Research Article

A Neuro-Augmented Observer for Robust Fault Detection in Nonlinear Systems

Huajun Gong and Ziyang Zhen

College of Automation Engineering, Nanjing University of Aeronautics & Astronautics, Nanjing 20016, China

Correspondence should be addressed to Huajun Gong, ghj301@nuaa.edu.cn

A new fault detection method using neural-networks-augmented state observer for nonlinear systems is presented in this paper. The novelty of the approach is that instead of approximating the entire nonlinear system with neural network, we only approximate the unmodeled part that is left over after linearization, in which a radial basis function (RBF) neural network is adopted. Compared with conventional linear observer, the proposed observer structure provides more accurate estimation of the system state. The state estimation error is proved to asymptotically approach zero by the Lyapunov method. An aircraft system demonstrates the efficiency of the proposed fault detection scheme, simulation results of which show that the proposed RBF neural network-based observer scheme is effective and has a potential application in fault detection and identification (FDI) for nonlinear systems.

1. Introduction

One of the basic requirements for successful residual-based fault detection and identification (FDI) is that during fault-free operation, the residuals should be zero in the deterministic case or zero-mean and uncorrelated in stochastic case. While this condition is easy to meet in linear systems, it is a difficult task for nonlinear systems. Reference [1] presents a fault-tolerant control (FTC) scheme for a class of multi-input-multi-output stochastic systems with actuator faults, in which the actuator fault diagnosis is based on the state estimation and the nominal controller is designed to compensate for the loss of actuator effectiveness. However, for nonlinear systems, the common approach of linearizing the nonlinear system with Taylor series expansion about an operating point may work reasonably well for control design, but the errors from the unmodeled nonlinear terms affect the residual in a way that renders residual-based FDI techniques unusable. Reference [2] proposes a fault-tolerant

tracking control method based on adaptive control technique for near-space-vehicle attitude dynamics, in which Takagi-Sugeno fuzzy model is used to describe the nonlinear system, and then an adaptive tracking control scheme is developed based on on-line actuator faults estimation.

One way of eliminating this problem is to develop state observers that estimate the actual state vector that includes the linear and nonlinear terms. State observers of nonlinear dynamic systems are, therefore, becoming a growing topic for fault detection and identification (FDI) [3–6]. Several observer design methods have been proposed in recent years for nonlinear dynamic systems [7–9]. However, effective and accurate observer design methods for generic nonlinear systems are still an open research issue. Neural networks (NN) provide one of the newer approaches to robust FDI for nonlinear systems [10–12]. The theory and application research of the neural networks is still a hot topic in intelligent control fields. Reference [13] uses three neural networks as parametric structures for facilitating the implementation of the iterative algorithm, to solve the near-optimal control problem of the nonlinear discrete-time systems with control constraints. Reference [14] develops a weighting-delay-based method for stability analysis of a class of recurrent neural networks with time-varying delay, and the optimization method is used to calculate the optimal weighting-delay parameters. Applications of neural networks in FDI can be grouped into two categories of tasks: (1) neural network observers for detecting the faults and (2) neural network classifiers for classifying fault patterns.

In this paper we focus on the first group of tasks: we present a new method for fault detection using a neural-networks-augmented state observer. The novelty of our approach is that instead of using the entire nonlinear system for designing the neural network-based observer, we only use the unmodeled part that is left over after a conventional linearization with Taylor series. Instead of the multilayered back-propagation neural networks, we choose radial basis function (RBF) neural network, which is faster to train and does not have the problem of local minima [15]. This allows us to build a state observer with an output tracking error that approaches zero asymptotically. The technique is applicable to systems that have nonlinearities, but their accurate nonlinear models are either not available or are difficult (or too time-consuming) to use. However, it assumes that the linearized models of these systems are available.

2. Problem Formulation

Consider the affine nonlinear dynamic system as follows:

$$\dot{x}(t) = f(x(t)) + g(x(t))u + \delta(t),$$
$$y(t) = Cx(t), \tag{2.1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^p$, $\delta \in R^n$ is a norm-bounded fault vector, $\|\delta(t)\| \leq D$, $C \in R^{p \times n}$, $f(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T$, $g(x) = [g_{ij}(x)]_{n \times m}$, and $f(\cdot)$, $g(\cdot)$ are continuously differentiable vector functions. Our goal is to design a nonlinear state observer that is a combination of a conventional observer and a neural network (the neuro-augmented observer).

System (2.1) is linearized at operating point (x_0, u_0), resulting in the state space model

$$\dot{x} = Ax + Bu + \phi(x, u, t) + \delta(t),$$
$$y(t) = Cx(t), \tag{2.2}$$

where $A = (\partial f / \partial x)_{x=x_0} \in R^{n \times n}$ and $B = g(x_0) \in R^{n \times m}$ are constant matrices. $f(x)$ is expanded in a first-order Taylor series approximation at $x = x_0$, and $f(x) = f(x_0) + Ax + \tilde{f}(x)$. Similarly, $g(x) = g(x_0) + \tilde{g}(x)$. Now we can get $\phi(x, u) = f(x_0) + \tilde{f}(x) + \tilde{g}(x)u$.

The nonlinear function $\phi(x, u)$ in system (2.2) includes an unmodeled nonlinear high-order term. If the system has no fault, that is, $\delta(t) = 0$, the state observer of system (2.2) is

$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu + \phi(\hat{x}, u), \tag{2.3}$$

where $\hat{x}$ is the state estimate, and $L \in R^{n \times p}$ is the observer gain matrix. Define the state estimation error $\varepsilon = x - \hat{x}$, and we get

$$\dot{\varepsilon} = (A - LC)\varepsilon + \phi(x, u) - \phi(\hat{x}, u). \tag{2.4}$$

The task of observer design is to select a suitable matrix L and make the state estimation error tend to zero asymptotically.

Reference [16] provides a method to check the stability of estimation error. However, it does not indicate how to choose the observer gain matrix L and how to approximate the nonlinear term. For the state estimation error equation (2.4), if $\phi_e = \phi(x, u) - \phi(\hat{x}, u)$ then the state estimation error at time t is

$$\varepsilon(t) = e^{(A-LC)t}\varepsilon(0) + \int_0^t e^{(A-LC)(t-\tau)}\phi_e(\tau)d\tau. \tag{2.5}$$

From (2.5) we can see that minimizing ϕ_e and driving it to zero would make the estimation error also approach zero asymptotically. Thus, the success of the observer depends on the successful approximation of the nonlinear term that resulted from the unmodeled portion of the original system. In order to achieve that goal, we propose the following structure shown as Figure 1, which incorporates an RBF neural network into the state estimation loop.

The RBF neural network is used to approximate the nonlinear term $\phi(x, u)$, thereby driving ϕ_e to zero, and thus improving the accuracy of state estimation. With that, we can expect the residuals during fault-free (healthy) situations to remain small, which is essential for successful FDI design.

3. Design of Neuro-Augmented Observer Based on RBF Neural Network

An RBF neural network observer is introduced for system (2.2), expressed by

$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu + \phi_{nn}(\hat{x}, u,), \tag{3.1}$$
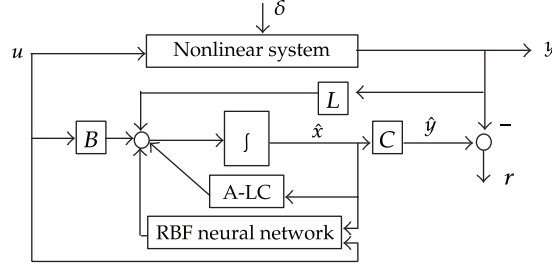
Figure 1: Robust fault detection based on neural network nonlinear observer.

where $\phi_{nn}(\hat{x}, u)$ is the neural network nonlinear estimator. It is realized by a three-layer RBF neural network. The first layer is direct input layer, and the hidden layer consists of several nodes. Each node contains a parameter vector called a center. The node calculates the Euclidean distance between the center and network input vector and then passes the result through a nonlinear function. The output layer is a set of linear combiners. The overall input-output response of the RBF neural network is a mapping $\phi_{nn} : R^{n+m} \rightarrow R^n$, then we get

$$\phi_{nn}(\hat{x}, u) = \left[\phi_{nn1}(\hat{x}, u), \phi_{nn2}(\hat{x}, u), \ldots, \phi_{nnn}(\hat{x}, u)\right], \tag{3.2}$$

$$\hat{\phi}(x, u) = \sum_{j=1}^{s} w_{ij}\sigma_j\left(\|x - c_j\|, \rho_j\right), \quad i = 1, 2, \ldots, n, \tag{3.3}$$

where $x \in R^{n+m}$ is input vector; $n + m, n$ are the number of inputs and outputs; s is the number of nodes in the hidden layer; $\{w_{ij}, i = 1 \sim n, j = 1 \sim s\}$ are the weights of the linear combiners of the output layer; $\{c_j, j = 1 \sim s\}$ are the centers of hidden nodes; $\{\rho_j, j = 1 \sim s\}$ are the widths of the Gaussian function. The nonlinear activation function $\sigma(\cdot)$ is selected as Gaussian function, and then $\sigma(x, \rho) = e^{-x^2/\rho}$.

Define

$$\sigma(x, \rho) = \left[\sigma_1(x, \rho), \sigma_2(x, \rho), \ldots, \sigma_s(x, \rho)\right]^T,$$

$$W = \left[w_{ij}\right], \quad i = 1, 2, \ldots, n, \ j = 1, 2, \ldots, s. \tag{3.4}$$

Then

$$\hat{\phi}(\hat{x}, u) = W\sigma(x, u). \tag{3.5}$$

Assumption 3.1. The RBF neural network can approximate the nonlinear function $\phi(x, u)$ arbitrarily closely, that is, there exists an ideal weight matrix W^*, satisfying $\phi(x, u) = W^*\sigma(x, \rho)$, where x is the input vector.

Assumption 3.2. For the system (2.2) and observer (3.3), the nonlinear term $\phi(x, u)$ satisfies the global Lipschitz condition, that is, $\|\phi(x, u) - \phi(\hat{x}, u)\| \leq \gamma\|x - \hat{x}\|$.

Theorem 3.3. *The state estimation error approaches zero asymptotically* $(\lim_{t \to \infty} \varepsilon = 0)$, *provided*

(i) $\gamma < \lambda_{\min}(Q)/2\lambda_{\max}(P)$, *where* P, Q *are positive definite symmetric matrices and* $\lambda_{\min}$ *and* $\lambda_{\max}$, *respectively, denote the minimum eigenvalue and maximum eigenvalue of the matrices,*

(ii) *the Lyapunov equation* $(A - LC)^T P + P(A - LC) = -Q$ *holds,*

(iii) *the weight tuning law of RBF neural network is adopted as follows:*

$$\dot{w}_i = \eta \sigma_i(\hat{x}, \rho) P \varepsilon, \tag{3.6}$$

where w_i *is the i th column of* W *and* η *is the learning rate in range of* $[0, 1]$.

Proof. Select the Lyapunov function

$$V = \frac{1}{2} \varepsilon^T P \varepsilon + \frac{1}{2\eta} \operatorname{tr}\left(\widetilde{W}^T \widetilde{W}\right), \tag{3.7}$$

where $\widetilde{W} = W^* - W$. Since the state estimation error is $\varepsilon = x - \hat{x}$, see from (2.4), we get

$$
\begin{aligned}
\dot{\varepsilon} &= (A - LC)\varepsilon + \phi(x, u, t) - \phi_{nn}(\hat{x}, u, t) \\
&= (A - LC)\varepsilon + W^* \sigma(x, \rho) - W\sigma(\hat{x}, \rho).
\end{aligned} \tag{3.8}
$$

Adding and subtracting $W^* \sigma(\hat{x}, \rho)$ in (3.8), we get

$$\dot{\varepsilon} = (A - LC)\varepsilon + W^* \sigma(x, \rho) - W\sigma(\hat{x}, \rho) + W^* \sigma(\hat{x}, \rho) - W^* \sigma(\hat{x}, \rho). \tag{3.9}$$

Considering $\tilde{\sigma} = \sigma(x, \rho) - \sigma(\hat{x}, \rho)$, $\widetilde{W} = W^* - W$, and $w(t) = W^* \tilde{\sigma}$, the above equation can be converted to

$$\dot{\varepsilon} = (A - LC)\varepsilon + \widetilde{W}\sigma(\hat{x}, \rho) + w(t). \tag{3.10}$$

Then we get

$$
\begin{aligned}
\dot{V} &= \frac{1}{2}\dot{\varepsilon}^T P \varepsilon + \frac{1}{2}\varepsilon^T P \dot{\varepsilon} + \frac{1}{\eta}\operatorname{tr}\left(\dot{\widetilde{W}}^T \widetilde{W}\right) \\
&= -\frac{1}{2}\varepsilon^T Q \varepsilon + \varepsilon^T P\left[\widetilde{W}\sigma(\hat{x}, \rho) + w(t)\right] + \frac{1}{\eta}\operatorname{tr}\left(\dot{\widetilde{W}}^T \widetilde{W}\right) \\
&= -\frac{1}{2}\varepsilon^T Q \varepsilon + \varepsilon^T P w(t) + \sum_{i=1}^{s}\varepsilon^T P \widetilde{w}_i \sigma_i(\hat{x}, \rho) + \frac{1}{\eta}\sum_{i=1}^{s}\dot{\widetilde{w}}_i^T \widetilde{w}_i,
\end{aligned} \tag{3.11}
$$

where $\widetilde{w}_i$ is the ith column of $\widetilde{W}$. And then

$$\dot{V} = -\frac{1}{2}\varepsilon^T Q\varepsilon + \varepsilon^T Pw(t) + \frac{1}{\eta}\sum_{i=1}^{s}\left[\left(\eta\varepsilon^T P\sigma_i(\hat{x},\rho) + \dot{\widetilde{w}}_i^T\right)\widetilde{w}_i\right]. \tag{3.12}$$

If the weight tuning law of the RBF neural network is $\dot{\widetilde{w}}_i = -\eta\sigma_i(\hat{x},\rho)P\varepsilon$ or $\dot{w}_i = \eta\sigma_i(\hat{x},\rho)P\varepsilon$, then

$$
\begin{aligned}
\dot{V} &= -\frac{1}{2}\varepsilon^T Q\varepsilon + \varepsilon^T Pw(t) \\[2mm]
&\le -\frac{1}{2}\|\varepsilon\|\lambda_{\min}(Q)\|\varepsilon\| + \|\varepsilon\|\lambda_{\max}(P)\|w(t)\| \\[2mm]
&= -\frac{1}{2}\|\varepsilon\|\lambda_{\min}(Q)\|\varepsilon\| + \|\varepsilon\|\lambda_{\max}(P)\|W^*\sigma(x,\rho) - W^*\sigma(\hat{x},\rho)\| \\[2mm]
&= -\frac{1}{2}\|\varepsilon\|\lambda_{\min}(Q)\|\varepsilon\| + \|\varepsilon\|\lambda_{\max}(P)\|\phi(x,u) - \phi(\hat{x},u)\| \\[2mm]
&\le -\frac{1}{2}\|\varepsilon\|\lambda_{\min}(Q)\|\varepsilon\| + \|\varepsilon\|\lambda_{\max}(P)\gamma\|x - \hat{x}\| \\[2mm]
&\le -\frac{1}{2}\|\varepsilon\|\lambda_{\min}(Q)\|\varepsilon\| + \|\varepsilon\|\lambda_{\max}(P)\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\|\varepsilon\| \\[2mm]
&\le 0,
\end{aligned}
\tag{3.13}
$$

then $\dot{V} \le 0$. Therefore, the RBF neural network observer is stable, and $\lim_{t\to\infty}\varepsilon = 0$.   □

4. Numerical Simulation Study

A fourth-order longitudinal aircraft model [15] is employed to demonstrate the effectiveness of the proposed method. The longitudinal perturbation equation is linearized in horizontal flight at a speed $v_0 = 300\,\text{m/sec}$ and at a height of 12000 m. The system matrices A, B, and C have the following forms:

$$
A = \begin{bmatrix} -0.017 & 0.026 & 0 & -9.81 \\ -0.0143 & -1.02 & 1 & 0 \\ 0 & 2.06 & -1.12 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 0 & 0 \\ -0.032 & -0.032 \\ -5.78 & -5.78 \\ 0 & 0 \end{bmatrix}, \qquad
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
\tag{4.1}
$$

where $x(t) = [v,\alpha,q,\theta]^T$ is state vector and the state variables v,α,θ,q are forward velocity, attack angle, pitch angle, and pitch angular velocity, respectively. $u = [\delta_{\text{el}},\delta_{\text{er}}]^T$ is control input vector, δ_{el} is the left elevator, and δ_{er} is the right elevator.
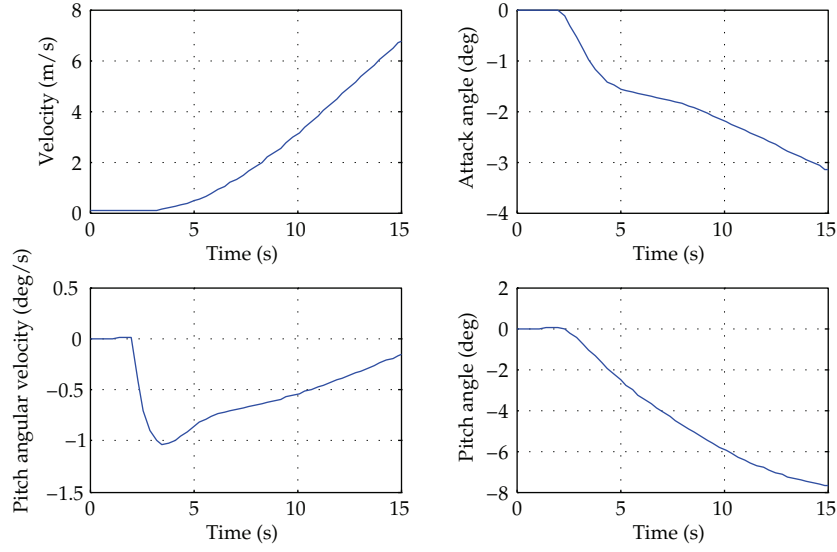
Figure 2: Residuals: left-elevator-stuck fault.

Nonlinear state observer (3.3) is designed by placing the poles of $(A - LC)$ at -2, -3, -4, and -5. The observer gain matrix L is

$$
L = \begin{bmatrix} 1.983 & 0.026 & 0 & -9.81 \\ -0.0143 & 1.98 & 1 & 0 \\ 0 & 2.06 & 2.88 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}.
\tag{4.2}
$$

The nonlinear term $\phi(x, u)$ is approximated by RBF neural network estimator $\phi_{nn}(\hat{x}, u)$. It is realized by a three-layer RBF neural network with a 6-8-4 structure. The input layer consists of 6 nodes, and the input vector is $[\hat{x}_1(t), \hat{x}_2(t), \hat{x}_3(t), \hat{x}_4(t), u_1(t), u_2(t)]^T$. The hidden layer consists of 8 nodes. The output layer is the estimation of nonlinear term $[\phi_{nn1}(t), \phi_{nn2}(t), \phi_{nn3}(t), \phi_{nn4}(t)]^T$. The RBF neural network can be off-line trained during fault-free operation. In practice, $\phi(x, u)$ will be variable with the changes of flight conditions. Therefore the off-line training and on-line learning can be combined to approximate the nonlinear term $\phi(x, u)$, in order to improve the adaptability of the neural network observer.

In the simulation, the left-elevator-stuck fault occurs from $t = 2$ sec. The residuals are shown in Figure 2. In Figure 2, we get that the state residuals change significantly at 2 sec and the fault-free residuals before 2 sec are zeros. Faults can be detected by comparing the output residuals with preselected thresholds, which are typically derived from the magnitudes of the residuals during fault-free cases.

5. Conclusions

A new nonlinear observer based on RBF neural network is designed in this paper. The novelty of this work is that instead of approximating the entire nonlinear system with neural network, only the unmodeled part of nonlinear system after linearization is approximated.

Furthermore, an RBF neural network that has better performance than the traditional neural networks is employed to design the state observer. The proposed neuro-augmented observer can generate residuals that are essential for fast fault detection. Simulation results of an aircraft model demonstrate the effectiveness of the proposed neuro-augmented observer and exhibit the application prospect of the robust fault detection scheme.

References

[1] B. Jiang and F. N. Chowdhury, "Fault estimation and accommodation for linear MIMO discrete-time systems," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 3, pp. 493–499, 2005.

[2] B. Jiang, Z. Gao, P. Shi, and Y. F. Xu, "Adaptive fault-tolerant tracking control of near-space vehicle using Takagi-Sugeno fuzzy models," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 1000–1007, 2010.

[3] B. Jiang and F. Chowdhury, "Observer-based fault diagnosis for a class of nonlinear systems," in *Proceedings of the American Control Conference (AAC'04)*, pp. 5671–5675, July 2004.

[4] H. Hammouri, M. Kinnaert, and E. H. El Yaagoubi, "Observer-based approach to fault detection and isolation for nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 10, pp. 1879–1884, 1999.

[5] H. Yang and M. Saif, "Fault detection in a class of nonlinear systems via adaptive sliding observer," in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, pp. 2199–2204, October 1995.

[6] W. Chen and M. Saif, "Robust fault detection in uncertain nonlinear systems via a second order sliding mode observer," in *Proceedings of the 40th IEEE Conference on Decision and Control (CDC'01)*, vol. 1, pp. 573–578, Orlando, Fla, USA, December 2001.

[7] D. N. Shields, S. A. Ashton, and S. Daley, "Robust fault detection observers for nonlinear polynomial systems," *International Journal of Systems Science*, vol. 32, no. 6, pp. 723–737, 2001.

[8] C. Edwards, S. K. Spurgeon, and R. J. Patton, "Sliding mode observers for fault detection and isolation," *Automatica*, vol. 36, no. 4, pp. 541–553, 2000.

[9] K. Adjallah, D. Maquin, and J. Ragot, "Non-linear observer-based fault detection," in *Proceedings of the 3rd IEEE Conference on Control Applications*, vol. 2, pp. 1115–1120, Glasgow, UK, August 1994.

[10] J. Wang, J. Zhao, and L. Ma, "A robust fault detection and isolation method via sliding mode observer," in *Proceedings of the 5th World Congress on Intelligent Control and Automation Conference (WCICA 2004)*, vol. 2, pp. 1727–1730, June 2004.

[11] C. W. Chan, K. C. Cheung, Y. Wang, and W. C. Chan, "On-line fault detection and isolation of nonlinear systems," in *Proceedings of the American Control Conference (ACC'99)*, pp. 3980–3984, June 1999.

[12] R. J. Schilling, J. J. Carroll, and A. F. Al-Ajlouni, "Approximation of nonlinear systems with radial basis function neural networks," *IEEE Transactions on Neural Networks*, vol. 12, no. 1, pp. 1–15, 2001.

[13] H. Zhang, Y. Luo, and D. Liu, "Neural-network-based near-optimal control for a class of discrete-time affine nonlinear systems with control constraints," *IEEE Transactions on Neural Networks*, vol. 20, no. 9, pp. 1490–1503, 2009.

[14] H. Zhang, Z. Liu, G. B. Huang, and Z. Wang, "Novel weighting-delay-based stability criteria for recurrent neural networks with time-varying delay," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 91–106, 2010.

[15] H. Shousong and Z. Chuan, "An approach to robust fault detection for nonlinear system based on RBF neural network observer," *Control Theory & Applications*, vol. 16, no. 6, pp. 853–857, 1999.

[16] Y. M. Cho and R. Rajamani, "A systematic approach to adaptive observer synthesis for nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 42, no. 4, pp. 534–537, 1997.