Research Article

H_∞ Fuzzy Control of DC-DC Converters with Input Constraint

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This paper proposes a method for designing H_∞ fuzzy control of DC-DC converters under actuator saturation. Because linear control design methods do not take into account the nonlinearity of the system, a T-S fuzzy model and a controller design approach is used. The designed control not only handles the external disturbance but also the saturation of duty cycle. The input constraint is first transformed into a symmetric saturation which is represented by a polytopic model. Stabilization conditions for the H_∞ state feedback system of DC-DC converters under actuator saturation are established using the Lyapunov approach. The proposed method has been compared and verified with a simulation example.

1. Introduction

The main task of DC-DC converters is the adaptation of the voltage and current levels between sources and loads while maintaining a low power loss in the conversion [1–3]. With the extensive use of DC-DC converters in different industry applications (e.g. power supplies for personal computers, DC-motor drive, telecommunications equipment, etc.), improving their performances has become an interesting problem in recent years [1–17]. Recently, different converter circuits (buck converter, boost converter, buck-boost converter, Cuk converter, etc.) are known. According to each application purpose (increase or decrease the magnitude of the DC voltage and/or invert its polarity), the converter circuit was chosen. Among them, we consider here, the control of the basic Pulse-Width-Modulation (PWM) buck converters, but it could be easily adapted for other converters.

The DC-DC switching power converters are highly nonlinear systems [4, 5, 15, 16]. Consequently, the conventional linear controls based on averaging and linearization techniques [12, 14] will result in poor dynamic performance or system instability. In order to resolve this problem, the fuzzy logic approach has been proposed as an alternative solution [4, 5, 7, 8, 11, 13, 15, 16]. Specifically, in [4, 5], the authors have reported very significant results on the modelling and control of DC-DC converters with T-S fuzzy systems. In [4, 5, 8, 11, 15, 16], authors have proposed methods for designing fuzzy control of DC-DC converters. Nevertheless, in the aforementioned papers, they have not taken into account the inherent nonlinearity of actuator saturation (duty cycle) and the external disturbance. Motivated by this observation, our aim is to use T-S fuzzy systems in order to control a nonlinear DC-DC converter subject to external disturbances and actuator saturation.

The actuator saturation can degrade the performance of the closed-loop system and often make the stable closed-loop system unstable [1, 18–21]. The T-S fuzzy system in the presence of saturation has been receiving increasing attention for control of nonlinear systems [18–29]. In these works, the saturation effect is considered as a symmetric function. This is not the case in DC-DC converter application where the saturation function constrained is between 1 and 0. The solution of this problem proposed in linear control case [2] has been leading to complexity analysis with the nonlinearity of DC-DC converter model. Here, we proposed a simple mathematical transformation to obtain a symmetric saturation.

The H_∞ approach is used to analyze and to synthesize controllers/observers achieving an optimal level of disturbance attenuation and to guarantee the stability of the closed-loop system. To achieve this goal, the idea is to minimize the H_∞ norm which represents the maximum value ratio between the output signal energy (controlled output) and the input signal energy (disturbance input). Recently, many researchers have used this approach for control design of T-S fuzzy systems (see for example [20, 21, 30, 31] and their references). In most cases, the quadratic Lyapunov function and linear matrix inequality (LMI) techniques are used to analyze and synthesize of stabilization of T-S fuzzy systems [17–21, 30, 31]. Based on these works, we address the control problem of DC-DC converters via PDC controller with actuator saturation and external disturbance. In this paper, we will use T-S fuzzy systems to represent the DC-DC converters. The control design is based on the parallel distributed compensation (PDC) scheme [32]. The idea is that for each local linear model, a linear feedback control is designed. The control problem is formulated and solved as a LMI optimization problem [33].

This paper is organized as follows. Section 2 gives the averaged model of basic PWM buck converter. Section 3 presents the T-S fuzzy model of the DC-DC converter. Section 4 formulates the conditions for the H_∞ stabilization of fuzzy control with actuator saturation problem in terms of LMI. The simulation results to show the effectiveness of the proposed method are given in Section 4.

Notation 1. I_r denotes the set $\{1, 2, \ldots, r\}$, $\Re$ denotes the set of real number and $\Re^{n \times m}$ the set of all $n \times m$ real matrix. $M > (\geq, , <, \leq)$ is used to denote a symmetric positive definite (positive semidefinite, negative definite, negative semidefinite, resp.) matrix. * denotes the symmetric bloc matrix, $X + (*)$ denotes $X + X^T$, × denotes the multiplication, co denotes the convex hull, and $\bigcap_{j=1}^{r}$ is used to denote the intersection of r sets.

2. Averaged Model of Basic PWM Buck Converter

Figure 1 shows the basic circuit of the nonlinear PWM buck converter proposed in [4, 5] with an external disturbance $i_{\text{load}}(t)$ as proposed in [1, 2]. R_m is the on-state resistance of the
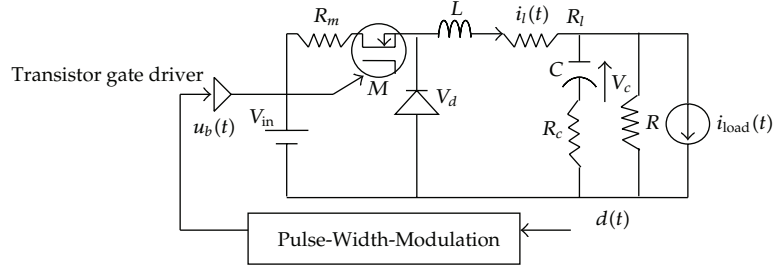
Figure 1: Schematic of a basic PWM buck converter.

MOSFET transistor, R_L is the winding resistance of inductor, V_d is the threshold voltage of the diode, and R_c is the equivalent series resistance of the filter capacitor.

By applying the Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) in on-state of the MOSFET transistor case, we obtain

$$
\begin{bmatrix} \dot{i}_l(t) \\ \dot{v}_c(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{L}\left[R_l + \dfrac{RR_c}{R + R_c}\right] & -\dfrac{R}{L(R + R_c)} \\ \dfrac{R}{C(R + R_c)} & -\dfrac{1}{C(R + R_c)} \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix}
$$

$$
+ \begin{bmatrix} \dfrac{1}{L}[V_{in} + R_m i_l(t)] \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{RR_c}{L(R + R_c)} \\ -\dfrac{R}{C(R + R_c)} \end{bmatrix} i_{\text{load}}(t).
$$

(2.1)

Now, in off-state of the MOSFET transistor case and by applying of KVL and KCL, we get

$$
\begin{bmatrix} \dot{i}_l(t) \\ \dot{v}_c(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{L}\left[R_l + \dfrac{RR_c}{R + R_c}\right] & -\dfrac{R}{L(R + R_c)} \\ \dfrac{R}{C(R + R_c)} & -\dfrac{1}{C(R + R_c)} \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix}
$$

$$
+ \begin{bmatrix} -\dfrac{V_d}{L} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{RR_c}{L(R + R_c)} \\ -\dfrac{R}{C(R + R_c)} \end{bmatrix} i_{\text{load}}(t).
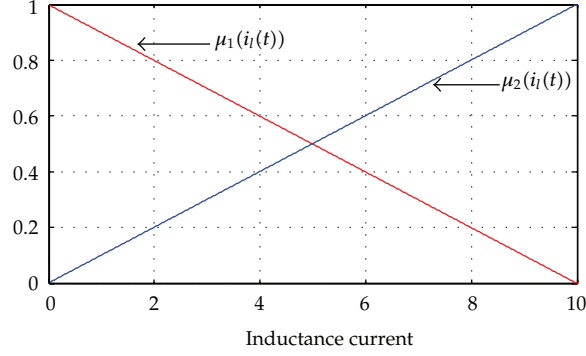$$

(2.2)

Figure 2: Membership functions of the T-S fuzzy model.

Using averaging method for one-time scale discontinuous system (AM-OTS-Ds) [9], the global dynamical behavior of the DC-DC converter is modeled as follows:

$$
\begin{bmatrix} \dot{i}_l(t) \\ \dot{v}_c(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{L}\left[R_l + \dfrac{RR_c}{R+R_c}\right] & -\dfrac{R}{L(R+R_c)} \\ \dfrac{R}{C(R+R_c)} & -\dfrac{1}{C(R+R_c)} \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix}
$$

$$
+ \begin{bmatrix} \dfrac{1}{L}[V_{in} + V_d + R_m i_l(t)] \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} -\dfrac{V_d}{L} \\ 0 \end{bmatrix} + \begin{bmatrix} \dfrac{RR_c}{L(R+R_c)} \\ -\dfrac{R}{C(R+R_c)} \end{bmatrix} i_{\text{load}}(t),
$$

(2.3)

where $d(t)$ is the duty cycle.

## 3. T-S Fuzzy Model of DC-DC Converter

Our control approach is based on T-S fuzzy models and the robustness of this control depends on the ability of the fuzzy model to represent the real system. In this section our objective is to show the effectiveness of T-S fuzzy model to represent the DC-DC converter.

The T-S fuzzy model has been successfully used to approximate nonlinear systems by interpolation of numerous local linear models [34].

R_i: If $\xi_1(t)$ is about M_{1i} and $\xi_q(t)$ is about M_{qi} then

$$
\begin{aligned}
\dot{x}(t) &= A_i x(t) + B_{1i} w(t) + B_{2i}\sigma(t) \\
z(t) &= C_{1i} x(t) + D_{1i} w(t) + D_{2i}\sigma(t)
\end{aligned} \qquad \text{For } i \in I_r
$$

(3.1)

in which M_{ji} is the fuzzy set of ξ_i in rule R_i, r is the number of if-then fuzzy rules and $\xi_i(t)$ are the decision variable assumed measurable, $x(t) \in \mathcal{R}^n$ is the system state vector, $\sigma(t) \in \mathcal{R}^m$ is the saturate control input, $y(t) \in \mathcal{R}^p$ is the measurable output, $z(t) \in \mathcal{R}^{nz}$ is the controlled output variable, and $w(t) \in \mathfrak{F}_2$ is the disturbance variable with $\mathfrak{F}_2 = \{w \in \mathcal{R}^{nw} \mid \|w\|_2 \leq \overline{w}, \overline{w} > 0\}$.

(a)

(b)
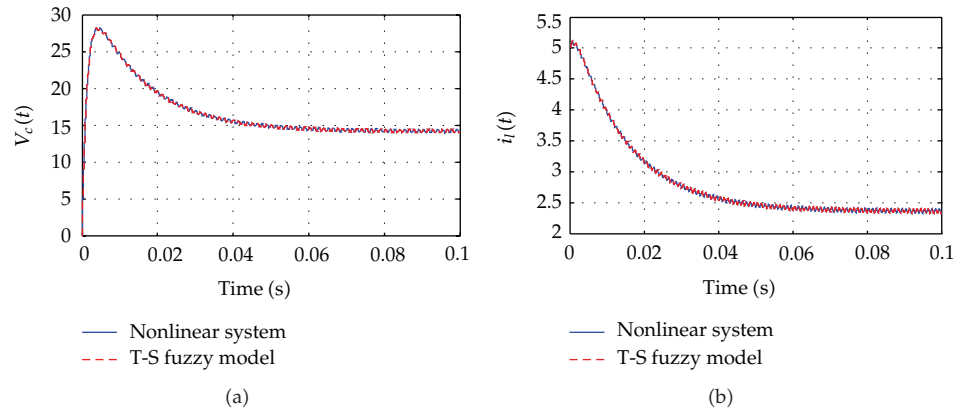
Figure 3: The nonlinear DC-DC converter responses and its T-S fuzzy representation with initial conditions $(v_c(0) = 0\,\text{V},\ i_l(0) = 5\,\text{A})$.

(a)

(b)
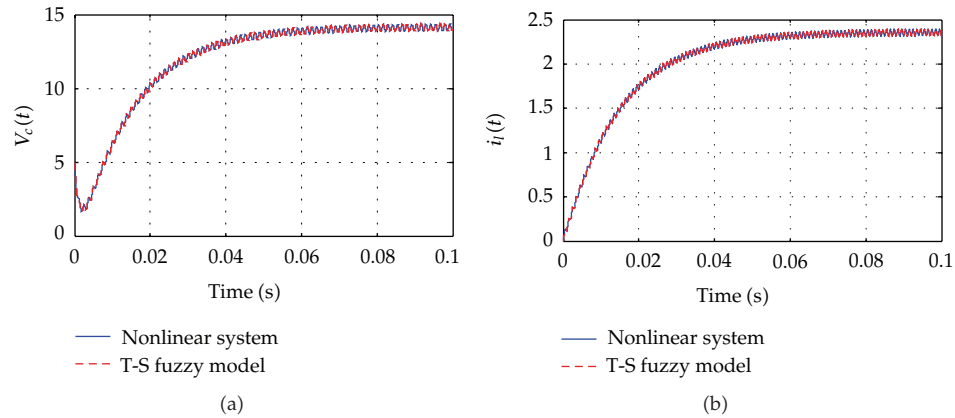
Figure 4: The nonlinear DC-DC converter responses and its T-S fuzzy representation with initial conditions $(v_c(0) = 5\,\text{V},\ i_l(0) = 0\,\text{A})$.

The global dynamic system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_{1i} w(t) + B_{2i}\sigma(t)),$$

$$z(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(C_{1i} x(t) + D_{1i} w(t) + D_{2i}\sigma(t)),$$

(3.2)

where

$$\mu_i(\xi(t)) = \frac{\prod_{j=1}^{q} M_{ji}(\xi_j(t))}{\sum_{i=1}^{r}\left(\prod_{j=1}^{q} M_{ji}(\xi_j(t))\right)},$$
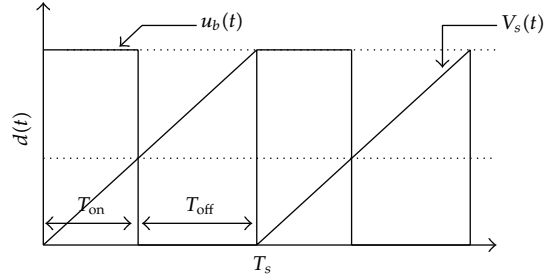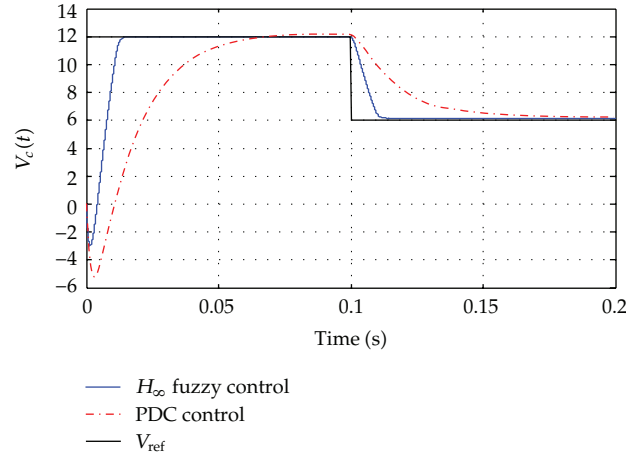
(3.3)

Figure 5: Waveforms of the PWM process.

Figure 6: Response of the output voltage $v_c(t)$ of PWM buck converter.

$M_{ji}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ji}. The normalized activation function $\mu_i(\xi(t))$ in relation with the ith submodel is such that

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \quad 0 \leq \mu_i(\xi(t)) \leq 1 \ \forall i \in I_r, \tag{3.4}$$

Among different methods (identification method, linearization around different operating points, or by transformation in nonlinear sector), the method by transformation in nonlinear sector is based on bounded function and gives a minimum number of local models. Here we will use this method to represent the DC-DC converter.

The following Lemma will be used in the sequel of the paper.

Lemma 3.1. *Let $f(x(t)) : R \rightarrow R$ is a bounded function, it always exist tow functions, $\eta_1(x(t))$ and $\eta_2(x(t))$ and two scalars α and β such that:*

$$f(x(t)) = \alpha \times \eta_1(x(t)) + \beta \times \eta_2(x(t)) \tag{3.5}$$

with

$$\eta_1(x(t)) + \eta_2(x(t)) = 1, \quad \eta_1(x(t)) \geq 0, \ \eta_2(x(t)) \geq 0. \tag{3.6}$$

In this case, the obtained T-S fuzzy model represents exactly the nonlinear system for $x \in R^n$ with 2^{nl} locals models where nl represents the number of local models.

Assuming that, $I_{l\min} \leq i_l(t) \leq I_{l\max}$ and by Lemma 3.1, the system described by (2.3) is modeled with a two-rules T-S fuzzy system as follows:

Rule 1: if $i_l(t)$ is about $I_{\min}$ then: $\dot{x}(t) = A_m x(t) + B_{m1} i_{\text{load}}(t) + B_{m21} d(t) + D_m$.

Rule 2: if $i_l(t)$ is about $I_{\max}$ then: $\dot{x}(t) = A_m x(t) + B_{m1} i_{\text{load}}(t) + B_{m22} d(t) + D_m$.

The overall model of T-S fuzzy system can be given by the following:

$$\dot{x}(t) = A_m x(t) + B_{m1} i_{\text{load}}(t) + B_{m2\mu} d(t) + D_m, \tag{3.7}$$

with $x_1(t) = i_l(t)$, $x_2(t) = v_c(t)$, $B_{m2\mu} = \sum_{i=1}^{2} \mu_i(i_l(t)) B_{m2i}$ and

$$A_m = \begin{bmatrix} -\dfrac{1}{L}\left[R_l + \dfrac{RR_c}{R+R_c}\right] & -\dfrac{R}{L(R+R_c)} \\ \dfrac{R}{C(R+R_c)} & -\dfrac{1}{C(R+R_c)} \end{bmatrix}, \qquad B_{m21=} \begin{bmatrix} \dfrac{1}{L}(V_{in} + V_d + R_m I_{l\min}) \\ 0 \end{bmatrix},$$

$$B_{m1} = \begin{bmatrix} \dfrac{RR_c}{L(R+R_c)} \\ -\dfrac{R}{C(R+R_c)} \end{bmatrix}, \qquad B_{m22=} \begin{bmatrix} \dfrac{1}{L}(V_{in} + V_d + R_m I_{l\max}) \\ 0 \end{bmatrix}, \qquad D_m = \begin{bmatrix} -\dfrac{V_d}{L} \\ 0 \end{bmatrix}. \tag{3.8}$$

The membership function is such that

$$\mu_1(i_l(t)) = \dfrac{-i_l(t) + I_{l\max}}{I_{l\max} - I_{l\min}}, \qquad \mu_2(i_l(t)) = \dfrac{i_l(t) - I_{l\min}}{I_{l\max} - I_{l\min}}. \tag{3.9}$$

To show the effectiveness of the fuzzy model to represent the nonlinear system, we simulate the fuzzy model and the nonlinear system for the same inputs with different initial conditions. The simulation parameters used in this work are as follows [4]:

$$R = 6\,\Omega, \qquad R_l = 48.5\,\text{m}\Omega, \qquad R_c = 0.16\,\Omega, \qquad R_m = 0.27\,\Omega, \qquad V_{in} = 30\,\text{V},$$

$$L = 98.58\,\text{mH}, \qquad C = 202.5\,\mu\text{F}, \qquad f = 1\,\text{KH}, \qquad I_{l\min} = 0\,\text{A}, \qquad I_{l\max} = 10\,\text{A}, \tag{3.10}$$

with $d(t) = 0.5$ and the input current $i_{\text{load}}(t) = 0.25 \sin(5000t)$.

Figure 2 shows the membership functions of the fuzzy model. The DC-DC converter responses of nonlinear system (2.3) and T-S fuzzy system (3.7) with initial conditions ($v_c(0) = 0\,\text{V}$, $i_l(0) = 5\,\text{A}$) and ($v_c(0) = 5\,\text{V}$, $i_l(0) = 0\,\text{A}$) are shown in Figures 3 and 4, respectively.

These figures demonstrate that, with different initial conditions, the T-S fuzzy system (2.3) has the same behaviour as the nonlinear system (3.7). This means the satisfactory approximation ability of the fuzzy model.

4. H_∞ **Fuzzy Control of DC-DC Converter**

In this section, we present an H_∞ fuzzy approach to the control design of DC-DC converter.

4.1. Saturated Control Analysis

In order to control the output voltage of the DC-DC converter, we define the following variables:

$$e_1(t) = v_c(t) - V_{\text{ref}}, \qquad e_2(t) = \dot{e}_1(t), \tag{4.1}$$

where V_{ref} is the reference voltage of $v_c(t)$.

The time-derivative of $e_2(t)$ is as follows:

$$\dot{e}_2(t) = \ddot{v}_c(t) = \frac{R}{C(R + R_c)}\dot{i}_l(t) - \frac{1}{C(R + R_c)}\dot{v}_c(t) - \frac{R}{C(R + R_c)}\dot{i}_{\text{load}}(t). \tag{4.2}$$

Using the converter's model defined in (2.3), we have

$$\dot{x}(t) = Ax(t) + B_w w_1(t) + B_2(x)d(t) + D,$$

$$x_1(t) = e_1(t), \qquad x_2(t) = e_2(t), \qquad w_1(t) = \begin{bmatrix} i_{\text{load}}(t) \\ \dot{i}_{\text{load}}(t) \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{(R + R_l)}{LC(R + R_c)} & -\dfrac{L + R_l C(R + R_c) + CRR_c}{LC(R + R_c)} \end{bmatrix}, \tag{4.3}$$

$$B_2(x) = \begin{bmatrix} 0 \\ \dfrac{R}{LC(R + R_c)}[V_{in} + V_d - R_m i_l(t)] \end{bmatrix},$$

$$B_w = \begin{bmatrix} 0 & 0 \\ -\dfrac{R_l R}{LC(R + R_c)} & -\dfrac{R}{C(R + R_c)} \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ -\dfrac{R(V_d + V_{\text{ref}}) + R_l V_{\text{ref}}}{LC(R + R_c)} \end{bmatrix}.$$

By Lemma 3.1, this system can be modeled with a two-rule T-S fuzzy system as follows:

Rule 1: if $i_l(t)$ is about I_{min} then: $\dot{x}(t) = Ax(t) + B_w w_1(t) + B_{21}d(t) + D$.

Rule 2: if $i_l(t)$ is about I_{max} then: $\dot{x}(t) = Ax(t) + B_w w_1(t) + B_{22}d(t) + D$.

and the global T-S fuzzy model can be given by the following:

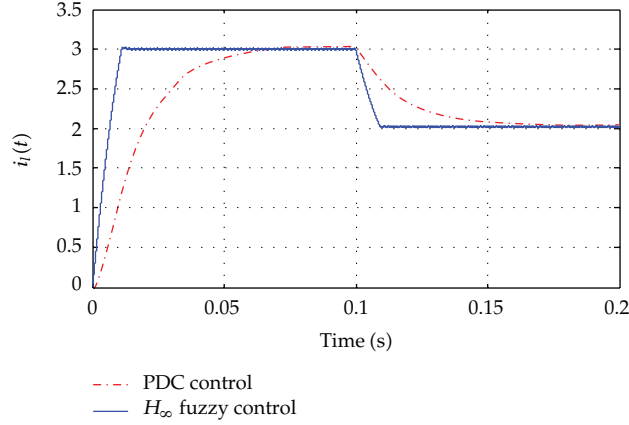$$\dot{x}(t) = Ax(t) + B_w w_1(t) + B_{2\mu}d(t) + D, \tag{4.4}$$

Figure 7: Response of the inductance current $i_l(t)$ of PWM buck converter.

with:

$$w_1(t) = \begin{bmatrix} i_{\text{load}}(t) \\ \dot{i}_{\text{load}}(t) \end{bmatrix}, \qquad B_{2\mu} = \sum_{i=1}^{2} \mu_i(i_l(t))B_{2i},$$

$$B_{21} = \begin{bmatrix} 0 \\ \dfrac{R}{LC(R+R_c)}[V_{in}+V_d-R_m I_{l\min}] \end{bmatrix}, \qquad B_{22} = \begin{bmatrix} 0 \\ \dfrac{R}{LC(R+R_c)}[V_{in}+V_d-R_m I_{l\max}] \end{bmatrix},$$

$$(4.5)$$

The output voltage of the DC-DC converter can be controlled by means of variation of duty cycle (Figure 1). The duty cycle is defined by the ratio T_{on}/Ts, where Ts is the frequency of the PWM circuit and T_{on} is the time when the MOSFET is on (Figure 5). A transistor gate driver (Figure 1) switches the MOSFET between the conducting (on) and blocking (off) states using a binary signal $u_b(t)$. This signal is produced by the modulator. In this operation, the duty cycle is compared with a sawtooth signal $V_s(t)$ of amplitude equals to 1. Consequently, the duty-cycle is constrained in amplitude between 0 and 1.

Thus, in this application, the control input is subject to actuator saturation with the following saturation:

$$0 \le d(t) \le 1, \tag{4.6}$$

which is not a symmetric saturation. However, most results reported in open literature have treated the controller design analysis with symmetric saturation [18–21]. To transform this saturation into a symmetric saturation, authors in [2] have proposed a change of variables to the linear converter model. This idea cannot be used in our case (nonlinear converter model) because it increases the complexity of the control analysis. Here, we will propose another idea that allows a simple analysis thereafter.
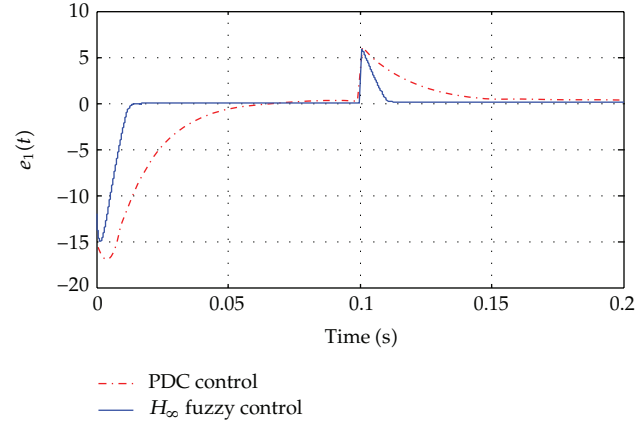
Figure 8: The trajectory of the error $e_1(t)$.
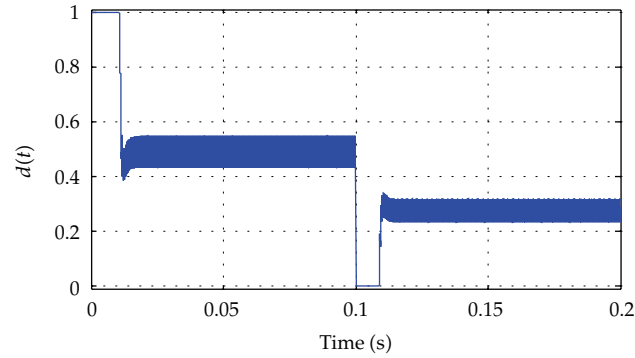
Figure 9: The trajectory of the saturated control (duty cycle $d(t)$).

The system described in (4.4) can be written as follows:

$$\dot{x}(t) = Ax(t) + B_w w_1(t) + B_2(x)\left(d(t) - \frac{1}{2}\right) + \frac{1}{2}B_2(x) + D,$$

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2(x)\delta(t),$$

(4.7)

with

$$w(t) = \frac{1}{2}\frac{R}{LC(R + R_c)}[V_{in} + V_d - R_m i_l(t)] - \frac{R_l R}{LC(R + R_c)}i_{\text{load}}(t)$$

$$- \frac{R}{C(R + R_c)}\dot{i}_{\text{load}}(t) - \frac{R(V_d + V_{\text{ref}}) + R_l V_{\text{ref}}}{LC(R + R_c)}$$

(4.8)

is the external disturbance, $B_1 = [0 \ 1]^T$, and $\delta(t) = d(t) - 1/2$ is the new control input.

This last system can be represented with the following T-S fuzzy model:

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_{2\mu}\delta(t). \tag{4.9}$$

Thus, the new control input $\delta(t)$ is constrained as follows:

$$\frac{-1}{2} \le \delta(t) \le \frac{1}{2}. \tag{4.10}$$

In this work, the controller is a nonlinear state feedback which shares the same activation functions as the T-S fuzzy model (3.2) of the following form [32]:

$$u(t) = K_\mu x(t),$$

$$K_\mu = \sum_{j=1}^{r} \mu_j(\xi(t)) K_j, \tag{4.11}$$

where and $K_j \in \Re^{m \times n}$ is the local controller matrix to be determined.

The control input is subject to actuator saturation the saturation function means:

$$\sigma(t) = \mathrm{sat}(u(t), \overline{u}) = [s_1, s_2, \ldots, s_m]^T,$$

$$s_i = \mathrm{sign}(u_i) \min\{\overline{u}_i, |u_i|\}, \tag{4.12}$$

$\overline{u} \in R^m$ denotes the saturation level and $\overline{u}_i$ and u_i denote the ith element of $\overline{u}$ and $u(t)$. In our case:

$$\overline{u}_i = \overline{u} = 0.5. \tag{4.13}$$

The saturation obtained in (3.6) is a symmetric saturation and the following lemma can be used to the stability analysis.

Lemma 4.1 (see [18]). *Let E be the set of $m \times m$ diagonal matrices whose diagonal elements are 1 or 0. Suppose that $|v_i| \le \overline{u}_i$ for all $i \in I_m$, where v_i and u_i denote the ith element of $v \in \Re^m$ and $u \in \Re^m$, respectively. If $x \in \bigcap_{j=1}^{r} \Im(H_j)$ for $x \in \Re^n$, then:*

$$\mathrm{sat}(u, \overline{u}) = \sum_{s=1}^{2^m} \alpha_s \left(E_s u + \overline{E}_s v \right),$$

$$\sum_{s=1}^{2^m} \alpha_s = 1, \quad 0 \le \alpha_s \le 1,$$

$$v = \sum_{j=1}^{r} \mu_j H_j x, \tag{4.14}$$

$$\Im(H_j) = \left\{ x \in \Re^n \mid \left| h_i^j x \right| \le \overline{u}_i \right\},$$

where E_s denotes all elements of E, $\overline{E}_s = I - E_s$, H_j is $m \times n$ matrix and h_i^j is the ith row of the matrix H_j.

In our case $m = 1$, $E_s \in \text{co}\{0, 1\}$, $\overline{u}_1$ is the saturation level ($\overline{u}_1 = 1/2$) and $r(r = 2)$ is the number of local models. Consequently, the saturation function in (4.10) can be rewritten as follows:

$$\text{sat}(K_\mu x(t)) = \left(E_\alpha K_\mu + \overline{E}_\alpha H_\mu\right)x(t), \tag{4.15}$$

with

$$\left|h_i^j x\right| \leq \frac{1}{2}, \quad \forall i \in I_m, \; j \in I_r, \tag{4.16}$$

and $H_\mu = \sum_{i=1}^{2} \mu_i H_i$, $E_\alpha = \sum_{s=1}^{2} \alpha_s E_s$ and $\overline{E}_\alpha = \sum_{s=1}^{2} \alpha_s \overline{E}_s$.

4.2. Quadratic Lyapunov Stability

For a constant $\rho > 0$ and a symmetric positive matrix P, define an ellipsoid as follows:

$$\varepsilon(P, \rho) = \left\{x \in \mathfrak{R}^n \mid x^T P x \leq \rho\right\}. \tag{4.17}$$

This ellipsoid can be rewritten as follows:

$$\varepsilon(P, \rho) = \left\{x \in \mathfrak{R}^n \mid x^T \mathbf{P} x \leq 1\right\} \tag{4.18}$$

$\rho > 0$, this implied that $\mathbf{P} = P/\rho \in \mathfrak{R}^{n \times n}$ is a symmetric positive matrix.

We define the Lyapunov function as follows:

$$V(x) = x(t)^T \mathbf{P} x(t). \tag{4.19}$$

An ellipsoid $\varepsilon(p, \rho)$ is said to be contractively invariant set if $\dot{V}(x) < 0$, for all $x \in \varepsilon(P, \rho) \mid \{0\}$. Therefore, if an ellipsoid is contractively invariant, it is inside the domain of attraction.

An ellipsoid $\varepsilon(P, \rho) = \{x \in R^n \mid x^T \mathbf{P} x \leq 1\}$ is inside $\bigcap_{j=1}^{r} \mathfrak{I}(H_j)$ if and only if for all $i \in I_m, \; j \in I_r$:

$$\left(h_i^j\right)^T \mathbf{P} h_i^j \leq \overline{u}_i^2 I. \tag{4.20}$$

In order to design a DC-DC converter to perform adequately in the presence of external disturbances, the H_∞ attenuation performance is chosen as the performance measure, which is defined as follows:

$$\int_0^T z(\tau)^T z(\tau) d\tau < \gamma^2 \int_0^T w(\tau)^T w(\tau) d\tau, \tag{4.21}$$

where γ is a positive scalar and small as possible, $w(t) \in L_2[0 \; 1]$ and $x(0) = 0$.
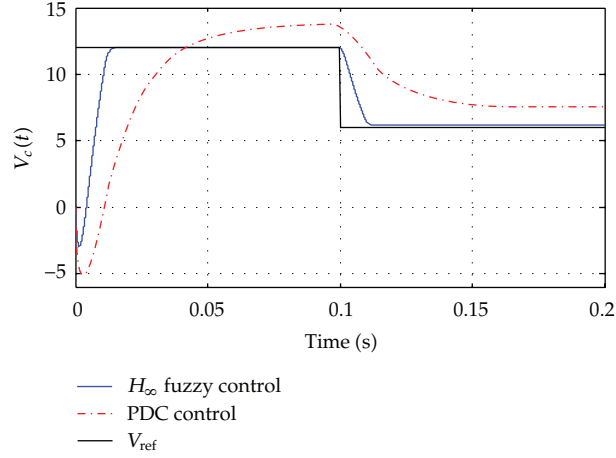
Figure 10: Response of the output voltage $v_c(t)$ of PWM buck converter for 1 kHz.

In our problem, the controlled output is the error $e_1(t)$, that is:

$$z(t) = C_1 x(t), \tag{4.22}$$

where $C_1 = [0 \ 1]$. The closed-loop system composed of (4.9), (4.22), and (4.15) is given by the following:

$$\dot{x}(t) = \left(A + B_{2\mu}\left(E_\alpha K_\mu + \overline{E}_\alpha H_\mu \right) \right) x(t) + B_{m1} w(t), \tag{4.23}$$

$$z(t) = C_1 x(t).$$

Theorem 4.2. *The ellipsoid $\varepsilon(P, \rho)$ is contractively invariant set of the closed-loop system* (4.15) *and achieves in a disturbance rejection level γ, if there exist a symmetric positive definite matrix Q and matrices $F_j \in \mathfrak{R}^{m \times n}$, solutions of the following LMI problem:*

$$\min_{Q, F_j, Z_j}^{\gamma}, \tag{4.24}$$

$$\begin{bmatrix} \dfrac{1}{4} & z_i^j \\[2mm] \left(z_i^j \right)^T & Q \end{bmatrix} \geq 0, \quad \forall i \in I_m, \ j \in I_r \tag{4.25}$$

$$\begin{bmatrix} AQ + B_{2i}E_s F_j + B_{2i}\overline{E}_s Z_j + (*) & * & * \\ B_1^T & -\gamma I & * \\ C_1 Q & 0 & -\gamma I \end{bmatrix} < 0, \quad \forall i \in I_r, \ j \in I_r. \tag{4.26}$$

Then one gets

$$K_j = F_j Q^{-1}, \qquad H_j = Z_j Q^{-1}. \tag{4.27}$$

Using Lemma 3.1 for inequality (4.20), one has:

$$\left(h_i^j\right)^T \mathbf{P} h_i^j \le \frac{1}{4}, \quad \forall i \in I_m, \; j \in I_r. \tag{4.28}$$

Let the following change of variables:

$$Q = \mathbf{P}^{-1}, \tag{4.29}$$

$$Z_j = H_j Q. \tag{4.30}$$

Consequently, the inequality (4.28) can be rewritten as follows

$$\frac{Q}{4} - \left(z_i^j\right)^T z_i^j \ge 0, \tag{4.31}$$

where z_i^j is the ith row of the matrix Z_j. By Schur complement [33], this last inequality can be transformed as LMI (4.25).

Proposition 4.3 (see [30]). *If the Lyapunov function defined in (4.19) satisfies the following Hamilton-Jacobi-Bellman inequality:*

$$\dot{V}(x(t)) + \gamma^{-1} z(t)^T z(t) - \gamma w(t)^T w(t) < 0, \tag{4.32}$$

along (4.23) for all $x(t) \neq 0$ and $w(t) \in L_2[0 \;\; 1] \triangleq \{w \in \mathfrak{R}^{nw} \mid \int_0^T \|w\|^2 dt \le \overline{w}, \overline{w} > 0\}$, then (4.21) is verified. Using (4.23), one has:

$$\dot{V}(x(t)) - \gamma w(t)^T w(t) + \gamma^{-1} z(t)^T z(t)$$

$$= \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}^T \left\{ \begin{bmatrix} \left(A + B_{2\mu} E_s K_\mu + B_{2\mu} \overline{E}_s H_\mu\right)^T \mathbf{P} + (*) & * \\ B_1^T \mathbf{P} & -\gamma I \end{bmatrix} + \gamma^{-1} \begin{bmatrix} C_1^T \\ 0 \end{bmatrix} \begin{bmatrix} C_1 & 0 \end{bmatrix} \right\} \begin{bmatrix} x(t) \\ w(t) \end{bmatrix}. \tag{4.33}$$

By Schur complement [13], the condition (4.32) holds if

$$\Sigma_\mu = \begin{bmatrix} \mathbf{P}^T \left(A + B_{2\mu} E_s K_\mu + B_{2\mu} \overline{E}_s H_\mu\right) + (*) & * & * \\ B_1^T \mathbf{P} & -\gamma I & * \\ C_1 & 0 & -\gamma I \end{bmatrix} < 0. \tag{4.34}$$

Pre- and postmultiplying this last inequality by $\Gamma = \mathrm{diag}(Q, I, I)$ one has

$$
\Gamma \Sigma_\mu \Gamma = \begin{bmatrix} A_i Q + B_{2i} E_s K_j Q + B_{2i} \overline{E}_s H_j Q + (*) & * & * \\ B_1^T & -\gamma I & * \\ C_1 Q & 0 & -\gamma I \end{bmatrix} < 0. \tag{4.35}
$$

Taking account (4.27) and (4.30), one gets the following LMI:

$$
\Gamma \Sigma_\mu \Gamma = \begin{bmatrix} A_\mu Q + B_{2\mu} F_\mu + (*) & * & * \\ B_1^T & -\gamma I & * \\ C_1 Q & 0 & -\gamma I \end{bmatrix} < 0, \tag{4.36}
$$

which ends the proof.

4.3. The Initial Conditions Constraint

The main purpose of the DC-DC converter control is to maintain the voltage level $v_c(t)$ equal to the desired level V_{ref}. The initial condition for the state vector is as follows:

$$
x_0 = \begin{bmatrix} x_1(0) & x_2(0) \end{bmatrix}^T = \begin{bmatrix} e_1(0) & e_2(0) \end{bmatrix}^T = \begin{bmatrix} V_{\text{ref}} & 0 \end{bmatrix}^T. \tag{4.37}
$$

However, it is not possible to assure that any initial condition will belong to the ellipsoid of stability given in Theorem 4.2. In this section, the objective is to guarantee the start up stability with maximum elimination of external disturbances in the presence of saturation.

The point $x_0 \in \varepsilon(P, \rho)$ is equivalent to the following:

$$
x_0^T P x_0 \leq 1. \tag{4.38}
$$

Or in LMI form by the following:

$$
\begin{bmatrix} 1 & x_0^T \\ x_0 & Q \end{bmatrix} \geq 0. \tag{4.39}
$$

The optimization problem proposed in Theorem 4.2 can be formulated as follows:

$$
\min_{Q, F_j, Z_j}^\gamma.
$$
$$
\text{LMI}(4.27), \; \text{LMI}(4.28), \; \text{LMI}(4.41) \tag{4.40}
$$

4.4. Simulation Results

In order to demonstrate the effectiveness of the proposed method, the DC-DC converter is controlled by the proposed H_∞-based state feedback and the PDC controller proposed in [4].

In this simulation, the current perturbation step equals to 1 A and the PWM frequency equals to 5 kHz.

Solving the optimization problem described in (4.40) for initial condition $x_0 = [12V \quad 0]^T$, we have:

$$Q = \begin{bmatrix} 817.3975 & -6.1238 \times 10^5 \\ -6.1238 \times 10^5 & 5.5691 \times 10^5 \end{bmatrix}, \quad H_1 = H_2 = [-0.0354 \quad 0],$$

$$\gamma = 0.7255, \quad K_1 = [-1.4681 \quad -0.0016], \quad K_2 = [-1.5030 \quad -0.0017]. \tag{4.41}$$

The control is defined by $d(t) = \text{sat}(K_\mu x(t)) + 1/2$.

Figures 6, 7, and 8 show the DC-DC converter responses (output voltage and inductance current, resp.). Figure 9 shows the trajectory of the saturated control input signal (duty cycle $d(t)$). Figure 8 shows, with the presence of saturation, the H_∞ control can guarantee the stability of all initial conditions in the interval $e_1(0) \in [-12V \quad 12V]$ in presence of actuator saturation and external disturbances (see initial time and 0.1 s time).

Simulation results (Figures 6–8) demonstrate that our controller guarantees better stabilization performance, better perturbation rejection (see initial time in Figure 6) and better time response (see Figures 6–8) Moreover, the proposed controller is robust with respect to frequency change. Figure 10 shows the simulation results for a frequency of 1 kHz. It shows that our controller achieves better performance even with different frequency values.

Despite that we take into account not only the actuators saturation but also the external disturbance in the designing control, our controller gives better stabilization performance (perturbation rejection and better time response) of the system as reported in the open literature [4]. Moreover, with different frequency values, our saturated control gives robust control signal (guarantees the same performances despite the change of frequency). These results demonstrate the effectiveness of proposed method.

5. Conclusion

This work has presented an H_∞ controller design for DC-DC converters via state feedback under actuator saturation and external disturbances. The T-S fuzzy system is first used to describe the DC-DC converter. Then, the state controller is designed to guarantee the stability of the closed loop system with H_∞ performance. The saturation effect is represented by a polytopic model. Based on Lyapunov approach, the problem of H_∞ stabilization in the presence of actuator saturation was formulated as an LMI optimization and solved easily by using existing numerical tools. Finally, the simulation results on a DC-DC converter have demonstrated the effeteness of the proposed control.
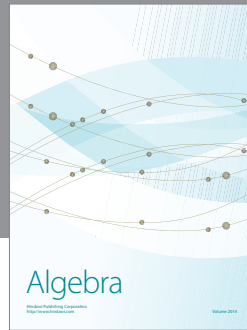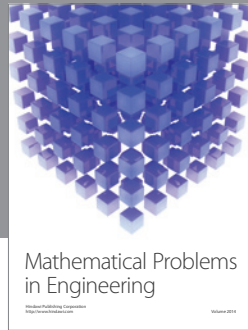
The analysis given here is applied to the buck converter; we can extend this work to other converters (boost converter, buck-boost converter, Cuk converter, etc.). Finally, the results developed in the paper can be extended to the case that the underlying systems are involved with any switching dynamics such as a nondeterministic or stochastic switching system (see [35–37]).

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