific systems, it is also common to introduce nonpositive variables in modeling the management problems or the system behavior, such as investment decisions, stresses, temperatures, electrical currents, velocities, and accelerations. For treating free variable x, Pörn et al. [52] suggested a simple translation, $x + \tau = e^X$. However, inserting the transformed result into the original signomial term will bring additional signomial terms and therefore increase the computation burden. Tsai et al. [11] proposed an approach to treat zero boundary signomial discrete programming problems and suggested some convexification rules. Li and Tsai [53], Tsai and Lin [54–56], Tsai et al. [57], Tsai [58], and Li and Lu [59] applied convexification strategies and piecewise linearization techniques to solve SP problems with free discrete/continuous variables. However, the optimal solution obtained is an approximate solution by the piecewise linearization approach. Lin and Tsai [60] presented a generalized method to solve signomial discrete programming problems with free variables for finding exactly alternative optima. Tsai and Lin [61] also integrated the convexification techniques and the bounding schemes to solve a posynomial geometric program with separable functions for finding a global optimal solution efficiently.

Convexification strategies for signomial terms are important techniques in global optimization for SP problems. With different convexification approaches, an SP problem can be reformulated into another convex program solvable to obtain an approximately global optimum. For solving SP problems, Pörn et al. [52] integrated the exponential transformation and piecewise linear approximations for reformulating nonconvex signomial problems. The results were extended by Björk [62], Björk et al. [63], and Pörn et al. [64], by including certain power transformations for convexification of nonconvex signomial terms. They discussed that the right choice of transformation for convexifying nonconvex signomial terms has a clear influence on the efficiency of the optimization approach. The concept of power convex functions is introduced to improve the solution efficiency for certain SP problems. T. Westerlund and J. Westerlund [65] proposed the generalized geometric programming extended cutting plane (GGPECP) algorithm for nonconvex optimization problems by using the cutting plane and transformation techniques. The GGPECP algorithm was described in more detail in Westerlund [66]. Lundell and Westerlund [67] and Lundell et al. [68] combined the GGPECP algorithm with an optimization framework for the transformations used to convexify the signomial terms into a signomial global optimization algorithm. The signomial global optimization algorithm was further extended by Lundell and Westerlund [69, 70]. Lin and Tsai [71] and Tsai and Lin [56] also presented similar reformulation and range reduction techniques to enhance the computational efficiency for solving SP problems.

For solving an SP problem, the above-mentioned convexification techniques are used to reformulate the original SP problem into a convex and underestimating problem solvable by a standard mixed-integer nonlinear programming (MINLP) solver [72–81]. Different transformations for positive and negative signomial terms have been proposed and discussed by Björk et al. [63], Westerlund [66], Lundell and Westerlund [67], Pörn et al. [64], Lundell et al. [68], and Lundell and Westerlund [69, 70]. For a positive signomial term $cx_1^{\alpha_1} x_2^{\alpha_2} \cdots x_m^{\alpha_m} x_{m+1}^{\alpha_{m+1}} \cdots x_n^{\alpha_n}$ ($c > 0, \alpha_1, \ldots, \alpha_m > 0$ and $\alpha_{m+1}, \ldots, \alpha_n < 0$), they suggested either the

exponential transformation (ET) or the power convex transformation (PCT) is applied based on the characteristics of the problems. The ET strategy and the PCT strategy are described as follows [63].

The ET strategy:

$$cx_1^{\alpha_1} x_2^{\alpha_2} \cdots x_m^{\alpha_m} x_{m+1}^{\alpha_{m+1}} \cdots x_n^{\alpha_n} \iff \begin{cases} x_i = e^{X_i}, & i = 1, \ldots, m, \\ c \dfrac{e^{\alpha_1 X_1 + \cdots + \alpha_m X_m}}{x_{m+1}^{|\alpha_{m+1}|} \cdots x_n^{|\alpha_n|}}. \end{cases} \tag{2.1}$$

The PCT strategy: this technique aims at constructing 1-convex signomial terms. First, transform all variables with positive exponents by an inverse transformation (IT), $x = X^{-1}$, except the one with the greatest exponent denoted as $\alpha_{\max}$. Let S be defined as $S = \sum_{i=1}^{n} |\alpha_i| - \alpha_{\max}$. If $\alpha_{\max} < S + 1$, then transform the variable with the exponent $\alpha_{\max}$ to that with the exponent $S + 1$. If $\alpha_{\max} > S + 1$, then change one of the ITs, with the exponent α_j to $x_j = X_j^{-\tau}$, where $\tau > 1$ so that $\alpha_{\max} = S + 1 + (\tau - 1)\alpha_j$.

Since some negative signomial terms may exist in SP problems, they suggested the potential transformation (PT) for a negative signomial term $cx_1^{\alpha_1} x_2^{\alpha_2} \cdots x_m^{\alpha_m} x_{m+1}^{\alpha_{m+1}} \cdots x_n^{\alpha_n}$ ($c < 0$, $\alpha_1, \ldots, \alpha_m > 0$ and $\alpha_{m+1}, \ldots, \alpha_n < 0$) expressed as follows.

The PT strategy:

$$cx_1^{\alpha_1} x_2^{\alpha_2} \cdots x_m^{\alpha_m} x_{m+1}^{\alpha_{m+1}} \cdots x_n^{\alpha_n} \iff \begin{cases} x_i = X_i^{1/R}, & i = 1, \ldots, m, \\ x_i = X_i^{-1/R}, & i = m + 1, \ldots, n, \\ cX_1^{\alpha_1/R} \cdots X_m^{\alpha_m/R} X_{m+1}^{|\alpha_{m+1}|/R} \cdots X_n^{|\alpha_n|/R}, & R = \displaystyle\sum_{i=1}^{n} |\alpha_i|. \end{cases} \tag{2.2}$$

In addition to convexification strategies, convex envelopes and convex underestimators of nonconvex functions are frequently applied in global optimization algorithms such as the αBB algorithm [19, 82, 83] to underestimate the nonconvex functions. A good convex underestimator should be as tight as possible and contain minimal number of new variables and constraints thus to improve the computational effect of processing a node in a branch-bound tree [40].

Tawarmalani and Sahinidis [84] developed the convex envelope and concave envelope for x/y over a unit hypercube, proposed a semidefinite relaxation of x/y, and suggested convex envelopes for functions of the form $f(x)y^2$ and $f(x)/y$. Ryoo and Sahinidis [85] studied the use of arithmetic intervals, recursive arithmetic intervals, logarithmic transformation, and exponential transformation for multilinear functions. Tawarmalani et al. [86] studied the role of disaggregation in leading to tighter linear programming relaxations. Tawarmalani and Sahinidis [38] introduced the convex extensions for lower semicontinuous functions, proposed a technique for constructing convex envelopes for nonlinear functions, and studied the maximum separation distance for functions such as x/y. Tawarmalani et al. [87] studied 0-1 hyperbolic programs and developed eight mixed-integer convex reformulations. Liberti and Pantelides [88] proposed a nonlinear continuous and differentiable convex envelope for monomials of odd degree, derived its linear relaxation, and compared to other relaxations. Björk et al. [63] studied convexifications for signomial terms, introduced properties of power convex functions, compared the effect of the convexification schemes for heat exchanger network problems, and studied quasiconvex convexifications. Meyer and Floudas [89]

studied trilinear monomials with positive or negative domains, derived explicit expressions for the facets of the convex and concave envelopes, and showed that these outperform the previously proposed relaxations based on arithmetic intervals or recursive arithmetic intervals. Meyer and Floudas [90] presented explicit expressions for the facets of convex and concave envelopes of trilinear monomials with mixed-sign domains. Tardella [91] studied the class of functions whose convex envelope on a polyhedron coincides with the convex envelope based on the polyhedron vertices and proved important conditions for a vertex polyhedral convex envelope. Meyer and Floudas [92] described the structure of the polyhedral convex envelopes of edge-concave functions over polyhedral domains using geometric arguments and proposed an algorithm for computing the facets of the convex envelopes.

Caratzoulas and Floudas [93] proposed novel convex underestimators for trigonometric functions, which are trigonometric functions themselves. Akrotirianakis and Floudas [94, 95] introduced a new class of convex underestimators for twice continuously differentiable nonlinear programs, studied their theoretical properties, and proved that the resulting convex relaxation is improved compared to the αBB one. Meyer and Floudas [90] proposed two new classes of convex underestimators for general C^2 nonlinear programs, which combine the αBB underestimators within a piecewise quadratic perturbation, derived properties for the smoothness of the convex underestimators, and showed the improvements over the classical αBB convex underestimators for box-constrained optimization problems.

Three popular convex underestimation methods, arithmetic intervals (AIs) [96], recursive arithmetic intervals (rAIs) [50, 85, 96], and explicit facets (EFs) for convex envelopes of trilinear monomials [89, 90], are effective to underestimate a trilinear term $x_1 x_2 x_3$ for x_i to be bounded variables. However, these existing methods have difficulty to treat a posynomial function. According to Ryoo and Sahinidis [85], for underestimating a multilinear function $x_1 x_2 \cdots x_n$ with n variables, the AI scheme needs to use $\prod_{k=2}^{n-1} \Theta_k^{\binom{n}{k}} \sum_{i=1}^{\lfloor n/2 \rfloor} \binom{n}{2i}$ linear constraints maximally. Θ_k denotes the number of linear functions that the AI scheme generates to lower bound k-cross-product terms, $k = 2, 3, \ldots, n-1$. Since the number of linear constraints of convex envelopes for a multilinear function with n variables grows doubly exponentially in n, AI bounding scheme may only treat $n \leq 3$ cases. It is more difficult for AI to treat a posynomial function for $n > 3$ cases. Moreover, applying rAI scheme to underestimate a multilinear function $x_1 x_2 \cdots x_n$ needs to use the maximum of exponentially many 2^{n-1} linear inequalities. Therefore, the rAI bounding scheme has difficulty to treat posynomial functions as well as the AI scheme. EF [89, 90] provided the explicit facets of the convex and concave envelopes of trilinear monomials and demonstrated that these result in tighter bounds than the AI and rAI techniques. An important difference between EF and other bounding schemes is that these explicit facets are linear cuts, which were proven to define the convex envelope. Explicit facets (EFs) of the convex envelope are effective in treating general trilinear monomials, but the derivation of explicit facets for the convex envelope of general multilinear monomials and signomials is an open problem. Li et al. [97] and Lu et al. [98] proposed a novel method for the convex relaxation of posynomial functions. The approach is different from the work of Maranas and Floudas [50], which provided an alternative way of generating convex underestimators for generalized geometric programming problems via the exponential transformation and linear underestimation of the concave terms. Applications of this approach include the area of process synthesis and design of separations, phase equilibrium, nonisothermal complex reactor networks, and molecular conformation problems (e.g., [99–101]).

3. Mixed-Integer Nonlinear Programming

Mixed-integer nonlinear programming (MINLP) problems involving both continuous and discrete variables arise in many applications of engineering design, chemical engineering, operations research, and management. Biegler and Grossmann [102] provided a retrospective on optimization techniques that have been applied in process systems engineering. They indicated that design and synthesis problems have been dominated by NLP and MINLP models. With the increasing reliance on modeling optimization problems in practical problems, a number of theoretical and algorithmic contributions of MINLP have been proposed. Many deterministic methods for convex MINLP problems have been reviewed by Biegler and Grossmann [102], Grossmann [103], and Grossmann and Biegler [104]. The methods include branch-and-bound (BB) [53, 72, 77], generalized benders decomposition (GBD) [76], outer-approximation (OA) [73, 74, 79], extended cutting plane method (ECP) [81], and generalized disjunctive programming (GDP) [78]. The BB method can find the global solution only when each subproblem can be solved to global optimality. The GBD method, the OA method, and the ECP method cannot solve optimization problems with nonconvex constraints or nonconvex objective functions because the subproblems may not have a unique optimum in the solution process. The GDP models address discrete/continuous optimization problems that involve disjunctions with nonlinear inequalities and logic propositions. The objective functions and the constraints in the GDP problem are assumed to be convex and bounded [56].

For deterministic optimization methods, these optimization problems are characterized by the convexity of the feasible domain or the objective function and may involve continuous and/or discrete variables. Although continuous and discrete optimization problems constitute two classes of global optimization problems, they primarily differ in the presence or absence of convexity rather than other features. Since the convexity of the objective function or the feasible domain is very important, understanding how to convexify the nonconvex parts is an essential area of research. As long as the formulated problem is a convex problem, efficient numerical methods are available to treat the optimization problem. However, optimization problems often include nonconvex functions that cannot be dealt with by the standard local optimization techniques to guarantee global optimality efficiently. For solving nonconvex or large-scale optimization problems, deterministic methods may not be easy to derive an optimal solution within reasonable time due to the high complexity of the problem.

Sherali et al. [105] presented an extension of the reformulation linearization technique (RLT) that is designed to exploit special structures and explored the strengthening of the RLT constraints through conditional logical expressions. Sinha et al. [106] studied a solvent design problem that is constructed as a nonconvex MINLP problem. They identified the sources of nonconvexities in the properties and solubility parameter design constraints and proposed linear underestimators based on a multilevel representation approach for the functions. A reduced space branch-and-bound global optimization algorithm was then presented for solving a single component blanket wash design problem. Pörn et al. [52] introduced different convexification strategies for nonconvex MINLP problems with both posynomial and negative binomial terms in the constraints. Harjunkoski et al. [107] studied the trim loss minimization problem for the paper converting industry and formulated the model as a nonconvex MINLP. They also proposed transformations for the bilinear terms based on linear representations and convex expressions. Pörn and Westerlund [108] proposed a cutting plane method for addressing global MINLP problems with pseudoconvex objective function

and constraints and tested the proposed method on several benchmark problems arising in process synthesis and scheduling applications. Parthasarathy and El-Halwagi [109] studied a systematic framework for the optimal design of condensation, which is an important technology for volatile organic compounds, and formulated the problem as a nonconvex MINLP model. They also proposed an iterative global optimization approach based on physical insights and active constraint principles that allow for decomposition and efficient solution and applied it to a case study for the manufacture of adhesive tapes. Adjiman et al. [82, 83, 110, 111] proposed two global optimization approaches, SMIN-αBB and GMIN-αBB, for nonconvex MINLP problems based on the concept of branch-and-bound. These two approaches rely on optimization or interval-based variable-bound updates to enhance efficiency. Although one possible approach to circumvent nonconvexities in nonlinear optimization models is reformulation, for instance, using the exponential transformation to treat the generalized geometric programming problems in which a signomial term $x_1^\alpha x_2^\beta$ is transferred into an exponential term $e^{\alpha \ln x_1 + \beta \ln x_2}$, the exponential transformation technique can only be applied to strictly positive variables and is thus unable to deal with nonconvex problems with free variables. Tsai et al. [11] proposed an approach to treat zero boundary optimization problems and suggested some convexification rules for the signomial terms with only three nonnegative discrete/integer variables. Björk and Westerlund [112] studied the global optimization of heat exchanger network synthesis through the simplified superstructure representation that allows only series and parallel schemes and applied convexification approaches for signomials by piecewise linear approximations. They also formulated convex MINLP lower bounding models using the Patterson formula for the log mean temperature difference considering both isothermal and nonisothermal mixing. Ostrovsky et al. [113] studied nonconvex MINLP models in which most variables are in the nonconvex terms and the number of linear constraints is much larger than the number of nonlinear constraints for solvent design and recovery problems. The work presents a tailored branch-and-bound approach using linear underestimators for tree functions based on a multilevel function representation and shows that there is a significant reduction in the branching variable space. Tawarmalani and Sahinidis [114] developed a branch and bound framework for the global optimization of MINLP problems. The framework involves novel linear relaxation schemes, a Lagrangian/linear duality-based theory for domain and range reduction, and branching strategies that guarantee finiteness of the solution sequence for certain classes of problems. They also discuss implementation issues and present computational results with a variety of benchmark problems. Kesavan et al. [115] presented outer-approximation algorithms for finding an optimal solution of a separable nonconvex MINLP program. Emet and Westerlund [116] conducted a computational comparison of solving a cyclic chromatographic separation problem using MINLP methods and reported that the extended cutting plane method compares favourably against traditional outer-approximation and branch-and-bound methods. A review of the recent advances in MINLP optimization of planning and design problems in the process industry was presented by Kallrath [117]. Tawarmalani and Sahinidis [118] introduced a polyhedral branch-and-cut approach in global optimization. Their algorithm exploits convexity in order to generate the polyhedral cuts and relaxations for multivariate nonconvex problems. Meyer and Floudas [119] studied superstructures of pooling networks, which are important to the petrochemical, chemical, and wastewater treatment industries, and formulated this generalized pooling problem as a nonconvex MINLP problem that involves many bilinear terms in the constraint functions. They proposed a global optimization algorithm based on a novel piecewise linear reformulation-linearization technique (RLT) formulation. Karuppiah and Grossmann [120]

addressed the problem of optimal synthesis of an integrated water system, where water using processes and water treatment operations are jointly considered. The designed MINLP model was solved with a new deterministic spatial branch and contract algorithm, in which piecewise under- and overestimators are used for constructing the relaxations at each node. Bergamini et al. [121] formulated an MINLP model for the global optimization of heat exchanger networks and presented a new solution methodology that is based on outer-approximation and utilizes piecewise underestimation. Rigorous constraints obtained from physical insights are also included in the formulation, and the authors reported computationally efficient global solutions for problems with up to nine process streams. Tsai and Lin [54, 56] proposed a method for solving a signomial MINLP problem with free variables by the convexification strategies and piecewise linearization techniques. However, the optimal solution obtained is an approximate solution by the piecewise linearization approach. Karuppiah et al. [122] presented an outer-approximation algorithm to globally solve a nonconvex MINLP formulation that corresponds to the continuous time scheduling of refinery crude oil operations. The solution procedure relies on effective mixed-integer linear relaxations that benefit from additional cuts derived after spatially decomposing the network. Foteinou et al. [123] presented a mixed-integer optimization framework for the synthesis and analysis of regulatory networks. Their approach integrates prior biological knowledge regarding interactions between genes and corresponding transcription factors, in an effort to minimize the complexity of the problem. Misener et al. [124] proposed an extended pooling problem to maximize the profit of blending reformulated gasoline on a predetermined network structure of feed stocks, intermediate storage tanks, and gasoline products subject to applicable environmental standards. They formulated the problem as a nonconvex MINLP model due to the presence of bilinear, polynomial, and fractional power terms. A mixed-integer linear programming relaxation of the extended pooling problem is proposed for several small- to large-scale test cases. Misener et al. [125] introduced a formulation for the piecewise linear relaxation of bilinear functions with a logarithmic number of binary variables and computationally compared their performance of this new formulation to the best performing piecewise relaxations with a linear number of binary variables. They also unified the new formulation into the computational tool APOGEE that globally optimizes standard, generalized, and extended pooling problems. Westerlund et al. [126] considered some special but fundamental issues related to convex relaxation techniques in nonconvex MINLP optimization, especially for optimization problems including nonconvex inequality constraints and their relaxations.

The alternative global optima of an MINLP problem can be found if more than one solution satisfies the same optimal value of the objective function. In practice, alternative optima are useful because they allow the decision maker to choose from many solutions without experiencing any deterioration in the objective function. For the case involving only 0-1 variables, Balas and Jeroslow [127] introduced the well-known binary cut with only one constraint and no additional variables. Duran and Grossmann [73] used this binary cut in their OA algorithm to exclude binary combinations. Tawarmalani and Sahinidis [38] mentioned that BARON can identify the K best solutions for a mixed-integer nonlinear program, where K is an option specified by the user. Tsai et al. [57] proposed a general integer cut to identify all alternative optimal solutions of a general integer linear programming problem. Lin and Tsai [60] proposed a generalized method to find multiple optimal solutions of an MINLP problem with free variables by means of variable substitution and convexification strategies. The problem is first converted into another convex MINLP problem solvable to obtain an exactly global optimum. Then, a general cut is utilized to

exclude the previous solution and an algorithm is developed to locate all alternative optimal solutions.

4. Conclusions

Given the rapid advances in computing technology over the past decades, large optimization theories and algorithms have been proposed to solve various real-world engineering and management problems. Therefore, to give a systematic overview of the extant literature is a challenge and motivates this study, particularly for that the field of optimization has grown and evolved rapidly. This work first reviewed methods for continuous variable optimization and survey advances in signomial programming. Then, mixed-integer nonlinear programming methods for optimization problems with discrete components were introduced. Contributions related to theoretical and algorithmic developments, formulations, and applications for these two classes of optimization problems were also discussed.

Although deterministic approaches take advantage of analytical properties of the problem to generate a sequence of points that converge to a global solution, heuristic approaches have been found to be more flexible and efficient than deterministic approaches. For solving nonconvex or large-scale optimization problems, deterministic methods may not be easy to derive a globally optimal solution within reasonable time due to the high complexity of the problem. Heuristic approaches therefore are presented to reduce the computational time of solving an optimization problem, but the obtained solution is not guaranteed to be a feasible or globally optimal solution. These two types of optimization methods have different pros and cons. Therefore, integrating deterministic and heuristic approaches may be a good way of solving large-scale optimization problems for finding a global optimum. It is hoped that this paper will stimulate further research on developing more advanced deterministic and heuristic methods to enhance the computational efficiency of finding a globally optimal solution for various real application problems.

Acknowledgment

References

[1] W. Sharpe, "A linear programming approximation for the general portfolio analysis," *Journal of Financial and Quantitative Analysis*, vol. 6, pp. 1263–1275, 1971.

[2] B. Stone, "A linear programming formulation of the general portfolio selection model," *Journal of Financial and Quantitative Analysis*, vol. 8, pp. 621–636, 1973.

[3] M. R. Young, "A minimax portfolio selection rule with linear programming solution," *Management Science*, vol. 44, no. 5, pp. 673–683, 1998.

[4] C. S. Chen, S. M. Lee, and Q. S. Shen, "An analytical model for the container loading problem," *European Journal of Operational Research*, vol. 80, no. 1, pp. 68–76, 1995.

[5] L. Faina, "A global optimization algorithm for the three-dimensional packing problem," *European Journal of Operational Research*, vol. 126, no. 2, pp. 340–354, 2000.

[6] H. L. Li, C. T. Chang, and J. F. Tsai, "Approximately global optimization for assortment problems using piecewise linearization techniques," *European Journal of Operational Research*, vol. 140, no. 3, pp. 584–589, 2002.

[7] R. J. Duffin, E. L. Peterson, and C. Zener, *Geometric programming: Theory and application*, John Wiley & Sons Inc., New York, 1967.

[8] J. S. Arora, *Introduction to Optimum Design*, McGraw-Hill, New York, NY, USA, 1989.

[9] E. Sandgren, "Nonlinear integer and discrete programming in mechnical design optimization," *Journal of Mechanisms, Transmissions, and Automation in Design*, vol. 112, no. 2, pp. 223–229, 1990.

[10] J. F. Fu, R. G. Fenton, and W. L. Cleghorn, "A mixed integer-discrete -continuous programming method and its application to engineering design optimization," *Engineering Optimization*, vol. 17, pp. 263–280, 1991.

[11] J. F. Tsai, H. L. Li, and N. Z. Hu, "Global optimization for signomial discrete programming problems in engineering design," *Engineering Optimization*, vol. 34, no. 6, pp. 613–622, 2002.

[12] J. F. Tsai, "Global optimization for signomial discrete programming problems in engineering design," *Engineering Optimization*, vol. 42, no. 9, pp. 833–843, 2010.

[13] D. Rotem, G. A. Schloss, and A. Segev, "Data allocation for multidisk databases," *IEEE Transactions on Knowledge and Data Engineering*, vol. 5, no. 5, pp. 882–887, 1993.

[14] X. Lin and M. Orlowska, "An integer linear programming approach to data allocation with the minimum total communication cost in distributed database systems," *Information Sciences*, vol. 85, no. 1–3, pp. 1–10, 1995.

[15] R. Sarathy, B. Shetty, and A. Sen, "A constrained nonlinear 0-1 program for data allocation," *European Journal of Operational Research*, vol. 102, no. 3, pp. 626–647, 1997.

[16] J. F. Tsai and H. L. Li, "Technical note—on optimization approach for multidisk vertical allocation problems," *European Journal of Operational Research*, vol. 165, no. 3, pp. 835–842, 2005.

[17] M. H. Lin, "An optimal workload-based data allocation approach for multidisk databases," *Data and Knowledge Engineering*, vol. 68, no. 5, pp. 499–508, 2009.

[18] H. S. Ryoo and N. V. Sahinidis, "Global optimization of nonconvex NLPs and MINLPs with applications in process design," *Computers and Chemical Engineering*, vol. 19, no. 5, pp. 551–566, 1995.

[19] C. A. Floudas, "Global optimization in design and control of chemical process systems," *Journal of Process Control*, vol. 10, no. 2, pp. 125–134, 2000.

[20] C. A. Floudas, "Recent advances in global optimization for process synthesis, design and control: enclosure of all Solutions," *Computers and Chemical Engineering*, vol. 23, no. 1, pp. S963–S973, 1999.

[21] C. A. Floudas, *Deterministic Global Optimization: Theory, Methods and Application*, Kluwer Academic Publishers, Boston, Mass, USA, 2000.

[22] C. D. Maranas and C. A. Floudas, "Global optimization in generalized geometric programming," *Computers and Chemical Engineering*, vol. 21, no. 4, pp. 351–369, 1997.

[23] J. G. Ecker, M. Kupferschmid, C. E. Lawrence, A. A. Reilly, and A. C. H. Scott, "An application of nonlinear optimization in molecular biology," *European Journal of Operational Research*, vol. 138, no. 2, pp. 452–458, 2002.

[24] J. Mockus, W. Eddy, L. Mockus, and G. Reklaitis, *Bayesian Heuristic Approach to Discrete and Global Optimization*, Kluwer Academic Publishers, London, UK, 1996.

[25] I.E. Grossmann, *Global Optimization in Engineering Design*, Kluwer Academic Publishers, London, UK, 1996.

[26] I. M. Bomze, T. Csendes, R. Horst, and P. M. Pardalos, *Developments in Global Optimization*, Kluwer Academic Publishers, London, UK, 1997.

[27] A. Migdalas, P. M. Pardalos, and P. Varbrand, *Multilevel Optimization: Algorithms and Applications*, Kluwer Academic Publishers, London, UK, 1997.

[28] E. S. Mistakidis and G. E. Stavroulakis, *Nonconvex Optimization in Mechanics: Algorithm, Heuristics and Engineering Applications by the F.E.M*, Kluwer Academic Publishers, London, UK, 1997.

[29] R. De Leone, A. Murli, P. M. Pardalos, and G. Toraldo, *High Performance Software for Nonlinear Optimization*, Kluwer Academic Publishers, London, UK, 1998.

[30] E. M. T. Hendrix, *Global optimization at work*, Ph.D. thesis, Agriculture University Wageningen, Wageningen, The Netherlands, 1998.

[31] G. F. Corliss and R. B. Kearfott, "Rigorous global search: industrial applications," in *Developments in Reliable Computing*, T. Csendes, Ed., pp. 1–16, Kluwer Academic Publishers, London, UK, 1999.

[32] C. A. Floudas and P. M. Pardalos, *Optimization in Computational Chemistry and Molecular Biology-Local and Global Approaches*, Kluwer Academic Publishers, London, UK, 2000.

[33] M. Laguna and J. L. González-Velarde, *Computing Tools for Modeling Optimization and Simulation*, Kluwer Academic Publishers, London, UK, 2000.

[34] P. M. Papalambros and D. J. Wilde, *Principles of Optimal Design-modeling and Computation*, Cambridge University Press, Cambridge, UK, 2nd edition, 2000.

[35] A. Atkinson, B. Bogacka, and A. A. Zhigljavsky, *Optimum Design 2000*, Kluwer Academic, London, UK, 2001.

[36] T. F. Edgar, D. M. Himmelblau, and L. S. Lasdon, *Optimization of Chemical Processes*, McGraw-Hill, Boston, Mass, USA, 2nd edition, 2001.

[37] P.M. Pardalos and H.E. Romeijn, *Handbook of Global Optimization*, vol. 2, Kluwer Academic Publishers, London, UK, 2002.

[38] M. Tawarmalani and N. V. Sahinidis, "Convex extensions and envelopes of lower semi-continuous functions," *Mathematical Programming A*, vol. 93, no. 2, Ser. A, pp. 247–263, 2002.

[39] N. Hadjisavvas, S. Komlósi, and S. S. Schaible, *Handbook of Generalized Convexity and Generalized Monotonicity*, Kluwer Academic Publishers, London, UK, 2005.

[40] C. A. Floudas, I. G. Akrotirianakis, S. Caratzoulas, C. A. Meyer, and J. Kallrath, "Global optimization in the 21st century: advances and challenges," *Computers and Chemical Engineering*, vol. 29, pp. 1185–1202, 2005.

[41] J. D. Pintér, *Global Optimization: Scientific and Engineering Case Studies*, Kluwer Academic Publishers, London, UK, 2006.

[42] S. K. Mishra and G. Giorgi, *Invexity and Optimization*, Kluwer Academic Publishers, London, UK, 2008.

[43] S. K. Mishra, S. Wang, and K. K. Lai, *Generalized Convexity and Vector Optimization*, Kluwer Academic Publishers, London, UK, 2009.

[44] C. A. Floudas and C. E. Gounaris, "A review of recent advances in global optimization," *Journal of Global Optimization*, vol. 45, no. 1, pp. 3–38, 2009.

[45] R. J. Duffin and E. L. Peterson, "Duality theory for geometric programming," *SIAM Journal on Applied Mathematics*, vol. 14, pp. 1307–1349, 1966.

[46] R. J. Duffin, "Linearizing geometric programs," *SIAM Review*, vol. 12, pp. 211–227, 1970.

[47] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, 2007.

[48] R. J. Duffin and E. L. Peterson, "Geometric programming with signomials," *Journal of Optimization Theory and Applications*, vol. 11, pp. 3–35, 1973.

[49] J. G. Ecker, "Geometric programming: methods, computations and applications," *SIAM Review*, vol. 22, no. 3, pp. 338–362, 1980.

[50] C. D. Maranas and C. A. Floudas, "Finding all solutions of nonlinearly constrained systems of equations," *Journal of Global Optimization*, vol. 7, no. 2, pp. 143–182, 1995.

[51] C. A. Floudas and P. M. Pardalos, *State of the Art in Global Optimization: Computational Methods and Applications*, Kluwer Academic Publishers, Boston, MAss, USA, 1996.

[52] R. Pörn, I. Harjunkoski, and T. Westerlund, "Convexification of different classes of non-convex MINLP problems," *Computers and Chemical Engineering*, vol. 23, no. 3, pp. 439–448, 1999.

[53] H. L. Li and J. F. Tsai, "Treating free variables in generalized geometric global optimization programs," *Journal of Global Optimization*, vol. 33, no. 1, pp. 1–13, 2005.

[54] J. F. Tsai and M. H. Lin, "An optimization approach for solving signomial discrete programming problems with free variables," *Computers and Chemical Engineering*, vol. 30, no. 8, pp. 1256–1263, 2006.

[55] J. F. Tsai and M.-H. Lin, "Finding all solutions of systems of nonlinear equations with free variables," *Engineering Optimization*, vol. 39, no. 6, pp. 649–659, 2007.

[56] J. F. Tsai and M.-H. Lin, "Global optimization of signomial mixed-integer nonlinear programming problems with free variables," *Journal of Global Optimization*, vol. 42, no. 1, pp. 39–49, 2008.

[57] J. F. Tsai, M.-H. Lin, and Y.-C. Hu, "On generalized geometric programming problems with non-positive variables," *European Journal of Operational Research*, vol. 178, no. 1, pp. 10–19, 2007.

[58] J. F. Tsai, "Treating free variables in generalized geometric programming problems," *Computers and Chemical Engineering*, vol. 33, no. 1, pp. 239–243, 2009.

[59] H. L. Li and H. C. Lu, "Global optimization for generalized geometric programs with mixed free-sign variables," *Operations Research*, vol. 57, no. 3, pp. 701–713, 2009.

[60] M.-H. Lin and J. F. Tsai, "Finding multiple optimal solutions of signomial discrete programming problems with free variables," *Optimization and Engineering*, vol. 12, no. 3, pp. 425–443, 2011.

[61] J. F. Tsai and M.-H. Lin, "An efficient global approach for posynomial geometric programming problems," *INFORMS Journal on Computing*, vol. 23, no. 3, pp. 483–492, 2011.

[62] K. M. Björk, *A global optimization method with some heat exchanger network applications*, Ph.D. thesis, Åbo Akademi University, 2002.

[63] K. M. Björk, P. O. Lindberg, and T. Westerlund, "Some convexifications in global optimization of problems containing signomial terms," *Computers and Chemical Engineering*, vol. 27, no. 5, pp. 669–679, 2003.

[64] R. Pörn, K.-M. Björk, and T. Westerlund, "Global solution of optimization problems with signomial parts," *Discrete Optimization*, vol. 5, no. 1, pp. 108–120, 2008.

[65] T. Westerlund and J. Westerlund, "GGPECP—An algorithm for solving non-convex MINLP problems by cutting plane and transformation techniques," *Chemical Engineering Transactions*, vol. 3, pp. 1045–1050, 2003.

[66] T. Westerlund, "Some transformation techniques in global optimization," in *Global Optimization*, vol. 84, pp. 45–74, Springer, New York, NY, USA, 2006.

[67] A. Lundell and T. Westerlund, "Optimization of power transformations in global optimization," *Chemical Engineering Transactions*, vol. 11, pp. 95–100, 2007.

[68] A. Lundell, J. Westerlund, and T. Westerlund, "Some transformation techniques with applications in global optimization," *Journal of Global Optimization*, vol. 43, no. 2-3, pp. 391–405, 2009.

[69] A. Lundell and T. Westerlund, "Convex underestimation strategies for signomial functions," *Optimization Methods & Software*, vol. 24, no. 4-5, pp. 505–522, 2009.

[70] A. Lundell and T. Westerlund, "On the relationship between power and exponential transformations for positive signomial functions," *Chemical Engineering Transactions*, vol. 17, pp. 1287–1292, 2009.

[71] M.-H. Lin and J. F. Tsai, "Range reduction techniques for improving computational efficiency in global optimization of signomial geometric programming problems," *European Journal of Operational Research*, vol. 216, no. 1, pp. 17–25, 2012.

[72] B. Borchers and J. E. Mitchell, "An improved branch and bound algorithm for mixed integer nonlinear programs," *Computers & Operations Research*, vol. 21, no. 4, pp. 359–367, 1994.

[73] M. A. Duran and I. E. Grossmann, "An outer-approximation algorithm for a class of mixed-integer nonlinear programs," *Mathematical Programming*, vol. 36, no. 3, pp. 307–339, 1986.

[74] R. Fletcher and S. Leyffer, "Solving mixed integer nonlinear programs by outer approximation," *Mathematical Programming*, vol. 66, no. 3, Ser. A, pp. 327–349, 1994.

[75] C. A. Floudas, *Nonlinear and Mixed Integer Optimization:Fundamentals and Applications*, Oxford University Press, New York, NY, USA, 1995.

[76] A. M. Geoffrion, "Generalized Benders decomposition," *Journal of Optimization Theory and Applications*, vol. 10, pp. 237–260, 1972.

[77] S. Leyffer, "Integrating SQP and branch-and-bound for mixed integer nonlinear programming," *Computational Optimization and Applications*, vol. 18, no. 3, pp. 295–309, 2001.

[78] S. Lee and I. E. Grossmann, "New algorithms for nonlinear generalized disjunctive programming," *Computers and Chemical Engineering*, vol. 24, no. 9-10, pp. 2125–2142, 2000.

[79] I. Quesada and I. E. Grossmann, "An LP/NLP based branch and bound algorithm for convex MINLP optimization problems," *Computers and Chemical Engineering*, vol. 16, no. 10-11, pp. 937–947, 1992.

[80] R. A. Stubbs and S. Mehrotra, "A branch-and-cut method for 0–1 mixed convex programming," *Mathematical Programming A*, vol. 86, no. 3, pp. 515–532, 1999.

[81] T. Westerlund and F. Pettersson, "An extended cutting plane method for solving convex MINLP problems," *Computers and Chemical Engineering*, vol. 19, no. 1, supplement, pp. S131–S136, 1995.

[82] C. S. Adjiman, S. Dallwig, C. A. Floudas, and A. Neumaier, "A global optimization method, αBB, for general twice-differentiable constrained NLPs-I. Theoretical advances," *Computers and Chemical Engineering*, vol. 22, no. 9, pp. 1137–1158, 1998.

[83] C. S. Adjiman, I. P. Androulakis, and C. A. Floudas, "A global optimization method, αBB, for general twice-differentiabe constrained NLPs-II. Implementation and computational results," *Computers and Chemical Engineering*, vol. 22, no. 9, pp. 1159–1179, 1998.

[84] M. Tawarmalani and N. V. Sahinidis, "Semidefinite relaxations of fractional programs via novel convexification techniques," *Journal of Global Optimization*, vol. 20, no. 2, pp. 137–158, 2001.

[85] H. S. Ryoo and N. V. Sahinidis, "Analysis of bounds for multilinear functions," *Journal of Global Optimization*, vol. 19, no. 4, pp. 403–424, 2001.

[86] M. Tawarmalani, S. Ahmed, and N. V. Sahinidis, "Product disaggregation in global optimization and relaxations of rational programs," *Optimization and Engineering*, vol. 3, no. 3, pp. 281–303, 2002.

[87] M. Tawarmalani, S. Ahmed, and N. V. Sahinidis, "Global optimization of 0-1 hyperbolic programs," *Journal of Global Optimization*, vol. 24, no. 4, pp. 385–416, 2002.

[88] L. Liberti and C. C. Pantelides, "Convex envelopes of monomials of odd degree," *Journal of Global Optimization*, vol. 25, no. 2, pp. 157–168, 2003.

[89] C. A. Meyer and C. A. Floudas, "Trilinear Monomials with Positive or Negative Domains: Facets of Convex and Concave Envelopes," in *Frontiers in global optimization*, C. A. Floudas and P. M. Pardalos, Eds., pp. 327–352, Kluwer Academic Publishers, Santorini, Greece, 2003.

[90] C. A. Meyer and C. A. Floudas, "Trilinear monomials with mixed sign domains: facets of the convex and concave envelopes," *Journal of Global Optimization*, vol. 29, no. 2, pp. 125–155, 2004.

[91] F. Tardella, "On the existence of polyhedral convex envelopes," in *Frontiers in Global Optimization*, C. A. Floudas and P. M. Pardalos, Eds., pp. 563–573, Kluwer Academic Publishers, Santorini, Greece, 2003.

[92] C. A. Meyer and C. A. Floudas, "Convex envelopes for edge-concave functions," *Mathematical Programming B*, vol. 103, no. 2, pp. 207–224, 2005.

[93] S. Caratzoulas and C. A. Floudas, "Trigonometric convex underestimator for the base functions in Fourier space," *Journal of Optimization Theory and Applications*, vol. 124, no. 2, pp. 339–362, 2005.

[94] I. G. Akrotirianakis and C. A. Floudas, "Computational experience with a new class of convex underestimators: box-constrained NLP problems," *Journal of Global Optimization*, vol. 29, no. 3, pp. 249–264, 2004.

[95] I. G. Akrotirianakis and C. A. Floudas, "A new class of improved convex underestimators for twice continuously differentiable constrained NLPs," *Journal of Global Optimization*, vol. 30, no. 4, pp. 367–390, 2004.

[96] A. S. E. Hamed, *Calculation of bounds on variables and underestimating convex functions for nonconvex functions*, Ph.D. thesis, The George Washington University, 1991.

[97] H. L. Li, J. F. Tsai, and C. A. Floudas, "Convex underestimation for posynomial functions of positive variables," *Optimization Letters*, vol. 2, no. 3, pp. 333–340, 2008.

[98] H. C. Lu, H. L. Li, C. E. Gounaris, and C. A. Floudas, "Convex relaxation for solving posynomial programs," *Journal of Global Optimization*, vol. 46, no. 1, pp. 147–154, 2010.

[99] A. Aggarwal and C. A. Floudas, "Synthesis of general distillation sequences-nonsharp separations," *Computers and Chemical Engineering*, vol. 14, no. 6, pp. 631–653, 1990.

[100] A. R. Ciric and C. A. Floudas, "A retrofit approach for heat exchanger networks," *Computers and Chemical Engineering*, vol. 13, no. 6, pp. 703–715, 1989.

[101] A. C. Kokossis and C. A. Floudas, "Optimization of complex reactor networks-II. Nonisothermal operation," *Chemical Engineering Science*, vol. 49, no. 7, pp. 1037–1051, 1994.

[102] L. T. Biegler and I. E. Grossmann, "Retrospective on optimization," *Computers and Chemical Engineering*, vol. 28, no. 8, pp. 1169–1192, 2004.

[103] I. E. Grossmann, "Review of nonlinear mixed-integer and disjunctive programming techniques," *Optimization and Engineering*, vol. 3, no. 3, pp. 227–252, 2002.

[104] I. E. Grossmann and L. T. Biegler, "Part II. Future perspective on optimization," *Computers and Chemical Engineering*, vol. 28, no. 8, pp. 1193–1218, 2004.

[105] H. D. Sherali, W. P. Adams, and P. J. Driscoll, "Exploiting special structures in constructing a hierarchy of relaxations for mixed integer problems," *Operations Research*, vol. 46, no. 3, pp. 396–405, 1998.

[106] M. Sinha, L. E. K. Achenie, and G. M. Ostrovsky, "Environmentally benign solvent design by global optimization," *Computers and Chemical Engineering*, vol. 23, no. 10, pp. 1381–1394, 1999.

[107] I. Harjunkoski, T. Westerlund, and R. Pörn, "Numerical and environmental considerations on a complex industrial mixed integer non-linear programming (MINLP) problem," *Computers and Chemical Engineering*, vol. 23, no. 10, pp. 1545–1561, 1999.

[108] R. Pörn and T. Westerlund, "A cutting plane method for minimizing pseudo-convex functions in the mixed integer case," *Computers and Chemical Engineering*, vol. 24, no. 12, pp. 2655–2665, 2000.

[109] G. Parthasarathy and M. M. El-Halwagi, "Optimum mass integration strategies for condensation and allocation of multicomponent VOCs," *Chemical Engineering Science*, vol. 55, no. 5, pp. 881–895, 2000.

[110] C. S. Adjiman, I. P. Androulakis, and C. A. Floudas, "Global optimization of MINLP problems in process synthesis and design," *Computers and Chemical Engineering*, vol. 21, no. 1, pp. S445–S450, 1997.

[111] C. S. Adjiman, I. P. Androulakis, and C. A. Floudas, "Global optimization of mixed-integer nonlinear problems," *AIChE Journal*, vol. 46, no. 9, pp. 1769–1797, 2000.

[112] K. M. Björk and T. Westerlund, "Global optimization of heat exchanger network synthesis problems with and without the isothermal mixing assumption," *Computers and Chemical Engineering*, vol. 26, no. 11, pp. 1581–1593, 2002.

[113] G. M. Ostrovsky, L. E. K. Achenie, and M. Sinha, "On the solution of mixed-integer nonlinear programming models for computer aided molecular design," *Computers and Chemistry*, vol. 26, no. 6, pp. 645–660, 2002.

[114] M. Tawarmalani and N. V. Sahinidis, "Global optimization of mixed-integer nonlinear programs: a theoretical and computational study," *Mathematical Programming. A* , vol. 99, no. 3, pp. 563–591, 2004.

[115] P. Kesavan, R. J. Allgor, E. P. Gatzke, and P. I. Barton, "Outer approximation algorithms for separable nonconvex mixed-integer nonlinear programs," *Mathematical Programming A*, vol. 100, no. 3, pp. 517–535, 2004.

[116] S. Emet and T. Westerlund, "Comparisons of solving a chromatographic separation problem using MINLP methods," *Computers and Chemical Engineering*, vol. 28, no. 5, pp. 673–682, 2004.

[117] J. Kallrath, "Solving planning and design problems in the process industry using mixed integer and global optimization," *Annals of Operations Research*, vol. 140, pp. 339–373, 2005.

[118] M. Tawarmalani and N. V. Sahinidis, "A polyhedral branch-and-cut approach to global optimization," *Mathematical Programming. B*, vol. 103, no. 2, pp. 225–249, 2005.

[119] C. A. Meyer and C. A. Floudas, "Global optimization of a combinatorially complex generalized pooling problem," *AIChE Journal*, vol. 52, no. 3, pp. 1027–1037, 2006.

[120] R. Karuppiah and I. E. Grossmann, "Global optimization for the synthesis of integrated water systems in chemical processes," *Computers and Chemical Engineering*, vol. 30, no. 4, pp. 650–673, 2006.

[121] M. L. Bergamini, N. J. Scenna, and P. A. Aguirre, "Global optimal structures of heat exchanger networks by piecewise relaxation," *Industrial and Engineering Chemistry Research*, vol. 46, no. 6, pp. 1752–1763, 2007.

[122] R. Karuppiah, K. C. Furman, and I. E. Grossmann, "Global optimization for scheduling refinery crude oil operations," *Computers and Chemical Engineering*, vol. 32, no. 11, pp. 2745–2766, 2008.

[123] P. T. Foteinou, E. Yang, G. K. Saharidis, M. G. Ierapetritou, and I. P. Androulakis, "A mixed-integer optimization framework for the synthesis and analysis of regulatory networks," *Journal of Global Optimization*, vol. 43, no. 2-3, pp. 263–276, 2009.

[124] R. Misener, C. E. Gounaris, and C. A. Floudas, "Mathematical modeling and global optimization of large-scale extended pooling problems with the (EPA) complex emissions constraints," *Computers and Chemical Engineering*, vol. 34, no. 9, pp. 1432–1456, 2010.

[125] R. Misener, J. P. Thompson, and C. A. Floudas, "Apogee: global optimization of standard, generalized, and extended pooling problems via linear and logarithmic partitioning schemes," *Computers and Chemical Engineering*, vol. 35, no. 5, pp. 876–892, 2011.

[126] T. Westerlund, A. Lundelf, and J. Westerlund, "On convex relaxations in nonconvex optimization," *Chemical Engineering Transactions*, vol. 24, pp. 331–336, 2011.

[127] E. Balas and R. Jeroslow, "Canonical cuts on the unit hypercube," *SIAM Journal on Applied Mathematics*, vol. 23, pp. 61–69, 1972.

Submit your manuscripts at
http://www.hindawi.com