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Research Article

Availability Equivalence Analysis of a Repairable Series-Parallel System

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This paper studies the availability equivalence of different designs of a repairable series-parallel system. Under the assumption that the system components have constant failure rates and repair rates, we derive the availability of the original and improved systems according to reduction, increase, hot duplication, warm duplication and cold duplication methods, respectively. The availability equivalence factor is introduced to compare different system designs. Two types of availability equivalence factors of the system are obtained. Numerical examples are provided to interpret how to utilize the obtained results.

1. Introduction

The study of repairable systems is an important topic in reliability. System availability is a very meaningful measure, and achieving a high or required level of availability is an essential requisite. In some engineering systems, the availability depends on the system structure as well as on the component availability. To maintain the availability of sophisticated systems to a higher level, the system's structural design or system components of higher availability should be required, or both of them are performed simultaneously [1]. In reliability analysis, sometimes different system designs should be comparable based on a reliability characteristic such as the reliability function in the case of no repairs [2] or the availability of repairable systems. In general, using components with high availability or increasing redundant components can improve a system design. Sarhan [3] introduced two main methods to improve a nonrepairable series-parallel system design. The first is the reduction method and the second is the redundancy method. It is assumed in the reduction method that the system design can be improved by reducing the failure rates of a set of its components by a factor ρ, $0 < \rho < 1$ [2–9]. The redundancy method is divided into some other types such as hot, warm, and cold redundancy. For a repairable system, the system design can be improved according to the methods mentioned previously, and can also be improved by doubling the maintenance staff in maintenance of its components (which can increase the repair rates) [10].

The system structure is virtually designed under the limitations such as weight, volume, or other technologies, so the availability cannot be further improved by using the redundancy method. In this case, using highly reliable components or doubling the maintenance resource can improve the system availability. In such situations, more emphasis must be placed on robust design, manufacturing quality control, maintenance intensities and on controlling the operating environment. Therefore, Råde [5] introduced the concept of reliability equivalence. In such a concept, the design of the system which is improved according to reduction method should be equivalent to the design of the system improved according to the redundancy method. The comparison of the designs produces the so-called reliability equivalence factors. Råde [6, 7], Sarhan [2–4, 8, 9, 11], and Xia and Zhang [12] investigated various systems by applying such concept. That is, in this concept, one may say that the reliability of a system can be improved through an alternative design. Sarhan [4, 9] provided more general methods of improving the reliability of a system. In such methods, the reliability of a system can be improved by using one of the following four different methods: (1) improving the quality of some components by reducing their failure rates by a factor ρ, $0 < \rho < 1$, (2) assuming warm duplications of some components, (3) assuming cold duplications of some components, and (4) assuming cold redundant standby components connected with some components (one for each) by random switches.

However, these systems in the above-mentioned literatures were investigated in the case of no repairs only and studied reliability equivalence according to the reduction and redundancy methods. In this paper, we focus on the equivalence analysis of different designs of a reparable series-parallel system. Series-parallel system indicates subsystems in which several components are connected in parallel, and then in series, or subsystems that several components are connected in series, and then in parallel [13]. The reliability of series-parallel system has drawn continuous attention in both problem characteristics and solution methodologies [2, 3, 13–18]. The reliability of a series-parallel system can be improved by four methods [19]: (1) use more reliable components, (2) increase redundant components in parallel, (3) utilize both (1) and (2), and (4) enable repeatedly the allocation of entire system framework. Generally, designers may introduce different technologies in a series-parallel system in order to improve the system reliability. For a reparable series-parallel system, the system reliability (availability) can also be improved by doubling the maintenance resource, this method is called as increase method in this paper. It is assumed in the increase method that the system design can be improved by increasing the repair rates of a set of its components by introducing a factor σ, $\sigma > 1$.

Comparable work on repairable series-parallel systems with the concept of availability equivalence is rarely found in the literature. This motivates us to develop the availability equivalence factors of a repairable series-parallel system consisting of some subsystems connected in series, and each subsystem consisting of multiple components connected in parallel. For the series-parallel system in the case of no repairs, Sarhan [11] studied the reliability equivalence factors of this system. In this paper, we consider a repairable series-parallel system. It is assumed that all components of each parallel subsystem are identical with constant failure rates and repair rates. Each parallel subsystem works when at least

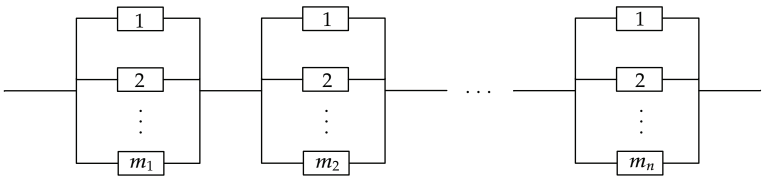

Figure 1: Repairable series-parallel system structure.

one of its components work and the entire system works if and only if all subsystems work. Our aim is to calculate the reliability equivalence factors of this system and to compare the design of the original system and that for the improved systems by using the availability as a performance measure. So, the reliability equivalence factor is called availability equivalence factor in this work.

The structure of this paper is organized as follows. In Section 2, we introduce the description of the system considered here and present the availability of the original system. Section 3 derives availability of the systems improved according to the five different methods. The availability equivalence factors of the system are obtained in Section 4. Numerical results and conclusions are given in Sections 5 and 6, respectively.

2. Repairable Series-Parallel System

The system we consider here consists of n subsystems connected in series, and each subsystem i has m_i components connected in parallel for $i = 1, 2, \ldots, n$. The common structure of the series-parallel system is illustrated in Figure 1. Each parallel subsystem works when at least one of its components works and the entire system works if and only if all subsystems work.

In each subsystem i $(i = 1, 2, \ldots, n)$, the components are assumed to be independent and identical. The life time of each component is exponential with failure rate λ_i, and the repair time of each component is also exponential with repair rate μ_i. Let A_{ij} be the availability of the component j $(j = 1, 2, \ldots, m_i)$ in subsystem i $(i = 1, 2, \ldots, n)$ and let A_i be the availability of the subsystem i. That is, A_{ij} and A_i can be expressed by:

$$A_{ij} = \frac{\mu_i}{\mu_i + \lambda_i} = \frac{1}{1 + \eta_i}, \tag{2.1}$$

$$A_i = 1 - \prod_{j=1}^{m_i} \left(1 - A_{ij}\right) = 1 - \left(\frac{\eta_i}{1 + \eta_i}\right)^{m_i}, \tag{2.2}$$

where $\eta_i = \lambda_i / \mu_i$. Since the subsystems are connected in series, the availability of this system is obtained:

$$A_S = \prod_{i=1}^{n} A_i = \prod_{i=1}^{n} \left[1 - \left(\frac{\eta_i}{1 + \eta_i}\right)^{m_i}\right]. \tag{2.3}$$

3. Different Designs of Improved Systems

In this section, we will present five different designs to improve the repairable series-parallel system as shown in Figure 1, with n parallel subsystems connected in series.

The system can be improved according to one of the following five different methods:

(1) *Reduction Method*. In this method it is assumed that the component can be improved by reducing its failure rate by a factor ρ, $0 < \rho < 1$.

(2) *Increase Method*. It is assumed in this method that the component can be improved by increasing its repair rate by a factor σ, $\sigma > 1$.

(3) *Hot Duplication Method*. This method assumed that the component is duplicated by a hot redundant standby component.

(4) *Warm Duplication Method*. In this method it is assumed that the component is duplicated by a warm redundant standby component.

(5) *Cold Duplication Method*. It is assumed in this method that the component is duplicated by a cold redundant standby component.

In the remainder of this section, the availability of the systems improved according to the methods mentioned previously are derived hereinafter in detail.

3.1. The Reduction Method

In the reduction method, it is assumed that the system can be improved by reducing the failure rates of the set A of system components by a factor ρ $(0 < \rho < 1)$. Here, the set A consists of r $(0 \leq r \leq M)$ components, and $M = \sum_{i=1}^{n} m_i$ denotes the total number of the system components. Furthermore, we assume that the components belonging to A can be distributed into the n subsystems of the system such that r_i components of the subsystem i belong to the set A where $0 \leq r_i \leq m_i$ $(i = 1, 2, \ldots, n)$. That is, $|A| = r = \sum_{i=1}^{n} r_i$. We denote such a set by $A_{|A|}^{(|A_1|,|A_2|,\ldots,|A_n|)}$, where A_i denotes the set of r_i out-of m_i components from subsystem i whose failure rates are reduced, that is, $A = \sum_{i=1}^{n} A_i$ and $|A_i| = r_i$ $(i = 1, 2, \ldots, n)$.

Let $A_{r,\rho}$ denote the availability of the system improved by reducing the failure rates of the set A components by the factor ρ. The availability of component j in the subsystem i after reducing its failure rate λ_i by ρ is

$$A_{ij,\rho} = \frac{\mu_i}{\mu_i + \rho\lambda_i} = \frac{1}{1 + \rho\eta_i}, \tag{3.1}$$

where $\eta_i = \lambda_i/\mu_i$. One can obtain the availability $A_{i,\rho}$ of the subsystem i improved by reduction method to be

$$A_{i,\rho} = 1 - \prod_{j=1}^{r_i} \left(1 - A_{ij,\rho}\right) \prod_{j=1}^{m_i - r_i} \left(1 - A_{ij}\right). \tag{3.2}$$

Using (2.1), (3.1), and (3.2), the availability $A_{i,\rho}$ of the subsystem i can be obtained:

$$A_{i,\rho} = 1 - \left(\frac{\rho\eta_i}{1 + \rho\eta_i}\right)^{r_i} \left(\frac{\eta_i}{1 + \eta_i}\right)^{m_i - r_i}, \tag{3.3}$$

from which we immediately have the availability of the system improved by the reduction method:

$$A_{r,\rho} = \prod_{i=1}^{n} A_{i,\rho} = \prod_{i=1}^{n} \left[1 - \left(\frac{\rho \eta_i}{1 + \rho \eta_i} \right)^{r_i} \left(\frac{\eta_i}{1 + \eta_i} \right)^{m_i - r_i} \right]. \tag{3.4}$$

3.2. The Increase Method

In this subsection, we will compute the availability of the system when it is improved by increasing the repair rates of some of its components by a factor σ, $\sigma > 1$.

Let S denote the set of system components whose repair rates are increased and denote by s their number, so that $|S| = s$ $(0 \le s \le M)$. Since these components may be arbitrarily chosen in the system, let S_i denote the set of s_i out-of m_i components from subsystem i whose repair rates are increased, that is, $|S_i| = s_i$ $(i = 1, 2, \ldots, n)$ and $S = \sum_{i=1}^{n} S_i$ with $s = \sum_{i=1}^{n} s_i$. Here, we denote the set S by $S_{|S|}^{(|S_1|, |S_2|, \ldots, |S_n|)}$.

Let $A_{s,\sigma}$ denote the availability of the system improved by increasing the repair rates of the set S components by the factor σ. The availability of component j in the subsystem i after increasing its repair rate μ_i by σ is

$$A_{ij,\sigma} = \frac{\sigma \mu_i}{\sigma \mu_i + \lambda_i} = \frac{\sigma}{\sigma + \eta_i}, \tag{3.5}$$

where $\eta_i = \lambda_i / \mu_i$. One can obtain the availability $A_{i,\sigma}$ of the subsystem i improved by the increase method to be

$$A_{i,\sigma} = 1 - \prod_{j=1}^{s_i} \left(1 - A_{ij,\sigma} \right) \prod_{j=1}^{m_i - s_i} \left(1 - A_{ij} \right). \tag{3.6}$$

Using (2.1), (3.5) and (3.6), the availability $A_{i,\sigma}$ of the subsystem i can be obtained:

$$A_{i,\sigma} = 1 - \left(\frac{\eta_i}{\sigma + \eta_i} \right)^{s_i} \left(\frac{\eta_i}{1 + \eta_i} \right)^{m_i - s_i}, \tag{3.7}$$

from which we immediately have the availability of the system improved by the increase method:

$$A_{s,\sigma} = \prod_{i=1}^{n} A_{i,\sigma} = \prod_{i=1}^{n} \left[1 - \left(\frac{\eta_i}{\sigma + \eta_i} \right)^{s_i} \left(\frac{\eta_i}{1 + \eta_i} \right)^{m_i - s_i} \right]. \tag{3.8}$$

3.3. The Hot Duplication Method

Let us assume that each component of the set B is duplicated by a hot redundant identical standby component. Here, the set B consists of h $(0 \le h \le M)$ components. In addition, we assume that the components belonging to B can be distributed into the n subsystems of the system such that h_i components of the subsystem i belong to the set B where $0 \le h_i \le m_i$

$(i = 1, 2, \ldots, n)$. That is, $|B| = h = \sum_{i=1}^{n} h_i$. We denote such a set by $B_{|B|}^{(|B_1|,|B_2|,\ldots,|B_n|)}$, where B_i denotes the set of h_i out-of m_i components from subsystem i which are hot duplicated, that is $B = \sum_{i=1}^{n} B_i$ and $|B_i| = h_i$ $(i = 1, 2, \ldots, n)$.

We denote by $A_{B,h}$ the availability of the system improved by improving the components belonging to the B according to the hot duplication method. For this system, when the availability of the subsystem i is improved by improving h_i of its components by hot duplication, we can write the availability $A_{i,h}$ of the subsystem i in the following form:

$$A_{i,h} = 1 - \prod_{j=1}^{m_i+h_i} \left(1 - A_{ij}\right). \tag{3.9}$$

Using (2.1) and (3.9), the availability $A_{i,h}$ of the subsystem i can be written as

$$A_{i,h} = 1 - \left(\frac{\eta_i}{1+\eta_i}\right)^{m_i+h_i}. \tag{3.10}$$

Thus, the availability $A_{B,h}$ of the system improved by the hot duplication method can be derived as follows:

$$A_{B,h} = \prod_{i=1}^{n} A_{i,h} = \prod_{i=1}^{n} \left[1 - \left(\frac{\eta_i}{1+\eta_i}\right)^{m_i+h_i}\right]. \tag{3.11}$$

3.4. The Warm Duplication Method

It is assumed in the warm duplication method that each component of the set B is connected with an warm standby component via a perfect switch. Each warm standby component in the subsystem i has constant standby failure rate ν_i. If we assume that w_i out-of m_i components in the subsystem i are warm duplicated, and we denote by B_i the set of these w_i components, then we have $|B_i| = w_i$ and $B = \sum_{i=1}^{n} B_i$ with $|B| = w = \sum_{i=1}^{n} w_i$ $(0 \leq w \leq M)$. Here, we denote the set B by $B_{|B|}^{(|B_1|,|B_2|,\ldots,|B_n|)}$.

Let $A_{ij,w}$ denote the availability of the component j in the subsystem i when it is improved according to the warm duplication method. The availability $A_{ij,w}$ can be obtained by using Markov process theory [20]:

$$A_{ij,w} = \frac{\mu_i^2 + \lambda_i\mu_i + \nu_i\mu_i}{\mu_i^2 + \lambda_i\mu_i + \nu_i\mu_i + (1/2)\lambda_i^2 + (1/2)\lambda_i\nu_i} = \frac{1 + \eta_i + \xi_i}{1 + \eta_i + \xi_i + (1/2)\eta_i^2 + (1/2)\eta_i\xi_i}, \tag{3.12}$$

where $\eta_i = \lambda_i/\mu_i$ and $\xi_i = \nu_i/\mu_i$.

We denote by $A_{i,w}$ the availability of the subsystem i improved according to the warm duplication method, it can be derived as follows:

$$A_{i,w} = 1 - \prod_{j=1}^{w_i} \left(1 - A_{ij,w}\right) \prod_{j=1}^{m_i-w_i} \left(1 - A_{ij}\right). \tag{3.13}$$

Using (2.1), (3.12) and (3.13), $A_{i,w}$ can be written as:

$$A_{i,w} = 1 - \left(\frac{(1/2)\eta_i^2 + (1/2)\eta_i \xi_i}{1 + \eta_i + \xi_i + (1/2)\eta_i^2 + (1/2)\eta_i \xi_i} \right)^{w_i} \left(\frac{\eta_i}{1 + \eta_i} \right)^{m_i - w_i} \tag{3.14}$$

from which we immediately have the availability of the system improved by the warm duplication method:

$$A_{B,w} = \prod_{i=1}^{n} A_{i,w} = \prod_{i=1}^{n} \left[1 - \left(\frac{(1/2)\eta_i^2 + (1/2)\eta_i \xi_i}{1 + \eta_i + \xi_i + (1/2)\eta_i^2 + (1/2)\eta_i \xi_i} \right)^{w_i} \left(\frac{\eta_i}{1 + \eta_i} \right)^{m_i - w_i} \right]. \tag{3.15}$$

3.5. The Cold Duplication Method

It is assumed, in the cold duplication method, that each component of the set B is connected with an identical component via a perfect switch. Here, the set B consists of c $(0 \le c \le M)$ components, so $|B| = c$. That is, the set B can be written as a union of n disjoint subsets $B_1, B_2, \ldots, B_n$ such that the subset B_i contains c_i components belonging to the subsystem i $(i = 1, 2, \ldots, n)$. Then, we have $|B_i| = c_i$ and $B = \sum_{i=1}^{n} B_i$ with $c = \sum_{i=1}^{n} c_i$. We denote the set B by $B_{|B|}^{(|B_1|,|B_2|,\ldots,|B_n|)}$.

Let $A_{B,c}$ denote the availability of the system improved by improving the components belonging to the B according to the cold duplication method. For this system, when the availability of the subsystem i is improved by improving c_i of its components according to cold duplication method, the availability of the improved system $A_{B,c}$ can be obtain as follows:

$$A_{B,c} = \prod_{i=1}^{n} A_{i,c}, \tag{3.16}$$

where $A_{i,c}$ denotes the availability of the subsystem i improved according to the cold duplication method, and we have

$$A_{i,c} = 1 - \prod_{j=1}^{c_i} (1 - A_{ij,c}) \prod_{j=1}^{m_i - c_i} (1 - A_{ij}). \tag{3.17}$$

Here, $A_{ij,c}$ is the availability of the component j in the subsystem i when it is improved according to the cold duplication method. The Markov process theory can be used to determine the availability $A_{ij,c}$ [21]:

$$A_{ij,c} = \frac{\mu_i^2 + \lambda_i \mu_i}{\mu_i^2 + \lambda_i \mu_i + (1/2)\lambda_i^2} = \frac{1 + \eta_i}{1 + \eta_i + (1/2)\eta_i^2}, \tag{3.18}$$

where $\eta_i = \lambda_i / \mu_i$.

Using (2.1), (3.16), (3.17), and (3.18), the availability $A_{B,c}$ of the improved system can be derived in the following form:

$$A_{B,c} = \prod_{i=1}^{n}\left[1-\left(\frac{(1/2)\eta_i^2}{1+\eta_i+(1/2)\eta_i^2}\right)^{c_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-c_i}\right]. \tag{3.19}$$

4. Availability Equivalence Factors

In this section, we present the availability equivalence factors of the repairable series-parallel system studied here. According to the literatures [2–4, 8, 9, 12], we introduce the following definition.

Definition 4.1. A availability equivalence factor is defined as the factor by which the failure rates (the repair rates) of some of the system's components should be reduced (increased) in order to reach equality of the availability of another better system.

Next, two types of availability equivalence factors will be introduced. These types are called availability equivalence factor I and availability equivalence factor II, respectively.

The availability equivalence factor I, say $\rho = \rho_{A,B}^{d}$, $d = h(w,c)$ for hot (warm, cold), is defined as that factor ρ by which the failure rates of the set A components should be reduced in order to reach the availability of that system which improved by improving some of the original system's component according to hot (warm, cold) duplications of the set B components. That is, $\rho = \rho_{A,B}^{d}$ is the solution of the following equation:

$$A_{r,\rho} = A_{B,d} \qquad d = h, w, c \tag{4.1}$$

with respect to ρ.

Therefore, taking (3.4) and (3.11) in (4.1), we have the following nonlinear equation with respect to ρ:

$$\prod_{i=1}^{n}\left[1-\left(\frac{\rho\eta_i}{1+\rho\eta_i}\right)^{r_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-r_i}\right] = \prod_{i=1}^{n}\left[1-\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i+h_i}\right]. \tag{4.2}$$

Similarly, applying (3.4) and (3.15) to (4.1), we have

$$\begin{aligned}&\prod_{i=1}^{n}\left[1-\left(\frac{\rho\eta_i}{1+\rho\eta_i}\right)^{r_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-r_i}\right]\\&\quad=\prod_{i=1}^{n}\left[1-\left(\frac{(1/2)\eta_i^2+(1/2)\eta_i\xi_i}{1+\eta_i+\xi_i+(1/2)\eta_i^2+(1/2)\eta_i\xi_i}\right)^{w_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-w_i}\right].\end{aligned} \tag{4.3}$$

Applying (3.4) and (3.19) to (4.1), we have

$$\prod_{i=1}^{n}\left[1-\left(\frac{\rho\eta_i}{1+\rho\eta_i}\right)^{r_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-r_i}\right] = \prod_{i=1}^{n}\left[1-\left(\frac{(1/2)\eta_i^2}{1+\eta_i+(1/2)\eta_i^2}\right)^{c_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-c_i}\right]. \tag{4.4}$$

As it seems, (4.2), (4.3), and (4.4) have no closed-form solutions. Therefore, a numerical technique method is needed to get their solutions.

The availability equivalence factor II, say $\sigma = \sigma_{S,B}^{d}$, $d = h(w,c)$ for hot (warm, cold) is defined as that factor σ by which the repair rates of the set S components should be increased in order to reach the availability of that system which improved by improving some of the original system's component according to hot (warm, cold) duplications of the set B components. That is, $\sigma = \sigma_{S,B}^{d}$ is the solution of the following equation:

$$A_{s,\sigma} = A_{B,d} \qquad d = h, w, c \tag{4.5}$$

with respect to σ.

Substituting from (3.8) and (3.11) into (4.5), one gets that

$$\prod_{i=1}^{n}\left[1-\left(\frac{\eta_i}{\sigma+\eta_i}\right)^{s_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-s_i}\right] = \prod_{i=1}^{n}\left[1-\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i+h_i}\right], \tag{4.6}$$

where (4.6) is a non-linear equation with respect to σ.

Similarly, using (3.8), (3.15) and (4.5), we have

$$\begin{aligned}&\prod_{i=1}^{n}\left[1-\left(\frac{\eta_i}{\sigma+\eta_i}\right)^{s_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-s_i}\right]\\ &\quad=\prod_{i=1}^{n}\left[1-\left(\frac{(1/2)\eta_i^2+(1/2)\eta_i\xi_i}{1+\eta_i+\xi_i+(1/2)\eta_i^2+(1/2)\eta_i\xi_i}\right)^{w_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-w_i}\right].\end{aligned} \tag{4.7}$$

Using (3.8), (3.19) and (4.5), we have

$$\prod_{i=1}^{n}\left[1-\left(\frac{\eta_i}{\sigma+\eta_i}\right)^{s_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-s_i}\right] = \prod_{i=1}^{n}\left[1-\left(\frac{(1/2)\eta_i^2}{1+\eta_i+(1/2)\eta_i^2}\right)^{c_i}\left(\frac{\eta_i}{1+\eta_i}\right)^{m_i-c_i}\right]. \tag{4.8}$$

The above (4.6), (4.7) and (4.8) have no closed-form solutions in σ. Thus, to find out σ, we have to use a numerical technique method to solve the above equations.

5. Numerical Results

Some numerical results are given in this section to illustrate how to interpret the theoretical results previously obtained. We consider a repairable series-parallel system (as shown in Figure 1) consisting of $M = 3$ components with $n = 2$, $m_1 = 1$, and $m_2 = 2$. The parameters λ_i, μ_i, ν_i, η_i, and ξ_i $(i = 1,2)$ for each subsystem are presented in Table 1. The objective is to improve the repairable series-parallel system by improving the performance of some components instead of increasing the number of these components.

We first present two numerical results to show the availability of the system improved by improving some sets of components according to the reduction method by the factor ρ

Table 1: Parameters of the subsystem i ($i = 1, 2$).

λ_1	μ_1	ν_1	$\eta_1 = \lambda_1/\mu_1$	$\xi_1 = \nu_1/\mu_1$
0.1	1.0	0.04	0.1	0.04
λ_2	μ_2	ν_2	$\eta_2 = \lambda_2/\mu_2$	$\xi_2 = \nu_2/\mu_2$
0.24	1.2	0.12	0.2	0.1

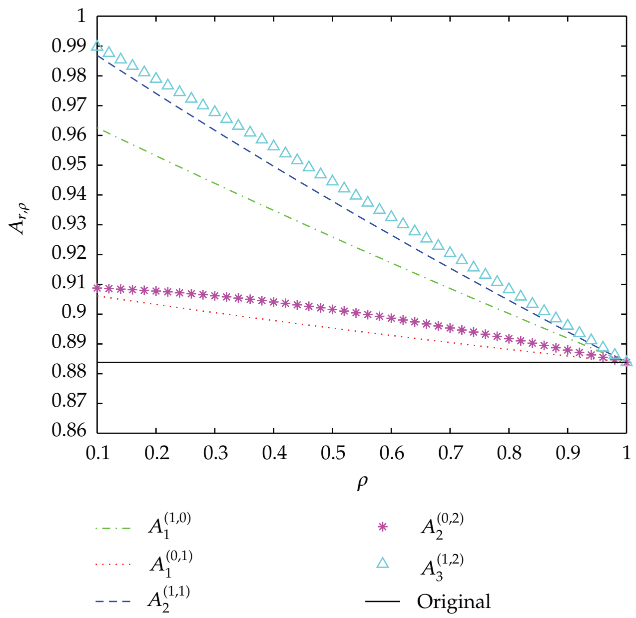

Figure 2: The availability of the system improved according to the reduction method.

($0 < \rho < 1$) and the increase method by the factor σ ($\sigma > 1$), respectively. The numerical results are depicted in Figures 2 and 3.

From the curves of Figures 2 and 3, we can see that the availability $A_{r,\rho}$ decreases with increasing ρ for all possible sets A and $A_{s,\sigma}$ increases with increasing σ for all possible sets S, and improving one component according to the reduction method or the increase method from the subsystem 1 with a smaller number of components gives a modified system with a higher availability than that of the system modified by improving one component according to the same method from the subsystem 2 with a larger number of components. It can also be seen from the two figures that improving two components selected from the subsystems 1 and 2 according to the reduction method or the increase method produces a better system than the system improved by improving two components selected from the subsystem 2. Moreover, the two figures show that improving all components of the system according to the reduction method or the increase method gives the best system.

Next, we give the values of availability of the original system and of the design obtained using the duplication methods for the example considered in this section. The availability A_S of the original system is 0.8838. Table 2 presents the availability of the system improved by the hot, warm, and cold duplication methods for all possible sets $B = B_{|B|}^{(|B_1|,|B_2|)}$. It seems from the results shown in Table 2 that: (1) $A_{B,h} < A_{B,w} < A_{B,c}$ for all possible sets B, (2) improving one component according to the duplication methods from the subsystem 1 with

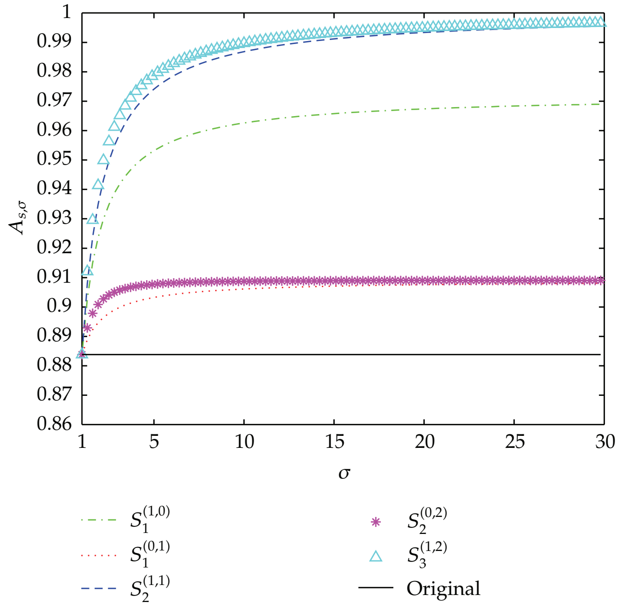

Figure 3: The availability of the system improved according to the increase method.

Table 2: The availability of the improved system according to the hot, warm, and cold duplication methods.

B	$A_{B,h}$	$A_{B,w}$	$A_{B,c}$
$B_1^{(1,0)}$	0.9642	0.9663	0.9678
$B_1^{(0,1)}$	0.9049	0.9057	0.9066
$B_2^{(1,1)}$	0.9871	0.9902	0.9928
$B_2^{(0,2)}$	0.9084	0.9086	0.9088
$B_3^{(1,2)}$	0.9910	0.9934	0.9952

a lower number of components gives a better design than what can be designed by improving one component according to the same methods from the subsystem 2 with a higher number of components, (3) improving two components selected from the subsystems 1 and 2 according to the duplication methods produces a better design than what can be designed by improving two components selected from the subsystem 2, and (4) improving all components of the system according to the duplication methods gives the best design.

Finally, the availability equivalence factor I $\rho^d_{A,B}$, $d = (h,w,c)$ and the availability equivalence factor II $\sigma^d_{S,B}$, $d = (h,w,c)$ for different sets $A = A^{(|A_1|,|A_2|)}_{|A|}$, $S = S^{(|S_1|,|S_2|)}_{|S|}$ and $B = B^{(|B_1|,|B_2|)}_{|B|}$ are calculated using Mathematica program system according to the previous theoretical formulae. Tables 3–5 give $\rho^d_{A,B}(d = h,w,c)$ and Tables 6–8 give $\sigma^d_{S,B}$ $(d = h,w,c)$, respectively. The negative values in Tables 3–8 mean that the availability of the system improved by the reduction method (the increase method) can not be increased to be equal to the availability of the system improved by the duplication methods.

Table 3: The availability equivalence factor $\rho^h_{A,B}$ for different sets A and B.

A	B $B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$A_1^{(1,0)}$	0.0832	0.7440	−ve	0.7026	−ve
$A_1^{(0,1)}$	−ve	0.1422	−ve	0.0229	−ve
$A_2^{(1,1)}$	0.2797	0.7976	0.0982	0.7649	0.0682
$A_2^{(0,2)}$	−ve	0.3642	−ve	0.1417	−ve
$A_3^{(1,2)}$	0.3313	0.8279	0.1247	0.7993	0.0878

Table 4: The availability equivalence factor $\rho^w_{A,B}$ for different sets A and B.

A	B $B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$A_1^{(1,0)}$	0.0613	0.7345	−ve	0.7002	−ve
$A_1^{(0,1)}$	−ve	0.1145	−ve	0.0163	−ve
$A_2^{(1,1)}$	0.2627	0.7901	0.0743	0.7631	0.0499
$A_2^{(0,2)}$	−ve	0.3252	−ve	0.1190	−ve
$A_3^{(1,2)}$	0.3129	0.8214	0.0954	0.7976	0.0648

According to the results summarized in Tables 2–8, we can say the following.

(1) Hot duplication of one component belonging to the subsystem 1 increases the system availability from 0.8838 to 0.9642 (see Table 2). The design with $A_{B,h} = 0.9642$ can be obtained by doing one of the following: (a) reducing the failure rate of each component belonging to the set $A_1^{(1,0)}$ by the factor $\rho^h_{A_1^{(1,0)},B_1^{(1,0)}} = 0.0832$, (b) reducing the failure rate of each component belonging to the set $A_2^{(1,1)}$ by the factor $\rho^h_{A_2^{(1,1)},B_1^{(1,0)}} = 0.2797$, (c) reducing the failure rate of each component belonging to the set $A_3^{(1,2)}$ by the factor $\rho^h_{A_3^{(1,2)},B_1^{(1,0)}} = 0.3313$ (see Table 3), (a$'$) increasing the repair rate of each component belonging to the set $S_1^{(1,0)}$ by the factor $\sigma^h_{S_1^{(1,0)},B_1^{(1,0)}} = 12.0191$, (b$'$) increasing the repair rate of each component belonging to the set $S_2^{(1,1)}$ by the factor $\sigma^h_{S_2^{(1,1)},B_1^{(1,0)}} = 3.5751$, and (c$'$) increasing the repair rate of each component belonging to the set $S_3^{(1,2)}$ by the factor $\sigma^h_{S_3^{(1,2)},B_1^{(1,0)}} = 3.0189$ (see Table 6).

(2) Warm duplication of one component belonging to the subsystem 1 increases the system availability from 0.8838 to 0.9663 (see Table 2). The design with $A_{B,w} = 0.9663$ can be obtained by doing one of the following: (a) reducing the failure rate of each component belonging to the set $A_1^{(1,0)}$ by the factor $\rho^w_{A_1^{(1,0)},B_1^{(1,0)}} = 0.0613$,

Table 5: The availability equivalence factor $\rho^c_{A,B}$ for different sets A and B.

A	B $B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$A_1^{(1,0)}$	0.0457	0.7238	−ve	0.6979	−ve
$A_1^{(0,1)}$	−ve	0.0836	−ve	0.0096	−ve
$A_2^{(1,1)}$	0.2505	0.7817	0.0544	0.7612	0.0362
$A_2^{(0,2)}$	−ve	0.2762	−ve	0.0911	−ve
$A_3^{(1,2)}$	0.2997	0.8140	0.0706	0.7960	0.0473

Table 6: The availability equivalence factor $\sigma^h_{S,B}$ for different sets S and B.

S	B $B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$S_1^{(1,0)}$	12.0191	1.3441	−ve	1.4233	−ve
$S_1^{(0,1)}$	−ve	7.0307	−ve	43.6596	−ve
$S_2^{(1,1)}$	3.5751	1.2538	10.1862	1.3073	14.6650
$S_2^{(0,2)}$	−ve	2.7456	−ve	7.0548	−ve
$S_3^{(1,2)}$	3.0189	1.2078	8.0200	1.2511	11.3880

(b) reducing the failure rate of each component belonging to the set $A_2^{(1,1)}$ by the factor $\rho^w_{A_2^{(1,1)},B_1^{(1,0)}} = 0.2627$, (c) reducing the failure rate of each component belonging to the set $A_3^{(1,2)}$ by the factor $\rho^w_{A_3^{(1,2)},B_1^{(1,0)}} = 0.3129$ (see Table 4), (a$'$) increasing the repair rate of each component belonging to the set $S_1^{(1,0)}$ by the factor $\sigma^w_{S_1^{(1,0)},B_1^{(1,0)}} = 16.3165$, (b$'$) increasing the repair rate of each component belonging to the set $S_2^{(1,1)}$ by the factor $\sigma^w_{S_2^{(1,1)},B_1^{(1,0)}} = 3.8071$, and (c$'$) increasing the repair rate of each component belonging to the set $S_3^{(1,2)}$ by the factor $\sigma^w_{S_3^{(1,2)},B_1^{(1,0)}} = 3.1963$ (see Table 7).

(3) Cold duplication of one component belonging to the subsystem 1 increases the system availability from 0.8838 to 0.9678 (see Table 2). The design with $A_{B,c} = 0.9678$ can be obtained by doing one of the following: (a) reducing the failure rate of each component belonging to the set $A_1^{(1,0)}$ by the factor $\rho^c_{A_1^{(1,0)},B_1^{(1,0)}} = 0.0457$, (b) reducing the failure rate of each component belonging to the set $A_2^{(1,1)}$ by the factor $\rho^c_{A_2^{(1,1)},B_1^{(1,0)}} = 0.2505$, (c) reducing the failure rate of each component belonging to the set $A_3^{(1,2)}$ by the factor $\rho^c_{A_3^{(1,2)},B_1^{(1,0)}} = 0.2997$ (see Table 5), (a$'$) increasing the repair rate of each component belonging to the set $S_1^{(1,0)}$ by the factor $\sigma^c_{S_1^{(1,0)},B_1^{(1,0)}} = 21.8849$,

Table 7: The availability equivalence factor $\sigma^{w}_{S,B}$ for different sets S and B.

S	B				
	$B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$S_1^{(1,0)}$	16.3165	1.3615	−ve	1.4281	−ve
$S_1^{(0,1)}$	−ve	8.7366	−ve	61.5284	−ve
$S_2^{(1,1)}$	3.8071	1.2657	13.4556	1.3105	20.0521
$S_2^{(0,2)}$	−ve	3.0747	−ve	8.4066	−ve
$S_3^{(1,2)}$	3.1963	1.2175	10.4791	1.2537	15.4342

Table 8: The availability equivalence factor $\sigma^{c}_{S,B}$ for different sets S and B.

S	B				
	$B_1^{(1,0)}$	$B_1^{(0,1)}$	$B_2^{(1,1)}$	$B_2^{(0,2)}$	$B_3^{(1,2)}$
$S_1^{(1,0)}$	21.8849	1.3815	−ve	1.4329	−ve
$S_1^{(0,1)}$	−ve	11.9655	−ve	103.9670	−ve
$S_2^{(1,1)}$	3.9914	1.2793	18.3687	1.3137	27.6279
$S_2^{(0,2)}$	−ve	3.6208	−ve	10.9803	−ve
$S_3^{(1,2)}$	3.3370	1.2285	14.1701	1.2563	21.1205

(b$'$) increasing the repair rate of each component belonging to the set $S_2^{(1,1)}$ by the factor $\sigma^{c}_{S_2^{(1,1)},B_1^{(1,0)}} = 3.9914$, and (c$'$) increasing the repair rate of each component belonging to the set $S_3^{(1,2)}$ by the factor $\sigma^{c}_{S_3^{(1,2)},B_1^{(1,0)}} = 3.3370$ (see Table 8).

In the same manner, one can interpret the rest of the results shown in Tables 2–8.

6. Conclusions

In this paper, we investigated the equivalence of different designs of a repairable series-parallel system based on the system availability. It is assumed that the failure rates and repair rates of the system's components are constant, and the system can be improved according to five different methods. We provided the availability of the original and the improved systems and obtained the availability equivalence factors of the system. Some numerical results are presented to illustrate how one can utilize the theoretical results obtained in this work and to compare the different availability factors of the system. Indeed, the problem studied in this paper can be extended to the following some cases: the components of each subsystem are not identical, limited repair teams are available for each subsystem, the failure rates and repair rates of the components are not constant and the components have multiple failure states.

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