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Research Article

Global Convergence of a Modified LS Method

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The LS method is one of the effective conjugate gradient methods in solving the unconstrained optimization problems. The paper presents a modified LS method on the basis of the famous LS method and proves the strong global convergence for the uniformly convex functions and the global convergence for general functions under the strong Wolfe line search. The numerical experiments show that the modified LS method is very effective in practice.

1. Introduction

The conjugate gradient method is one of the most common methods used in the optimization, which is especially effective in solving the unconstrained optimization problem:

$$\min_{x \in R^n} f(x), \tag{1.1}$$

where $f : R^n \to R$ is continuously differentiable nonlinear function.

The LS method introduced by Liu and Storey [1] is one of the conjugate gradient methods, and its iteration formulas are as follows:

$$x_{k+1} = x_k + \alpha_k d_k, \tag{1.2}$$

$$d_k = \begin{cases} -g_k, & \text{for } k = 1; \\ -g_k + \beta_k d_{k-1}, & \text{for } k \geq 2; \end{cases} \tag{1.3}$$

$$\beta_k = -\frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T g_{k-1}}, \tag{1.4}$$

where g_k is the gradient of f at x_k; $\alpha_k > 0$ is a step length which is determined by some line search; d_k is the search direction.

The convergence properties of the LS method have been studied extensively. For example, Yu et al. [2] proposed a modified LS method called the LS1 method in this paper, in which parameter β_k satisfies the following formula:

$$\beta_k^{\mathrm{LS1}} = \begin{cases} \dfrac{\|g_k\|^2 - |g_k^T g_{k-1}|}{\varsigma |g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} & \text{if } \|g_k\|^2 \geq |g_k^T g_{k-1}|, \\ 0 & \text{else.} \end{cases} \tag{1.5}$$

They proved the global convergence property of the LS1 method under the Wolfe line search and verified that the LS1 method was very effective in solving the large unconstrained optimization problems. Liu et al. [3] further studied the LS method on the basis of [2] and proposed the parameter β_k:

$$\beta_k^{\mathrm{LS2}} = \begin{cases} \dfrac{g_k^T (g_k - g_{k-1})}{\rho |g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} & \text{if } \min\{1, \rho - 1 - \xi\} \cdot \|g_k\|^2 > |g_k^T g_{k-1}|, \\ 0 & \text{else,} \end{cases} \tag{1.6}$$

where $\rho > 1 + \xi$, and ξ is sufficiently small positive number. The corresponding method is called the LS2 method in this paper. Jinkui Liu, among others, proved the global convergence properties of the LS2 method under the Wolfe line search and showed that the achievements of the LS2 method was comparable with the PRP^+ method.

In this paper, a modified LS method is proposed on the basis of [2], which can guarantee generate the sufficient descent direction in each step under the strong Wolfe line search. Moreover, the new method has the strong global convergence properties for uniformly convex functions and the global convergence properties for ordinary functions.

2. The Sufficient Descent Property of the New Method

MLS Method

Step 1. Data $x_1 \in R^n$, $\varepsilon \geq 0$. Set $d_1 = -g_1$, if $\|g_1\| \leq \varepsilon$, then stop.

Step 2. Compute α_k by the strong Wolfe line search ($\delta \in (0, 1/2), \sigma \in (\delta, 1)$):

$$\begin{gathered} f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k, \\ \left| g(x_k + \alpha_k d_k)^T d_k \right| \leq -\sigma g_k^T d_k. \end{gathered} \tag{2.1}$$

Step 3. Let $x_{k+1} = x_k + \alpha_k d_k$, $g_{k+1} = g(x_{k+1})$, if $\|g_{k+1}\| \leq \varepsilon$, then stop.

Step 4. Compute d_{k+1} by (1.3), and generate β_{k+1} by

$$\beta_{k+1} = \max\left\{ 0, \frac{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|}{u |g_{k+1}^T d_k| - g_k^T d_k} - \frac{\nu g_{k+1}^T s_{k+1}}{u |g_{k+1}^T d_k| - g_k^T d_k} \right\}, \tag{2.2}$$

where $s_{k+1} = x_{k+1} - x_k$, $u \geq 1$, $\nu \geq 0$.

Step 5. Set $k = k + 1$, go to Step 2.

Theorem 2.1. *Let the sequences* $\{g_k\}$ *and* $\{d_k\}$ *be generated by* MLS *method, then*

$$g_k^T d_k < -(1-\sigma)\|g_k\|^2, \quad \forall k \in N^+. \tag{2.3}$$

Proof. The conclusion can be proved by induction. Since $-g_1^T d_1/\|g_1\|^2 = 1$, the conclusion holds for $k = 1$. Now, we assume that the conclusion is true for $k-1$, for $k \geq 2$. We need to prove that the conclusion holds for k.

Multiplying (1.3) by g_k^T, we have

$$g_k^T d_k = -\|g_k\|^2 + \beta_k g_k^T d_{k-1}. \tag{2.4}$$

From (2.2), if $\beta_k = 0$, then $g_k^T d_k = -\|g_k\|^2 \leq -(1-\sigma)\|g_k\|^2$; if $\beta_k = (\|g_k\|^2 - |g_k^T g_{k-1}|)/(u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}) - \nu g_k^T s_k/(u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}) > 0$, the proof is divided into two parts.

(i) If $g_k^T d_{k-1} \leq 0$, then we have $g_k^T d_k \leq -\|g_k\|^2 \leq -(1-\sigma)\|g_k\|^2$.

If $g_k^T d_{k-1} > 0$, then we get

$$\begin{aligned}
g_k^T d_k &= -\|g_k\|^2 + \left(\frac{\|g_k\|^2 - |g_k^T g_{k-1}|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} - \frac{\nu g_k^T s_k}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}}\right) \cdot g_k^T d_{k-1} \\
&\leq -\|g_k\|^2 - \frac{\nu \alpha_{k-1}(g_k^T d_{k-1})^2}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} - \frac{|g_k^T g_{k-1}|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \cdot g_k^T d_{k-1} + \frac{\|g_k\|^2}{-g_{k-1}^T d_{k-1}} \cdot g_k^T d_{k-1} \\
&\leq -\|g_k\|^2 - \frac{\nu \alpha_{k-1}(g_k^T d_{k-1})^2}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} - \frac{|g_k^T g_{k-1}|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \cdot g_k^T d_{k-1} \\
&\quad + \frac{\|g_k\|^2}{-g_{k-1}^T d_{k-1}} \cdot \left(-\sigma g_{k-1}^T d_{k-1}\right) \\
&= -(1-\sigma)\|g_k\|^2 - \frac{\nu \alpha_{k-1}(g_k^T d_{k-1})^2}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} - \frac{|g_k^T g_{k-1}|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \cdot g_k^T d_{k-1} \\
&< -(1-\sigma)\|g_k\|^2.
\end{aligned} \tag{2.5}$$

From the above inequalities, we obtain that the conclusion holds for k. □

3. The Global Convergence Properties

In order to prove the global convergence of the MLS method, we assume that the objective function f satisfies the following assumption.

Assumption H. (i) The level set $L = \{x \in R^n \mid f(x) \leq f(x_1)\}$ is bounded, where x_1 is the starting point.

(ii) In a neighborhood V of L, f is continuously differentiable and its gradient g is Lipschitz continuous, namely, there exists a constant $M > 0$ such that

$$\|g(x) - g(y)\| \leq M\|x - y\|, \quad \forall x, y \in V. \tag{3.1}$$

From Assumption H, there exists a constant $\tilde{r} > 0$, such that

$$\|g(x)\| \leq \tilde{r} \quad \forall x \in V. \tag{3.2}$$

Firstly, we prove that the MLS method has the strong global convergence property for uniformly convex functions.

Lemma 3.1 (see [4]). *Suppose Assumption H holds. Consider any iteration of the form* (1.2)-(1.3), *where d_k satisfies $g_k^T d_k < 0$ for $k \in N^+$ and α_k satisfies the strong Wolfe line search. If*

$$\sum_{k \geq 1} \frac{1}{\|d_k\|^2} = +\infty, \tag{3.3}$$

then

$$\lim_{k \to +\infty} \inf \|g_k\| = 0. \tag{3.4}$$

Theorem 3.2. *Suppose Assumption H holds. Let the sequences $\{g_k\}$ and $\{d_k\}$ be generated by* MLS *method. If $f(x)$ is a uniformly convex function, that is, there exists $t > 0$, for all $x, y \in L$, subject to*

$$(g(x) - g(y))^T (x - y) \geq t\|x - y\|^2, \tag{3.5}$$

then

$$\lim_{k \to +\infty} \|g_k\| = 0. \tag{3.6}$$

Proof. From $f(x)$ is uniformly convex function and $u \geq 1$, we have

$$\begin{aligned} u\left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1} &\geq \left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1} \geq g_k^T d_{k-1} - g_{k-1}^T d_{k-1} \\ &= d_{k-1}^T (g_k - g_{k-1}) \geq t\alpha_{k-1}\|d_{k-1}\|^2. \end{aligned} \tag{3.7}$$

From Lipschitz condition and (1.2), we have

$$\begin{aligned} \left| \|g_k\|^2 - \left|g_k^T g_{k-1}\right| \right| &\leq \left| \|g_k\|^2 - g_k^T g_{k-1} \right| \\ &\leq \|g_k\| \cdot \|g_k - g_{k-1}\| \\ &\leq \|g_k\| \cdot M\alpha_{k-1} \cdot \|d_{k-1}\|. \end{aligned} \tag{3.8}$$

Then, from (1.3), (2.2), and (3.2), we have

$$\begin{aligned}
\|d_k\| &\le \|g_k\| + \left| \frac{\|g_k\|^2 - |g_k^T g_{k-1}|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} - \frac{\nu g_k^T s_k}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \right| \cdot \|d_{k-1}\| \\
&\le \|g_k\| + \frac{\left| \|g_k\|^2 - |g_k^T g_{k-1}| - \nu g_k^T s_k \right|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \cdot \|d_{k-1}\| \\
&\le \|g_k\| + \frac{\left| \|g_k\|^2 - g_k^T g_{k-1} \right| + |\nu g_k^T s_k|}{u|g_k^T d_{k-1}| - g_{k-1}^T d_{k-1}} \cdot \|d_{k-1}\| \qquad (3.9)\\
&\le \|g_k\| + \frac{\|g_k\| \cdot M\alpha_{k-1} \cdot \|d_{k-1}\| + \nu\|g_k\| \cdot \alpha_{k-1} \cdot \|d_{k-1}\|}{t\alpha_{k-1}\|d_{k-1}\|^2} \cdot \|d_{k-1}\| \\
&\le \tilde{r}\left(1 + \frac{M+\nu}{t}\right).
\end{aligned}$$

From the above inequality, we obtain that the conclusion (3.3) holds. Then, from Lemma 3.1, the conclusion (3.4) holds; and $f(x)$ is uniformly convex function, so the conclusion (3.6) holds.

Secondly, we prove that the MLS method has the global convergence for ordinary functions. □

Lemma 3.3 (see [5]). *Suppose Assumption H holds. Let the sequence $\{x_k\}$ be generated by the iteration of the form (1.2)-(1.3), where d_k satisfies $g_k^T d_k < 0$ for $k \in N^+$ and α_k satisfies the strong Wolfe line search. Then,*

$$\sum_{k\ge1} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < +\infty. \qquad (3.10)$$

Lemma 3.4 (see [6]). *Suppose Assumption H holds. Consider any iteration of the form (1.2)-(1.3), where $\beta_k \ge 0$, and α_k satisfies the strong Wolfe line search and (2.3) holds. If there exists a constant $r > 0$, such that*

$$\|g_k\| \ge r, \quad \forall k \ge 1, \qquad (3.11)$$

then

$$\sum_{k\ge2} \|u_k - u_{k-1}\|^2 < +\infty, \qquad (3.12)$$

where $u_k = d_k/\|d_k\|$.

Lemma 3.5. *Consider any iteration of the form (1.2)-(1.3), where β_k satisfies (2.2) and α_k satisfies the strong Wolfe line search. Suppose that*

$$0 < r \le \|g_k\| \le \tilde{r}, \quad \forall k \ge 1. \qquad (3.13)$$

Say that the MLS method has property (*), that is,

(1) *if there exists constant $b > 1$, such that $|\beta_k^*| \le b$,*

(2) *if there exists constant $\lambda > 0$, such that $\|x_k - x_{k-1}\| \le \lambda$, one has $|\beta_k^*| \le 1/2b$.*

Proof. Firstly, from the strong Wolfe line search, (2.3) and (3.13), we have

$$\begin{aligned} u\left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1} &\geq \left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1} \\ &\geq g_k^T d_{k-1} - g_{k-1}^T d_{k-1} \\ &\geq (\sigma - 1) g_{k-1}^T d_{k-1} \\ &\geq (1-\sigma)^2 \left\|g_{k-1}\right\|^2 \\ &\geq (1-\sigma)^2 r^2. \end{aligned} \tag{3.14}$$

From the Assumption H (i), there exists a positive constant a, such that $\|x\| \leq a$, for all $x \in L$. So, from (2.3) and Lipschitz condition, we have

$$\begin{aligned} \left|\beta_k\right| &\leq \frac{\left|\left\|g_k\right\|^2 - \left|g_k^T g_{k-1}\right|\right|}{u\left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1}} + \frac{\left|\nu g_k^T s_k\right|}{u\left|g_k^T d_{k-1}\right| - g_{k-1}^T d_{k-1}} \\ &\leq \frac{\left|\left\|g_k\right\|^2 - g_k^T g_{k-1}\right|}{(1-\sigma)^2 r^2} + \frac{\left|\nu g_k^T s_k\right|}{(1-\sigma)^2 r^2} \\ &\leq \frac{\left\|g_k\right\| \cdot \left\|g_k - g_{k-1}\right\|}{(1-\sigma)^2 r^2} + \frac{\nu \left\|g_k\right\| \cdot \|s_k\|}{(1-\sigma)^2 r^2} \\ &\leq \frac{(M+\nu)\left\|g_k\right\| \cdot \|s_k\|}{(1-\sigma)^2 r^2} \leq \frac{2a\widetilde{r}(M+\nu)}{(1-\sigma)^2 r^2} = b. \end{aligned} \tag{3.15}$$

Define $\lambda = (1-\sigma)^2 r^2 / 2b\widetilde{r}(M+\nu)$. Let $\|x_k - x_{k-1}\| \leq \lambda$, then from the above inequality, we also have

$$\left|\beta_k\right| \leq \frac{\lambda \widetilde{r}(M+\nu)}{(1-\sigma)^2 r^2} = \frac{1}{2b}. \tag{3.16}$$

□

Lemma 3.6 (see [6]). *Suppose Assumption H holds. Consider any iteration of the form* (1.2)-(1.3), *where* $\beta_k \geq 0$, *and* α_k *satisfies the strong Wolfe line search and* (2.3) *holds. If* β_k *has the property (*), and if there exists a constant* $r > 0$, *subject to*

$$\left\|g_k\right\| \geq r \quad \forall k \in N^+, \tag{3.17}$$

then there exits $\lambda > 0$, *for any* $\Delta \in Z^+$ *and* $k_0 \in Z^+$, *and for all* $k \geq k_0$, *such that*

$$\left|\Re_{k,\Delta}^{\lambda}\right| > \frac{\Delta}{2}, \tag{3.18}$$

where $\Re_{k,\Delta}^{\lambda} \triangleq \{i \in Z^+ : k \leq i \leq k + \Delta - 1, \|x_i - x_{i-1}\| \geq \lambda\}$, $|\Re_{k,\Delta}^{\lambda}|$ *denotes the number of the* $\Re_{k,\Delta}^{\lambda}$.

Lemma 3.7 (see [6]). *Suppose Assumption H holds. Consider any iteration of the form* (1.2)-(1.3), *where $\beta_k \geq 0$, and α_k satisfies the strong Wolfe line search and* (2.3) *holds. If β_k has the property (*), then*

$$\liminf_{k \to +\infty} \|g_k\| = 0. \tag{3.19}$$

From above Lemmas, we also have the following convergence results, that is, MLS method has the global convergence for ordinary functions.

Theorem 3.8. *Suppose Assumption H holds. Consider the method* (1.2)-(1.3), *where β_k is computed by* (2.2), *and α_k satisfies the strong Wolfe line search and* (2.3) *holds, then*

$$\liminf_{k \to +\infty} \|g_k\| = 0. \tag{3.20}$$

4. Numerical Results

In this section, we test the MLS method for problems from [7], and we compare its performance to that of the LS method, LS1 method, and LS2 method under the strong Wolfe line search. The parameters $\delta = 0.01$, $\sigma = 0.1$, $\varsigma = 1.25$, $\rho = 1.5$, $\xi = 0.001$, $u = 1.0$, $\nu = 0.35$. The termination condition is $\|g_k\| \leq 10^{-6}$ or It-max >9999. It-max denotes the Maximum number of iterations.

The numerical results of our tests are reported in Table 1. The column "Problem" represents the problem's name in [7]. "Dim" denotes the dimension of the test problems. The detailed numerical results are listed in the form NI/NF/NG, where NI, NF, NG denote the number of iterations, function evaluations, and gradient evaluations, respectively. "NaN" means the calculation failure.

In order to rank the average performance of all the above conjugate gradient methods, one can compute the total number of function and gradient evaluation by the following formula:

$$N_{\text{total}} = \text{NF} + l * \text{NG}, \tag{4.1}$$

where l is some integer. According to the results on automatic differentiation [8, 9], the value of l can be set to 5. That is to say, one gradient evaluation is equivalent to five function evaluations if automatic differentiation is used.

By (4.1), we compare the MLS method with LS method, LS1 method, and LS2 method as follows: for the ith problem, compute the total numbers of function evaluations and gradient evaluations required by the MLS method, LS method, LS1 method, and LS2 method, and denote them as $N_{\text{total},i}(\text{MLS})$, $N_{\text{total},i}(\text{LS})$, $N_{\text{total},i}(\text{LS1})$, and $N_{\text{total},i}(\text{LS2})$, respectively. Then, we calculate the ratio:

$$\begin{aligned} \gamma_i(\text{LS}) &= \frac{N_{\text{total},i}(\text{LS})}{N_{\text{total},i}(\text{MLS})}, \\ \gamma_i(\text{LS1}) &= \frac{N_{\text{total},i}(\text{LS1})}{N_{\text{total},i}(\text{MLS})}, \\ \gamma_i(\text{LS2}) &= \frac{N_{\text{total},i}(\text{LS2})}{N_{\text{total},i}(\text{MLS})}. \end{aligned} \tag{4.2}$$

Table 1: The numerical results of LS method, LS1 method, LS2 method, and MLS method.

Problem	Dim	LS	LS1	LS2	MLS
ROSE	2	25/101/77	25/125/98	37/163/138	26/125/103
FROTH	2	10/53/36	15/85/68	12/78/60	12/78/62
BADSCP	2	96/399/338	22/183/168	27/231/214	22/170/156
BADSCB	2	14/70/52	13/106/96	11/88/79	19/148/136
BEALE	2	12/47/33	15/59/46	12/56/43	18/68/53
JENSAM	8	10/40/17	NaN/NaN/NaN	11/51/28	12/51/28
HELIX	3	80/217/185	39/125/106	28/84/71	37/128/111
BRAD	3	18/67/52	28/98/81	17/57/46	21/75/60
SING	4	841/2515/2237	93/357/312	41/168/143	95/345/303
WOOD 14	4	125/413/341	43/189/153	33/161/129	57/237/200
KOWOSB	4	75/216/189	85/300/267	45/150/132	66/249/217
BD	4	72/238/193	27/136/105	24/126/93	30/144/109
BIGGS	6	171/445/395	201/754/664	187/559/500	143/520/462
OSB2	11	252/609/548	585/1562/1394	272/681/619	638/1694/1508
VARDIM	5	6/57/38	6/57/38	6/57/38	6/57/38
	10	7/81/52	7/81/52	7/81/52	7/81/52
WATSON	5	141/408/353	110/359/311	91/295/250	100/329/282
	15	4486/12738/11326	2045/7346/6517	1465/5138/4540	1769/6365/5648
PEN2	50	536/1835/1580	182/851/741	163/809/694	177/876/765
	100	72/220/185	79/252/208	76/217/186	103/300/251
PEN1	100	18/120/83	31/195/153	30/194/151	27/184/140
	200	18/157/114	30/209/159	29/211/160	30/208/157
TRIG	100	57/125/115	60/143/135	53/109/102	60/143/133
	200	68/163/155	61/138/125	61/136/128	50/107/99
ROSEX	500	25/101/77	26/127/99	37/163/138	27/131/109
	1000	25/101/77	26/127/99	37/163/138	27/131/109
SINGX	500	215/712/618	120/483/428	63/281/248	97/351/306
	1000	105/344/295	119/503/447	63/281/248	100/408/363
BV	500	1940/3247/3246	148/358/325	1722/3057/3056	135/339/313
	1000	214/347/346	16/32/30	158/273/272	16/35/33
IE	500	7/15/8	6/13/7	6/13/7	6/13/7
	1000	7/15/8	6/13/7	6/13/7	6/13/7
TRID	500	35/78/74	31/71/57	35/78/73	31/71/59
	1000	34/76/72	35/79/75	34/76/72	35/79/75

If the i_0th problem is not run by the method, we use a constant $\lambda = \max\{\gamma_i(\text{method}) \mid i \in S_1\}$ instead of γ_{i_0} (method), where S_1 denotes the set of test problems which can be run by the method.

The geometric mean of these ratios for VLS method over all the test problems are defined by

$$\gamma(\text{LS}) = \left(\prod_{i\in S}\gamma_i(\text{LS})\right)^{1/|S|},$$
$$\gamma(\text{LS1}) = \left(\prod_{i\in S}\gamma_i(\text{LS1})\right)^{1/|S|}, \tag{4.3}$$
$$\gamma(\text{LS2}) = \left(\prod_{i\in S}\gamma_i(\text{LS2})\right)^{1/|S|},$$

Table 2: Relative efficiency of the MLS method, LS method, LS1 method and LS2 method.

MLS	LS	LS1	LS2
1	1.3012	1.2549	1.0580

where S denotes the set of the test problems, and $|S|$ denotes the number of elements in S. One advantage of the above rule is that the comparison is relative and hence does not be dominated by a few problems for which the method requires a great deal of function evaluations and gradient functions.

From the above rule, it is clear that $\gamma(\text{MLS}) = 1$. The values of $\gamma(\text{LS})$, $\gamma(\text{LS1})$, and $\gamma(\text{LS2})$ are computed and listed in Table 2.

From Table 2, it is clear that the MLS method is superior to the LS method and the LS1 method, and it is comparable with the LS2 method for the given test problems. So, the MLS method has certain value of research.

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