FOURIER COEFFICIENTS OF A CLASS OF ETA QUOTIENTS

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Abstract

Recently, Williams, and then Yao, Xia and Jin, discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of one hundred and twenty-six eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$, and Yao, Xia and Jin, following the method of Williams' proof, expressed only the even coefficients of one hundred and four eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, by using the method of Williams' proof, we will express the odd Fourier coefficients of seventy-four eta quotients $f(q)$ in terms of $\sigma_5\left(2n-1\right)$ and $\sigma_5\left(\frac{2n-1}{3}\right)$, i.e., the Fourier coefficients of the difference $f(q) - f(-q)$ of seventy-four eta quotients; and we will express the even Fourier coefficients of sixty eta quotients, i.e., the Fourier coefficients of the sum $f(q) + f(-q)$ of sixty eta quotients, in terms of $\sigma_5\left(n\right), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right), \sigma_5\left(\frac{n}{6}\right)$ and $\sigma_5\left(\frac{n}{12}\right)$.

1. Introduction

The divisor function $\sigma_i\left(n\right)$ is defined for a positive integer i by

$$\sigma_i\left(n\right) = \begin{cases} \sum_{d \text{ positive integer}, d|n} d^i, & \text{if } n \text{ is a positive integer} \\ 0, & \text{if } n \text{ is not a positive integer} \end{cases}.$$

The Dedekind eta function is defined by

$$\eta\left(z\right) = q^{1/24} \prod_{n=1}^{\infty} \left(1 - q^n\right),$$

where

$$q = e^{2\pi i z}, z \in H = \left\{x + iy : y > 0\right\},$$

and an eta quotient of level n is defined by

$$f(z) = \prod_{m|n} \eta(mz)^{a_m}, n, m \in \mathbb{N}, a_m \in Z.$$

It is interesting and important to find explicit formulas for the Fourier coefficients of eta quotients since they are the building blocks of modular forms of level n and weight k. The book of Koehler (see [13], p. 39) describes such expansions by means of Hecke theta series, and it develops algorithms for the determination of suitable eta quotients. One can find more information in [4, 6, 14, 18, 17]. Additionally, the present author has determined the Fourier coefficients of the theta series associated with some quadratic forms (see [12], [11], [10], [8], [7] and [9]).

Recently, Williams [16] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of one hundred and twenty-six eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. An example is

$$\frac{\eta^2(2z)\,\eta^4(4z)\,\eta^6(6z)}{\eta^2(z)\,\eta^2(3z)\,\eta^4(12z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(n) = 2\sigma(n) - 3\sigma(n/2) + 4\sigma(n/4) + 9\sigma(n/6) - 36\sigma(n/12).$$

Then, Yao, Xia and Jin [15] expressed the even Fourier coefficients of one hundred and four eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. One example is

$$\frac{\eta^{25}(2z)\,\eta^4(3z)}{\eta^{12}(z)\,\eta^5(4z)\,\eta^3(6z)\,\eta(12z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(2n) = 65\sigma_3(n) - 68\sigma_3(n/2) - 81\sigma_3(n/3) + 324\sigma_3(n/6).$$

Motivated by these two results, we find that we can express the odd Fourier coefficients of seventy-four eta quotients in terms of $\sigma_5(2n+1)$ and $\sigma_5(\frac{2n+1}{3})$; see Table1/A. An example is

$$\frac{\eta^{23}(4z)\,\eta^7(6z)}{\eta^{13}(2z)\,\eta^5(12z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(2n-1) = 0.$$

We can also express the even Fourier coefficients of sixty eta quotients in terms of $\sigma_5(n), \sigma_5\left(\frac{n}{2}\right), \sigma_5\left(\frac{n}{3}\right), \sigma_5\left(\frac{n}{4}\right), \sigma_5\left(\frac{n}{6}\right)$ and $\sigma_5\left(\frac{n}{12}\right)$; see Table2/A. An example is

$$\frac{\eta^{24}(2z)\,\eta^{12}(12z)}{\eta^{12}(4z)\,\eta^{12}(6z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(2n) = 4\sigma_5(2n) - 132\sigma_5(n) + 128\sigma_5\left(\frac{n}{2}\right) + 792\sigma_5\left(\frac{n}{3}\right).$$

Now we define some rational numbers and functions for the theorem: let $b_1, b_2, \cdots, b_5$ be non-negative integers satisfying

$$b_1 + b_2 + \cdots + b_5 \leq 12.$$

Define the integers a_1, a_2, a_3, a_4, a_6 and a_{12} by

$$\begin{aligned}
a_1 &= -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 12,\\
a_2 &= 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 30,\\
a_3 &= 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 36,\\
a_4 &= -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 12,\\
a_6 &= -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 90,\\
a_{12} &= 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 36. \qquad (1)
\end{aligned}$$

Now define the integers $k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}$ and k_{12} by

$$\begin{aligned}
&\frac{1}{2^{b_1+b_5}} x^{b_1}(1-x)^{b_2}(1+x)^{b_3}(1+2x)^{b_4}(2+x)^{b_5} =\\
&k_0 + k_1x + k_2x^2 + k_3x + k_4x^4 + k_5x^5 + k_6x^6\\
&+k_7x^7 + k_8x^8 + k_9x^9 + k_{10}x^{10} + k_{11}x^{11} + k_{12}x^{12}. \qquad (2)
\end{aligned}$$

Let

$$\begin{aligned}
f_1 &= \sum_{n=0}^{\infty} f_1(n)\,q^n = \eta(2z)\,\eta(4z)\,\eta^5(6z)\,\eta^5(12z),\\
f_2 &= \sum_{n=0}^{\infty} f_2(n)\,q^n = \eta^5(2z)\,\eta^5(4z)\,\eta(6z)\,\eta(12z),\\
f_3 &= \sum_{n=0}^{\infty} f_3(n)\,q^n = \frac{\eta^9(2z)\,\eta^9(12z)}{\eta^3(4z)\,\eta^3(6z)},\\
f_4 &= \sum_{n=0}^{\infty} f_4(n)\,q^n = \frac{\eta^8(4z)\,\eta^{16}(6z)}{\eta^4(2z)\,\eta^8(12z)},\\
f_5 &= \sum_{n=0}^{\infty} f_5(n)\,q^n = \eta^{12}(2z),
\end{aligned}$$

$$f_6 = \sum_{n=0}^{\infty} f_6(n) q^n = \frac{\eta^5(2z)\eta^{13}(6z)}{\eta(4z)\eta^5(12z)},$$
$$f_7 = \sum_{n=0}^{\infty} f_7(n) q^n = \frac{\eta^{12}(4z)\eta^{24}(6z)}{\eta^{12}(2z)\eta^{12}(12z)}.$$

Now define integers $k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10,} k_{11}$ and k_{12} by

$$\frac{1}{2^{b_1+b_5}} x^{b_1}(1-x)^{b_2}(1+x)^{b_3}(1+2x)^{b_4}(2+x)^{b_5}$$
$$= k_0 + k_1 x + k_2 x^2 + k_3 x + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 + k_9 x^9 + k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12}.$$

Define the rational numbers $c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, r_3, r_4, r_5, r_6$ and r_7 by

$$\begin{aligned}
c_1 &= \frac{20732}{8067}k_0 - \frac{45998}{24201}k_1 + \frac{32588}{24201}k_2 - \frac{2430}{2689}k_3 \\
&\quad + \frac{13684}{24201}k_4 - \frac{7774}{24201}k_5 + \frac{1252}{8067}k_6 - \frac{1118}{24201}k_7 \\
&\quad - \frac{652}{24201}k_8 + \frac{518}{8067}k_9 - \frac{2456}{24201}k_{10} + \frac{2456}{24201}k_{11}, \\
c_2 &= -\frac{101678936}{734097}k_0 + \frac{65872684}{734097}k_1 - \frac{14225816}{244699}k_2 \\
&\quad + \frac{27033238}{734097}k_3 - \frac{2344028}{104871}k_4 + \frac{3081530}{244699}k_5 \\
&\quad - \frac{4620236}{734097}k_6 + \frac{1635406}{734097}k_7 + \frac{102820}{244699}k_8 \\
&\quad - \frac{1383458}{734097}k_9 + \frac{27016}{8067}k_{10} - \frac{934216}{244699}k_{11} + \frac{256}{273}k_{12}, \\
c_3 &= \frac{43804}{8067}k_0 - \frac{147610}{24201}k_1 + \frac{161020}{24201}k_2 - \frac{19082}{2689}k_3 \\
&\quad + \frac{179924}{24201}k_4 - \frac{185834}{24201}k_5 + \frac{63284}{8067}k_6 - \frac{192490}{24201}k_7 \\
&\quad + \frac{194260}{24201}k_8 - \frac{65054}{8067}k_9 + \frac{196064}{24201}k_{10} - \frac{196064}{24201}k_{11}, \\
c_4 &= \frac{99265280}{734097}k_0 - \frac{193319296}{2202291}k_1 + \frac{124840960}{2202291}k_2 \\
&\quad - \frac{26219264}{734097}k_3 + \frac{6725120}{314613}k_4 - \frac{25219328}{2202291}k_5 \\
&\quad + \frac{1100544}{244699}k_6 + \frac{2423552}{2202291}k_7 - \frac{1178624}{169407}k_8 \\
&\quad + \frac{10973696}{734097}k_9 - \frac{9282304}{314613}k_{10} + \frac{123832960}{2202291}k_{11} \\
&\quad - \frac{28928}{273}k_{12},
\end{aligned}$$

$$
\begin{aligned}
c_6 &= \frac{89933384}{734097}k_0 - \frac{59999908}{734097}k_1 + \frac{12268224}{244699}k_2 \\
&\quad - \frac{21160462}{734097}k_3 + \frac{1505060}{104871}k_4 - \frac{1123938}{244699}k_5 \\
&\quad - \frac{1252540}{734097}k_6 + \frac{4237370}{734097}k_7 - \frac{2060412}{244699}k_8 \\
&\quad + \frac{7256234}{734097}k_9 - \frac{91552}{8067}k_{10} + \frac{2891808}{244699}k_{11} \\
&\quad + \frac{139520}{273}k_{12}, \\
c_{12} &= \frac{276592384}{734097}k_0 + \frac{193319296}{2202291}k_1 - \frac{124840960}{2202291}k_2 \\
&\quad + \frac{26219264}{734097}k_3 - \frac{6725120}{314613}k_4 + \frac{25219328}{2202291}k_5 \\
&\quad - \frac{1100544}{244699}k_6 - \frac{2423552}{2202291}k_7 + \frac{1178624}{169407}k_8 \\
&\quad - \frac{10973696}{734097}k_9 + \frac{9282304}{314613}k_{10} - \frac{123832960}{2202291}k_{11} \\
&\quad - \frac{110848}{273}k_{12}, \\
r_1 &= -\frac{608}{91}k_0 + \frac{26672}{91}k_1 - \frac{26480}{91}k_2 + \frac{19792}{91}k_3 \\
&\quad - \frac{1824}{13}k_4 + \frac{7680}{91}k_5 - \frac{4208}{91}k_6 + \frac{2032}{91}k_7 \\
&\quad - \frac{64}{7}k_8 - \frac{32}{91}k_9 + \frac{32}{13}k_{10} - \frac{608}{91}k_{11} + \frac{1216}{91}k_{12},
\end{aligned}
$$

$$
\begin{aligned}
r_2 &= \frac{8272}{273}k_0 + \frac{456}{91}k_1 - \frac{2648}{273}k_2 + \frac{1912}{273}k_3 \\
&\quad - \frac{48}{13}k_4 + \frac{544}{273}k_5 - \frac{320}{273}k_6 + \frac{64}{91}k_7 \\
&\quad - \frac{128}{273}k_8 + \frac{64}{273}k_9 - \frac{128}{273}k_{11} + \frac{256}{273}k_{12}, \\
r_3 &= \frac{144}{13}k_0 - \frac{1624}{39}k_1 + 40k_2 - \frac{392}{13}k_3 + \frac{752}{39}k_4 \\
&\quad - \frac{128}{13}k_5 + \frac{16}{13}k_6 + \frac{272}{39}k_7 - \frac{192}{13}k_8 \\
&\quad + \frac{288}{13}k_9 - \frac{1120}{39}k_{10} + \frac{544}{13}k_{11} - \frac{1088}{13}k_{12}, \\
r_4 &= -\frac{149764}{2689}k_0 + \frac{428678}{8067}k_1 - \frac{361214}{8067}k_2 + \frac{90816}{2689}k_3 \\
&\quad - \frac{183448}{8067}k_4 + \frac{108562}{8067}k_5 - \frac{17726}{2689}k_6 + \frac{13724}{8067}k_7 \\
&\quad + \frac{13372}{8067}k_8 - \frac{9370}{2689}k_9 + \frac{42848}{8067}k_{10} - \frac{42848}{8067}k_{11},
\end{aligned}
$$

$$\begin{aligned}
r_5 &= \frac{38036}{8067}k_0 - \frac{98006}{24201}k_1 + \frac{80510}{24201}k_2 - \frac{6852}{2689}k_3 \\
&\quad + \frac{41560}{24201}k_4 - \frac{20314}{24201}k_5 - \frac{626}{8067}k_6 + \frac{24760}{24201}k_7 \\
&\quad - \frac{48076}{24201}k_8 + \frac{23942}{8067}k_9 - \frac{95576}{24201}k_{10} + \frac{95576}{24201}k_{11}, \\
r_6 &= -\frac{136}{2689}k_0 + \frac{364}{8067}k_1 - \frac{220}{8067}k_2 - \frac{30}{2689}k_3 \\
&\quad + \frac{676}{8067}k_4 - \frac{1714}{8067}k_5 + \frac{1156}{2689}k_6 - \frac{6290}{8067} \\
&\quad k_7 + \frac{10532}{8067}k_8 - \frac{5398}{2689}k_9 + \frac{21856}{8067}k_{10} - \frac{21856}{8067}k_{11}, \\
r_7 &= \frac{111864}{2689}k_0 - \frac{395572}{8067}k_1 + \frac{345460}{8067}k_2 - \frac{86364}{2689}k_3 \\
&\quad + \frac{173480}{8067}k_4 - \frac{102668}{8067}k_5 + \frac{17196}{2689}k_6 - \frac{16060}{8067}k_7 \\
&\quad - \frac{8096}{8067}k_8 + \frac{6960}{2689}k_9 - \frac{33664}{8067}k_{10} + \frac{33664}{8067}k_{11}.
\end{aligned}$$

Now we can state our main theorem:

Theorem 1. *The functions $f_1, f_2, \ldots, f_6$ are in $S_6(\Gamma_0(12))$, while f_7 is in $M_6(\Gamma_0(12))$, and*

$$\eta^{a_1}(z)\,\eta^{a_2}(2z)\,\eta^{a_3}(3z)\,\eta^{a_4}(4z)\,\eta^{a_6}(6z)\,\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n,$$

where for $n \in \mathbb{N}$,

$$\begin{aligned}
c(n) &= -c_1\sigma_5(n) - c_2\sigma_5\left(\frac{n}{2}\right) - c_3\sigma_5\left(\frac{n}{3}\right) - c_4\sigma_5\left(\frac{n}{4}\right) - c_6\sigma_5\left(\frac{n}{6}\right) - c_{12}\sigma_5\left(\frac{n}{12}\right) \\
&\quad + r_1f_1(n) + r_2f_2(n) + r_3f_3(n) + r_4f_4(n) + r_5f_5(n) + r_6f_6(n) + r_7f_7(n).
\end{aligned}$$

In particular,

$$\begin{aligned}
c(2n) &= -c_1\sigma_5(2n) - c_2\sigma_5(n) - c_4\sigma_5\left(\frac{n}{2}\right) - (33c_3 + c_6)\,\sigma_5\left(\frac{n}{3}\right) \\
&\quad - (c_{12} - 32c_3)\sigma_5\left(\frac{n}{6}\right) + r_1f_1(2n) + r_2f_2(2n) + r_3f_3(2n), \\
c(2n-1) &= -c_1\sigma_5(2n-1) - c_3\sigma_5\left(\frac{2n-1}{3}\right) \\
&\quad + r_4f_4(2n-1) + r_5f_5(2n-1) + r_6f_6(2n-1) + r_7f_7(2n-1),
\end{aligned}$$

for $n \in \mathbb{N}$.

Proof. It follows from (1) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1,$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 12$$

and

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - 2\frac{a_4}{3} - \frac{a_6}{3} - 2\frac{a_{12}}{3} = -b_1 - b_5.$$

Now we will use (p, k), the parametrization of Alaca, Alaca and Williams (see [2]):

$$p\left(q\right) = \frac{\varphi^2\left(q\right) - \varphi^2\left(q^3\right)}{2\varphi^2\left(q^3\right)}, k\left(q\right) = \frac{\varphi^3\left(q^3\right)}{\varphi\left(q\right)},$$

where the theta function $\varphi\left(q\right)$ is defined by

$$\varphi\left(q\right) = \sum_{-\infty}^{\infty} q^{n^2}.$$

Setting $x = p$ in (2) and multiplying both sides by k^6, we obtain

$$\begin{aligned} &\tfrac{k^6}{2^{b_1+b_5}} p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5} \\ &\quad = \quad (k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + k_6 p^6 \\ &\quad\quad\quad + k_7 p^7 + k_8 p^8 + k_9 p^9 + k_{10} p^{10} + k_{11} p^{11} + k_{12} p^{12}) k^6. \end{aligned}$$

Alaca, Alaca and Williams [1] have established the following representations in terms of p and k:

$$\eta\left(q\right) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \quad (3)$$

$$\eta\left(q^2\right) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \quad (4)$$

$$\eta\left(q^3\right) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \quad (5)$$

$$\eta\left(q^4\right) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \quad (6)$$

$$\eta\left(q^6\right) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \quad (7)$$

$$\eta\left(q^{12}\right) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}. \quad (8)$$

We also have the following:

$$\begin{aligned} E_6(q) = 1 - 504\sum_{n=1}^{\infty} \sigma_5\left(n\right) q^n \quad &= \quad (1 - 246p - 5532p^2 - 38614p^3 - 135369p^4 - 276084p^5 \\ &\quad -348024p^6 - 276084p^7 - 135369p^8 - 38614p^9 - 5532p^{10} \\ &\quad -246p^{11} + p^{12}) k^6, \end{aligned}$$

$$
\begin{aligned}
E_6(q^2) &= (1+6p-114p^2-625p^3-\frac{4059}{2}p^4-4302p^5 \\
&\quad -5556p^6-4302p^7-\frac{4059}{2}p^8-625p^9-114p^{10} \\
&\quad +6p^{11}+p^{12})k^6, \\
E_6(q^3) &= (1+6p+12p^2-58p^3-297p^4-396p^5-264p^6-396p^7 \\
&\quad -297p^8-58p^9+12p^{10}+6p^{11}+p^{12})k^6, \\
E_6(q^4) &= (1+6p+12p^2+5p^3-45p^4-144p^5-\frac{1167}{8}p^6+\frac{171}{8}p^7 \\
&\quad +\frac{2151}{32}p^8-\frac{739}{16}p^9-\frac{345}{8}p^{10} \\
&\quad +\frac{129}{32}p^{11}+\frac{1}{64}p^{12})k^6, \\
E_6(q^6) &= (1+6p+12p^2+5p^3-\frac{27}{2}p^4-18p^5 \\
&\quad -12p^6-18p^7-\frac{27}{2}p^8+5p^9+12p^{10}+6p^{11}+p^{12})k^6, \\
E_6(q^{12}) &= (1+6p+12p^2+5p^3-\frac{27}{2}p^4-18p^5 \\
&\quad -\frac{33}{8}p^6+\frac{45}{8}p^7+\frac{135}{32}p^8+\frac{17}{16}p^9+\frac{3}{16}p^{10} \\
&\quad +\frac{3}{32}p^{11}+\frac{1}{64}p^{12})k^6.
\end{aligned}
$$

It is easy to check the following expressions using (3)-(8):

$$
\begin{aligned}
f_1 &= \sum_{n=0}^{\infty} f_1(n)\,q^n = \eta(2z)\,\eta(4z)\,\eta^5(6z)\,\eta^5(12z) \\
&= \left(\frac{1}{16}p^4+\frac{1}{4}p^5+\frac{17}{64}p^6-\frac{5}{64}p^7-\frac{19}{64}p^8-\frac{11}{64}p^9-\frac{1}{32}p^{10}\right)k^6, \\
f_2 &= \sum_{n=0}^{\infty} f_2(n)\,q^n = \eta^5(2z)\,\eta^5(4z)\,\eta(6z)\,\eta(12z) \\
&= (\frac{1}{4}p^2+\frac{5}{4}p^3+\frac{13}{8}p^4-p^5-\frac{223}{64}p^6-\frac{109}{64}p^7 \\
&\quad +\frac{71}{64}p^8+\frac{89}{64}p^9+\frac{1}{2}p^{10}+\frac{1}{16}p^{11})k^6, \\
f_3 &= \sum_{n=0}^{\infty} f_3(n)\,q^n = \frac{\eta^9(2z)\,\eta^9(12z)}{\eta^3(4z)\,\eta^3(6z)} \\
&= (\frac{1}{16}p^4+\frac{1}{4}p^5+\frac{9}{64}p^6-\frac{29}{64}p^7 \\
&\quad -\frac{29}{64}p^8+\frac{9}{64}p^9+\frac{1}{4}p^{10}+\frac{1}{16}p^{11})k^6,
\end{aligned}
$$

$$\begin{aligned}
f_4 &= \sum_{n=0}^{\infty} f_4(n)\, q^n = \frac{\eta^8(4z)\,\eta^{16}(6z)}{\eta^4(2z)\,\eta^8(12z)} \\
&= (\frac{1}{2}p + \frac{11}{4}p^2 + \frac{43}{8}p^3 + \frac{57}{16}p^4 - \frac{9}{4}p^5 \\
&\quad -\frac{21}{4}p^6 - \frac{7}{2}p^7 - \frac{17}{16}p^8 - \frac{1}{8}p^9)k^6, \\
f_5 &= \sum_{n=0}^{\infty} f_5(n)\, q^n = \eta^{12}(2z) \\
&= (\frac{1}{2}p + \frac{11}{4}p^2 + \frac{27}{8}p^3 - \frac{87}{16}p^4 - \frac{51}{4}p^5 \\
&\quad +\frac{51}{4}p^7 + \frac{87}{16}p^8 - \frac{27}{8}p^9 - \frac{11}{4}p^{10} - \frac{1}{2}p^{11})k^6, \\
f_6 &= \sum_{n=0}^{\infty} f_6(n)\, q^n = \frac{\eta^5(2z)\,\eta^{13}(6z)}{\eta(4z)\,\eta^5(12z)} \\
&= (\frac{1}{2}p + \frac{11}{4}p^2 + \frac{17}{4}p^3 - \frac{3}{2}p^4 - 9p^5 \\
&\quad -\frac{21}{4}p^6 + \frac{13}{4}p^7 + 4p^8 + p^9)k^6, \\
f_7 &= \sum_{n=0}^{\infty} f_7(n)\, q^n = \frac{\eta^{12}(4z)\,\eta^{24}(6z)}{\eta^{12}(2z)\,\eta^{12}(12z)} \\
&= (\frac{1}{2}p + \frac{11}{4}p^2 + \frac{51}{8}p^3 + \frac{129}{16}p^4 + 6p^5 \\
&\quad +\frac{21}{8}p^6 + \frac{5}{8}p^7 + \frac{1}{16}p^8)k^6.
\end{aligned}$$

We see that $f_1, \dots f_6, \in S_6(\Gamma_0(12))$, $f_7 \in M_6(\Gamma_0(12))$ and

$$ord_{1/1} f_7 = 0,\ ord_{1/2} f_7 = 0,$$

by [5]. Now

$$\begin{aligned}
&\eta^{a_1}(z)\,\eta^{a_2}(2z)\,\eta^{a_3}(3z)\,\eta^{a_4}(4z)\,\eta^{a_6}(6z)\,\eta^{a_{12}}(12z) \\
= &\ q^{b_1} \prod_{n=1}^{\infty} (1-q^n)^{a_1} \left(1-q^{2n}\right)^{a_2} \left(1-q^{3n}\right)^{a_3} \left(1-q^{4n}\right)^{a_4} \left(1-q^{6n}\right)^{a_6} \left(1-q^{12n}\right)^{a_{12}} \\
= &\ 2^{-\frac{a_1}{6}-\frac{a_2}{3}-\frac{a_3}{6}-2\frac{a_4}{3}-\frac{a_6}{3}-2\frac{a_{12}}{3}} p^{\frac{a_1}{24}+\frac{a_2}{12}+\frac{a_3}{8}+\frac{a_4}{6}+\frac{a_6}{4}+\frac{a_{12}}{2}} (1-p)^{\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{6}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}} \\
&\ (1+p)^{\frac{a_1}{6}+\frac{a_2}{12}+\frac{a_3}{2}+\frac{a_4}{24}+\frac{a_6}{4}+\frac{a_{12}}{8}} (1+2p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}} (2+p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{2}+\frac{a_6}{12}+\frac{a_{12}}{6}} \\
&\ k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}} = \frac{k^6}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5}
\end{aligned}$$

$$
\begin{aligned}
&= k^6(k_0 + k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + k_6 p^6 \\
&\quad + k_7 p^7 + k_8 p^8 + k_9 p^9 + k_{10} p^{10} + k_{11} p^{11} + k_{12} p^{12}) \\
&= \frac{c_1}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^n\right) + \frac{c_2}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^{2n}\right) \\
&\quad + \frac{c_3}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^{3n}\right) + \frac{c_4}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^{4n}\right) \\
&\quad + \frac{c_6}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^{6n}\right) + \frac{c_{12}}{504}\left(1 - 504\sum_{n=1}^{\infty}\sigma_5(n)\, q^{12n}\right) \\
&\quad + r_1 q^4 \prod_{n=1}^{\infty}\left(1 - q^{2n}\right)\left(1 - q^{4n}\right)\left(1 - q^{6n}\right)^5\left(1 - q^{12n}\right)^5 \\
&\quad + r_2 q^2 \prod_{n=1}^{\infty}\left(1 - q^{2n}\right)^5\left(1 - q^{4n}\right)^5\left(1 - q^{6n}\right)\left(1 - q^{12n}\right) \\
&\quad + r_3 q^4 \prod_{n=1}^{\infty}\frac{\left(1 - q^{2n}\right)^9\left(1 - q^{12n}\right)^9}{\left(1 - q^{4n}\right)^3\left(1 - q^{6n}\right)^3} \\
&\quad + r_4 q \prod_{n=1}^{\infty}\frac{\left(1 - q^{4n}\right)^8\left(1 - q^{6n}\right)^{16}}{\left(1 - q^{2n}\right)^4\left(1 - q^{12n}\right)^8} \\
&\quad + r_5 . q \prod_{n=1}^{\infty}\left(1 - q^{2n}\right)^{12} + r_6 q \prod_{n=1}^{\infty}\frac{\left(1 - q^{2n}\right)^5\left(1 - q^{6n}\right)^{13}}{\left(1 - q^{4n}\right)\left(1 - q^{12n}\right)^5} \\
&\quad + r_7 q \prod_{n=1}^{\infty}\frac{\left(1 - q^{4n}\right)^{12}\left(1 - q^{6n}\right)^{24}}{\left(1 - q^{2n}\right)^{12}\left(1 - q^{12n}\right)^{12}} \\
&= \delta(b_1) - \sum_{n=1}^{\infty}(c_1\sigma_5(n) + c_2\sigma_5\left(\frac{n}{2}\right) + c_3\sigma_5\left(\frac{n}{3}\right) + c_4\sigma_5\left(\frac{n}{4}\right) \\
&\quad + c_6\sigma_5\left(\frac{n}{6}\right) + c_{12}\sigma_5\left(\frac{n}{12}\right))q^n + r_1 f_1(n) q^n + ... + r_7 f_7(n) q^n,
\end{aligned}
$$

where

$$
\delta(b_1) = \begin{cases} 0 \text{ if } b_1 \neq 0 \\ 1 \text{ if } b_1 = 0 \end{cases} .
$$

So

$$
\begin{aligned}
c(n) &= -(c_1\sigma_5(n) + c_2\sigma_5\left(\frac{n}{2}\right) + c_3\sigma_5\left(\frac{n}{3}\right) + c_4\sigma_5\left(\frac{n}{4}\right) \\
&\quad + c_6\sigma_5\left(\frac{n}{6}\right) + c_{12}\sigma_5\left(\frac{n}{12}\right)) + r_1 f_1(n) + ... + r_7 f_7(n).
\end{aligned}
$$

Therefore, for $n = 1, 2, \ldots,$

$$\begin{aligned} c(2n) &= -c_1\sigma_5(2n) - c_2\sigma_5(n) - c_4\sigma_5\left(\frac{n}{2}\right) - (33c_3 + c_6)\,\sigma_5\left(\frac{n}{3}\right) \\ &\quad -(c_{12} - 32c_3)\sigma_5\left(\frac{n}{6}\right) + r_1f_1(2n) + r_2f_2(2n) + r_3f_3(2n), \end{aligned}$$

$$\begin{aligned} c(2n-1) &= -c_1\sigma_5(2n-1) - c_3\sigma_5\left(\frac{2n-1}{3}\right) \\ &\quad +r_4f_4(2n-1) + r_5f_5(2n-1) + r_6f_6(2n-1) + r_7f_7(2n-1), \end{aligned}$$

since it is easy to see that

$$\sigma_5\left(\frac{2n}{3}\right) = 33\sigma_5\left(\frac{n}{3}\right) - 32\sigma_5\left(\frac{n}{6}\right),$$

and, for $n = 1, 2, \ldots,$

$$f_1(2n-1) = f_2(2n-1) = f_3(2n-1) = 0,$$

$$f_4(2n) = f_5(2n) = f_6(2n) = f_7(2n) = 0.$$

These formulas are valid for 6135 nontrivial eta quotients; see www.bariskendirli.com.tr\eta_quotients.

Among them, we have found 74 eta quotients (see Table1/A), such that

$$\begin{aligned} c(2n) &= -c_1\sigma_5(2n) - c_2\sigma_5(n) - c_4\sigma_5\left(\frac{n}{2}\right) - (33c_3 + c_6)\,\sigma_5\left(\frac{n}{3}\right) \\ &\quad -(c_{12} - 32c_3)\sigma_5\left(\frac{n}{6}\right) + r_1f_1(2n) + r_2f_2(2n) + r_3f_3(2n), \\ c(2n-1) &= -c_1\sigma_5(2n-1) - c_3\sigma_5\left(\frac{2n-1}{3}\right) = 0; \end{aligned}$$

and 60 eta quotients (see Table2/A), such that

$$\begin{aligned} c(2n) &= -c_1\sigma_5(2n) - c_2\sigma_5(n) - c_4\sigma_5\left(\frac{n}{2}\right) - (33c_3 + c_6)\,\sigma_5\left(\frac{n}{3}\right) \\ &\quad -(c_{12} - 32c_3)\sigma_5\left(\frac{n}{6}\right), \\ c(2n-1) &= -c_1\sigma_5(2n-1) - c_3\sigma_5\left(\frac{2n-1}{3}\right) + r_4f_4(2n-1) + r_5f_5(2n-1) \\ &\quad +r_6f_6(2n-1) + r_7f_7(2n-1). \end{aligned}$$

Remark 1: The coefficients $k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}$ and $r_1, r_2, r_3, r_4, r_5, r_6, r_7$ are given in Table1/B and Table2/B.

Remark 2: If f is an eta quotient, then $f(-q)$ is also an eta quotient, the coefficients of $\frac{1}{2}(f(q)-f(-q))$ are exactly the odd coefficients of f, and the coefficients of $\frac{1}{2}(f(q)+f(-q))$ are exactly the even coefficients of f. In particular, this means that we have obtained all coefficients of the difference of seventy-four eta quotients and the sum of sixty eta quotients.

Remark 3: The space $S_6(\Gamma_0(12))$ is seven-dimensional(see [3, Chapter 3 and 5]), and it is generated by

$$\Delta_{3,6}, \Delta_{3,6}(2z), \Delta_{3,6}(4z), \Delta_{4,6}, \Delta_{4,6}(3z), \Delta_{6,6}, \Delta_{6,6}(2z),$$

where $\Delta_{3,6}$ is the unique newform in $S_6(\Gamma_0(3))$, $\Delta_{4,6}$ is the unique newform in $S_6(\Gamma_0(4))$, and $\Delta_{6,6}$ is the unique newform in $S_6(\Gamma_0(6))$. By simple calculation, we see that

$$\begin{aligned}
f_1 &= -\frac{1}{18}\Delta_{3,6}(2z)+\frac{4}{9}\Delta_{3,6}(4z)+\frac{1}{18}\Delta_{6,6}(2z),\\
f_2 &= \frac{1}{2}\Delta_{3,6}(2z)-4\Delta_{3,6}(4z)+\frac{1}{2}\Delta_{6,6}(2z),\\
f_3 &= -\frac{1}{2}\Delta_{3,6}(2z)-4\Delta_{3,6}(4z)+\frac{1}{2}\Delta_{6,6}(2z),\\
f_4 &= \frac{4}{9}\Delta_{3,6}(z)+\frac{8}{3}\Delta_{3,6}(2z)+\frac{128}{9}\Delta_{3,6}(4z)+\frac{1}{3}\Delta_{4,6}(z)\\
&\quad +6\Delta_{4,6}(2z)+\frac{2}{9}\Delta_{6,6}(z)-\frac{8}{9}\Delta_{6,6}(2z),\\
f_5 &= \Delta_{4,6}(z),\\
f_6 &= \frac{4}{9}\Delta_{3,6}(z)+\frac{8}{3}\Delta_{3,6}(2z)+\frac{128}{9}\Delta_{3,6}(4z)+\frac{1}{3}\Delta_{4,6}(z)\\
&\quad -3\Delta_{4,6}(2z)+\frac{2}{9}\Delta_{6,6}(z)-\frac{8}{9}\Delta_{6,6}(2z).
\end{aligned}$$

Note that f_7 is in $M_6(\Gamma_0(12))\backslash S_6(\Gamma_0(12))$, so it can be written in the form

$$\begin{aligned}
f_7 &= \frac{55}{117}\Delta_{3,6}(z)+\frac{110}{39}\Delta_{3,6}(2z)+\frac{1760}{117}\Delta_{3,6}(4z)+\frac{1}{3}\Delta_{4,6}(z)+8\Delta_{4,6}(2z)\\
&\quad +\frac{11}{63}\Delta_{6,6}(z)-\frac{44}{63}\Delta_{6,6}(2z)-\frac{1}{22932}E_6(z)+\frac{11}{7644}E_6(2z)\\
&\quad +\frac{1}{22932}E_6(3z)-\frac{8}{5733}E_6(4z)-\frac{11}{7644}E_6(6z)+\frac{8}{5733}E_6(12z).
\end{aligned}$$

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Appendix

Table 1/A

No	a_1	a_2	a_3	a_4	a_6	a_{12}	c_2	c_4	c_6	c_{12}
1	0	−18	0	36	6	−12	−4905/1456	3897/1456	−6561/1456	741393/1456
2	0	−17	0	31	11	−13	−2179/728	1675/728	−3645/728	371061/728
3	0	−16	0	26	16	−14	−242/91	179/91	−486/91	46413/91
4	0	−15	0	21	21	−15	−215/91	152/91	−513/91	46440/91
5	0	−14	0	16	26	−16	−191/91	128/91	−537/91	46464/91
6	0	−13	0	11	31	−17	−509/273	320/273	−1675/273	139456/273
7	0	−12	0	6	36	−18	−452/273	256/273	−1732/273	139520/273
8	0	−9	0	27	3	−9	9/728	−513/728	−6561/728	373977/728
9	0	−8	0	22	8	−10	1/91	−64/61	−729/91	46656/91
10	0	−7	0	17	13	−11	1/91	−64/91	−729/91	46656/91
11	0	−6	0	12	18	−12	1/91	−64/91	−729/91	46656/91
12	0	−5	0	7	23	−13	1/91	−64/91	−729/91	46656/91
13	0	−4	0	2	28	−14	4/273	−256/273	−2188/273	140032/273
14	0	0	0	18	0	−6	0	−9/13	0	6561/13
15	0	1	0	13	5	−7	1/91	−64/91	−729/91	46656/91
16	0	2	0	8	10	−8	1/91	−64/91	−729/91	46656/91
17	0	3	0	3	15	−9	1/91	−64/91	−729/91	46656/91
18	0	4	0	−2	20	−10	4/91	−256/91	−732/91	46848/91
19	0	9	0	9	−3	−3	9/91	−72/91	−6561/91	52488/91
20	0	10	0	4	2	−4	1/91	−64/91	−729/91	46656/91
21	0	11	0	−1	7	−5	1/91	−64/91	−729/91	46656/91
22	0	12	0	−6	12	−6	4/13	−256/13	−108/13	6912/13
23	0	18	0	0	−6	0	−9/13	0	6561/13	0
24	0	19	0	−5	−1	−1	1/91	−64/91	−729/91	46656/91
25	0	20	0	−10	4	−2	244/91	−15616/91	−972/91	62208/91
26	0	27	0	−9	−9	3	513/91	−576/91	−373977/91	419904/91
27	0	28	0	−14	−4	2	2188/91	−140032/91	−2916/91	186624/91
28	0	36	0	−18	−12	6	15588/91	−1255680/91	2965572/91	−1679616/91
29	0	−14	0	28	2	−4	−547/1456	547/1456	729/1456	−729/1456
30	0	−13	0	23	7	−5	−243/728	243/728	243/728	−243/728
31	0	−12	0	18	12	−6	−27/91	27/91	27/91	−27/91
32	0	−11	0	13	17	−7	−24/91	24/91	24/91	−24/91
33	0	−10	0	8	22	−8	−64/273	64/273	64/273	−64/273
34	0	−9	0	3	27	−9	−19/91	64/273	19/91	−64/273
35	0	−5	0	19	−1	−1	1/728	−1/728	−729/728	729/728
36	0	−1	0	−1	19	−5	−1/273	64/273	1/273	−64/273
37	0	4	0	10	−4	2	−1/91	1/91	729/91	−729/91
38	0	7	0	−5	11	−1	−3/91	192/91	3/91	−192/91
39	0	13	0	1	−7	5	8/91	−8/91	−5832/91	5832/91
40	0	15	0	−9	3	3	−27/91	1728/91	27/91	−1728/91
41	0	22	0	−8	−10	8	−64/91	64/91	46656/91	−46656/91
42	0	23	0	−13	−5	7	−243/91	15552/91	243/91	−15552/91
43	0	31	0	−17	−13	11	−1675/91	139456/91	−371061/91	233280/91

44	0	−10	0	20	−2	4	−61/1456	61/1456	243/1456	−243/1456
45	0	−9	0	15	3	3	−27/728	27/728	27/728	−27/728
46	0	−8	0	10	8	2	−3/91	3/91	3/91	−3/910
47	0	−7	0	5	13	1	−8/273	8/273	8/273	−8/273
48	0	−6	0	0	18	0	−1/39	0	1/39	0
49	0	−1	0	11	−5	7	1/728	−1/728	−729/728	729/728
50	0	2	0	−4	10	4	1/273	−64/273	−1/273	64/273
51	0	8	0	2	−8	10	−1/91	1/91	729/91	−729/91
52	0	10	0	−8	2	8	3/91	−192/91	−3/91	192/91
53	0	17	0	−7	−11	13	8/91	−8/91	−5832/91	5832/91
54	0	18	0	−12	−6	12	27/91	−1728/91	−27/91	1728/91
55	0	26	0	−16	−14	16	179/91	−15488/91	46413/91	−31104/91
56	0	−6	0	12	−6	12	−1/208	1/208	27/208	−27/208
57	0	−5	0	7	−1	11	−3/728	3/728	3/728	−3/728
58	0	−4	0	2	4	10	−1/273	1/273	1/273	−1/273
59	0	−3	0	−3	9	9	−1/273	8/273	1/273	−8/273
60	0	3	0	3	−9	15	1/728	−1/728	−729/728	729/728
61	0	5	0	−7	1	13	−1/273	64/273	1/273	−64/273
62	0	12	0	−6	−12	18	−1/91	1/91	729/91	−729/91
63	0	13	0	−11	−7	17	−3/91	192/91	3/91	−192/91
64	0	21	0	−15	−15	21	−19/91	1720/91	−5805/91	4104/91
65	0	−2	0	4	−10	20	−1/1456	1/1456	183/1456	−183/1456
66	0	−1	0	−1	−5	19	−1/2184	1/2184	1/2184	−1/2184
67	0	0	0	−6	0	18	0	−1/39	0	1/39
68	0	7	0	−5	−13	23	1/728	−1/728	−729/728	729/728
69	0	8	0	−10	−8	22	1/273	−64/273	−1/273	64/273
70	0	16	0	−14	−16	26	2/91	−191/91	726/91	−537/91
71	0	2	0	−4	−14	28	−1/4368	1/4368	547/4368	−547/4368
72	0	3	0	−9	−9	27	−1/2184	19/728	1/2184	−19/728
73	0	11	0	−13	−17	31	−5/2184	509/2184	−2179/2184	1675/2184
74	0	6	0	−12	−18	36	1/4368	−113/4368	545/4368	−433/4368

Table 1/B

No	k_0	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	r_1	r_2	r_3
1	1	6	$\frac{33}{2}$	$\frac{55}{2}$	$\frac{495}{16}$	$\frac{99}{4}$	$\frac{231}{16}$	$\frac{99}{16}$	$\frac{495}{256}$	$\frac{55}{128}$	$\frac{33}{512}$	$\frac{3}{512}$	$\frac{1}{4096}$	$\frac{772011}{5824}$	$\frac{21303}{1456}$	$-\frac{12393}{832}$
2	1	6	$\frac{65}{4}$	$\frac{105}{4}$	$\frac{225}{8}$	21	$\frac{357}{32}$	$\frac{135}{32}$	$\frac{285}{256}$	$\frac{25}{128}$	$\frac{21}{1024}$	$\frac{1}{1024}$	0	$\frac{23085}{182}$	$\frac{10197}{728}$	$-\frac{1485}{104}$
3	1	6	16	25	$\frac{203}{8}$	$\frac{35}{2}$	$\frac{133}{16}$	$\frac{43}{16}$	$\frac{145}{256}$	$\frac{9}{128}$	$\frac{1}{256}$	0	0	$\frac{87723}{728}$	$\frac{1214}{91}$	$-\frac{1413}{104}$
4	1	6	$\frac{63}{4}$	$\frac{95}{4}$	$\frac{363}{16}$	$\frac{57}{4}$	$\frac{377}{64}$	$\frac{99}{64}$	$\frac{15}{64}$	$\frac{1}{64}$	0	0	0	$\frac{10350}{91}$	$\frac{1150}{91}$	$-\frac{167}{13}$
5	1	6	$\frac{31}{2}$	$\frac{45}{2}$	$\frac{321}{16}$	$\frac{45}{4}$	$\frac{31}{8}$	$\frac{3}{4}$	$\frac{1}{16}$	0	0	0	0	$\frac{9712}{91}$	$\frac{1083}{91}$	$-\frac{157}{13}$
6	1	6	$\frac{61}{4}$	$\frac{85}{4}$	$\frac{35}{2}$	$\frac{17}{2}$	$\frac{9}{4}$	$\frac{1}{4}$	0	0	0	0	0	$\frac{9064}{91}$	$\frac{3040}{273}$	$-\frac{440}{39}$
7	1	6	15	20	15	6	1	0	0	0	0	0	0	$\frac{8416}{91}$	$\frac{2824}{273}$	$-\frac{136}{13}$
8	1	6	$\frac{57}{4}$	$\frac{65}{4}$	$\frac{45}{8}$	-9	$-\frac{483}{32}$	$-\frac{369}{32}$	$-\frac{1395}{256}$	$-\frac{215}{128}$	$-\frac{339}{1024}$	$-\frac{39}{1024}$	$-\frac{1}{512}$	$\frac{59049}{728}$	$\frac{6561}{728}$	$-\frac{243}{26}$
9	1	6	14	15	$\frac{27}{8}$	$-\frac{21}{2}$	$-\frac{231}{16}$	$-\frac{153}{16}$	$-\frac{975}{256}$	$-\frac{119}{128}$	$-\frac{33}{256}$	$-\frac{1}{128}$	0	$\frac{50787}{728}$	$\frac{729}{91}$	$-\frac{837}{104}$
10	1	6	$\frac{55}{4}$	$\frac{55}{4}$	$\frac{19}{16}$	$-\frac{47}{4}$	$-\frac{863}{64}$	$-\frac{485}{64}$	$-\frac{155}{64}$	$-\frac{27}{64}$	$-\frac{1}{32}$	0	0	$\frac{5427}{91}$	$\frac{638}{91}$	$-\frac{90}{13}$
11	1	6	27/2	$\frac{25}{2}$	$-\frac{15}{16}$	$-\frac{51}{4}$	$-\frac{49}{4}$	$-\frac{45}{8}$	$-\frac{21}{16}$	$-\frac{1}{8}$	0	0	0	$\frac{4608}{91}$	$\frac{547}{91}$	$-\frac{77}{13}$
12	1	6	53/4	$\frac{45}{4}$	-3	$-\frac{27}{2}$	$-\frac{43}{4}$	$-\frac{15}{4}$	$-\frac{1}{2}$	0	0	0	0	$\frac{3880}{91}$	$\frac{456}{91}$	$-\frac{64}{13}$
13	1	6	13	10	-5	-14	-9	-2	0	0	0	0	0	$\frac{3232}{91}$	$\frac{1096}{273}$	$-\frac{152}{39}$
14	1	6	12	5	$-\frac{117}{8}$	$-\frac{45}{2}$	$\frac{147}{16}$	$\frac{99}{16}$	$\frac{2385}{256}$	$\frac{649}{128}$	$\frac{381}{256}$	$\frac{15}{64}$	$\frac{1}{64}$	$-\frac{2187}{104}$	0	$\frac{243}{104}$
15	1	6	$\frac{47}{4}$	$\frac{15}{4}$	$-\frac{261}{16}$	$-\frac{87}{4}$	$-\frac{375}{64}$	$\frac{531}{64}$	$\frac{555}{64}$	$\frac{233}{64}$	$\frac{3}{4}$	$\frac{1}{16}$	0	$-\frac{1944}{91}$	$-\frac{90}{91}$	$\frac{27}{13}$
16	1	6	$\frac{23}{2}$	$\frac{5}{2}$	$-\frac{287}{16}$	$-\frac{83}{4}$	$-\frac{19}{8}$	10	$\frac{121}{16}$	$\frac{9}{4}$	$\frac{1}{4}$	0	0	$-\frac{1944}{91}$	$-\frac{181}{91}$	$\frac{27}{13}$
17	1	6	$\frac{45}{4}$	$\frac{5}{4}$	$-\frac{39}{2}$	$-\frac{39}{2}$	$\frac{5}{4}$	$\frac{45}{4}$	6	1	0	0	0	$-\frac{1944}{91}$	$-\frac{272}{91}$	$\frac{40}{13}$
18	1	6	11	0	-21	-18	5	12	4	0	0	0	0	$-\frac{1952}{91}$	$-\frac{360}{91}$	$\frac{56}{13}$
19	1	6	$\frac{39}{4}$	$-\frac{25}{4}$	$-\frac{477}{16}$	$-\frac{63}{4}$	$\frac{1329}{64}$	$\frac{1611}{64}$	$\frac{225}{64}$	$-\frac{499}{64}$	$-\frac{165}{32}$	$-\frac{21}{16}$	$-\frac{1}{8}$	$-\frac{2187}{91}$	$-\frac{810}{91}$	0
20	1	6	$\frac{19}{2}$	$-\frac{15}{2}$	$-\frac{495}{16}$	$-\frac{51}{4}$	$\frac{51}{2}$	$\frac{189}{8}$	$-\frac{21}{16}$	$\frac{-71}{8}$	$\frac{-15}{4}$	$-\frac{1}{2}$	0	$-\frac{1944}{91}$	$\frac{909}{91}$	$\frac{27}{13}$
21	1	6	$\frac{37}{4}$	$-\frac{35}{4}$	-32	$-\frac{19}{2}$	$\frac{121}{4}$	$\frac{85}{4}$	$-\frac{13}{2}$	-9	-2	0	0	$-\frac{1944}{91}$	$-\frac{1000}{91}$	$\frac{144}{13}$
22	1	6	9	-10	-33	-6	35	18	-12	-8	0	0	0	$-\frac{288}{13}$	$-\frac{152}{13}$	$\frac{184}{13}$
23	1	6	$\frac{15}{2}$	$-\frac{35}{2}$	$-\frac{639}{16}$	$\frac{45}{4}$	$\frac{507}{8}$	$\frac{45}{4}$	$-\frac{639}{16}$	$-\frac{35}{2}$	$\frac{15}{2}$	6	1	0	$-\frac{243}{13}$	$\frac{243}{13}$
24	1	6	$\frac{29}{4}$	$-\frac{75}{4}$	$-\frac{81}{2}$	$\frac{33}{2}$	$\frac{273}{4}$	$\frac{9}{4}$	-48	-10	12	4	0	$-\frac{1944}{91}$	$-\frac{1728}{91}$	$\frac{1080}{13}$
25	1	6	7	-20	-41	22	73	-8	-56	0	16	0	0	$-\frac{2592}{91}$	$-\frac{1576}{91}$	$\frac{504}{13}$
26	1	6	$\frac{21}{4}$	$-\frac{115}{4}$	-45	$\frac{117}{2}$	$\frac{429}{4}$	$-\frac{279}{4}$	$-\frac{225}{2}$	50	48	-12	-8	$-\frac{17496}{91}$	$-\frac{1944}{91}$	$\frac{7776}{13}$
27	1	6	5	-30	-45	66	111	-90	-120	80	48	-32	0	$-\frac{7776}{91}$	$-\frac{360}{91}$	$-\frac{4104}{13}$
28	1	6	3	-40	-45	126	141	-252	-180	320	48	-192	64	$\frac{69984}{91}$	$\frac{12312}{91}$	$-\frac{99144}{13}$
29	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{45}{16}$	$\frac{15}{4}$	$\frac{105}{32}$	$\frac{63}{32}$	$\frac{105}{128}$	$\frac{15}{64}$	$\frac{45}{1024}$	$\frac{5}{1024}$	$\frac{1}{4096}$	$\frac{33291}{5824}$	$\frac{909}{1456}$	$\frac{-513}{832}$
30	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{11}{4}$	$\frac{7}{2}$	$\frac{91}{32}$	$\frac{49}{32}$	$\frac{35}{64}$	$\frac{1}{8}$	$\frac{17}{1024}$	$\frac{1}{1024}$	0	$\frac{4617}{728}$	$\frac{485}{728}$	$-\frac{9}{13}$
31	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{43}{16}$	$\frac{13}{4}$	$\frac{155}{64}$	$\frac{73}{64}$	$\frac{85}{256}$	$\frac{7}{128}$	$\frac{1}{256}$	0	0	$\frac{4923}{728}$	$\frac{64}{91}$	$-\frac{77}{104}$
32	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{21}{8}$	3	$\frac{129}{64}$	$\frac{51}{64}$	$\frac{11}{64}$	$\frac{1}{64}$	0	0	0	$\frac{638}{91}$	$\frac{67}{91}$	$-\frac{10}{13}$
33	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{41}{16}$	$\frac{11}{4}$	$\frac{13}{8}$	$\frac{1}{2}$	$\frac{1}{16}$	0	0	0	0	$\frac{648}{91}$	$\frac{209}{273}$	$-\frac{31}{39}$
34	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{4}$	$\frac{1}{4}$	0	0	0	0	0	$\frac{648}{91}$	$\frac{72}{91}$	$-\frac{32}{39}$
35	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{9}{4}$	$\frac{3}{2}$	$-\frac{21}{32}$	$-\frac{63}{32}$	$-\frac{105}{64}$	$-\frac{3}{4}$	$-\frac{207}{1024}$	$-\frac{31}{1024}$	$-\frac{1}{512}$	$\frac{4131}{364}$	$\frac{729}{728}$	$-\frac{135}{104}$
36	0	0	$\frac{1}{4}$	$\frac{5}{4}$	2	$\frac{1}{2}$	$-\frac{7}{4}$	$-\frac{7}{4}$	$-\frac{1}{2}$	0	0	0	0	$\frac{648}{91}$	$\frac{272}{273}$	$-\frac{40}{39}$
37	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{27}{16}$	$-\frac{3}{4}$	$-\frac{213}{64}$	$-\frac{135}{64}$	$\frac{165}{256}$	$\frac{183}{128}$	$\frac{189}{256}$	$\frac{11}{64}$	$\frac{1}{64}$	$\frac{243}{728}$	$\frac{90}{91}$	$\frac{27}{104}$
38	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{15}{4}$	$-\frac{3}{4}$	2	1	0	0	0	$\frac{8}{91}$	$\frac{88}{91}$	$-\frac{16}{13}$
39	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{9}{8}$	-3	$-\frac{303}{64}$	$\frac{99}{64}$	$\frac{309}{64}$	$\frac{69}{64}$	$-\frac{45}{32}$	$-\frac{13}{16}$	$-\frac{1}{8}$	$-\frac{243}{91}$	$\frac{99}{91}$	$-\frac{27}{13}$
40	0	0	$\frac{1}{4}$	$\frac{5}{4}$	1	$-\frac{7}{2}$	$-\frac{19}{4}$	$\frac{13}{4}$	$\frac{11}{2}$	-1	-2	0	0	$\frac{72}{91}$	$\frac{64}{91}$	$-\frac{40}{13}$
41	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{9}{16}$	$-\frac{21}{4}$	$-\frac{39}{8}$	9	$\frac{129}{16}$	$-\frac{15}{2}$	$-\frac{9}{2}$	2	1	$\frac{1944}{91}$	$\frac{27}{91}$	$\frac{-837}{13}$
42	0	0	$\frac{1}{4}$	$\frac{5}{4}$	$\frac{1}{2}$	$-\frac{11}{2}$	$-\frac{19}{4}$	$-\frac{41}{4}$	8	-10	-4	4	0	$\frac{648}{91}$	$-\frac{152}{91}$	$\frac{576}{13}$
43	0	0	$\frac{1}{4}$	$\frac{5}{4}$	0	$-\frac{15}{2}$	$-\frac{15}{4}$	$\frac{81}{4}$	$\frac{15}{2}$	-30	0	20	-8	$-\frac{9720}{91}$	$-\frac{1584}{91}$	$\frac{11880}{13}$
44	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{35}{128}$	$\frac{7}{64}$	$\frac{7}{256}$	$\frac{1}{256}$	$\frac{1}{4096}$	$\frac{-3645}{5824}$	$-\frac{61}{1456}$	$\frac{63}{832}$
45	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{27}{64}$	$\frac{25}{64}$	$\frac{55}{256}$	$\frac{9}{128}$	$\frac{13}{1024}$	$\frac{1}{1024}$	0	$-\frac{153}{364}$	$-\frac{27}{728}$	$\frac{5}{104}$
46	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{13}{32}$	$\frac{11}{32}$	$\frac{41}{256}$	$\frac{5}{128}$	$\frac{1}{256}$	0	0	$-\frac{181}{728}$	$-\frac{3}{91}$	$\frac{3}{104}$

47	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{25}{64}$	$\frac{19}{64}$	$\frac{7}{64}$	$\frac{1}{64}$	0	0	0	$-\frac{10}{91}$	$-\frac{8}{273}$	$\frac{1}{39}$
48	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$	0	0	0	0	0	$-\frac{1}{39}$	$\frac{1}{39}$
49	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{19}{64}$	$\frac{1}{64}$	$\frac{-65}{256}$	$-\frac{31}{128}$	$-\frac{107}{1024}$	$-\frac{23}{1024}$	$-\frac{1}{512}$	$\frac{891}{728}$	$\frac{1}{728}$	$-\frac{9}{52}$
50	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{5}{16}$	$-\frac{1}{8}$	0	0	0	$\frac{80}{91}$	$\frac{1}{273}$	$\frac{1}{39}$
51	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{5}{32}$	$-\frac{13}{32}$	$-\frac{119}{256}$	$\frac{5}{128}$	$\frac{61}{256}$	$\frac{7}{64}$	$\frac{1}{64}$	$\frac{243}{728}$	$-\frac{1}{91}$	$\frac{27}{104}$
52	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{8}$	$-\frac{1}{2}$	$-\frac{7}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$-\frac{8}{91}$	$\frac{3}{91}$	$\frac{3}{13}$
53	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{1}{64}$	$-\frac{53}{64}$	$-\frac{23}{64}$	$\frac{61}{64}$	$\frac{11}{32}$	$-\frac{5}{16}$	$-\frac{1}{8}$	$-\frac{243}{91}$	$\frac{8}{91}$	$\frac{90}{13}$
54	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	0	$-\frac{7}{8}$	$-\frac{5}{16}$	$\frac{9}{8}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{72}{91}$	$\frac{27}{91}$	$-\frac{77}{13}$
55	0	0	0	0	$\frac{1}{16}$	$\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{5}{4}$	$\frac{1}{16}$	$\frac{5}{2}$	$-\frac{1}{2}$	-2	1	$\frac{1296}{91}$	$\frac{179}{91}$	$-\frac{1413}{13}$
56	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{15}{256}$	$\frac{5}{128}$	$\frac{15}{1024}$	$\frac{3}{1024}$	$\frac{1}{4096}$	$-\frac{171}{832}$	$-\frac{1}{208}$	$\frac{28}{832}$
57	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{7}{128}$	$\frac{1}{32}$	$\frac{9}{1024}$	$\frac{1}{1024}$	0	$-\frac{125}{728}$	$-\frac{3}{728}$	$\frac{1}{52}$
58	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{13}{256}$	$\frac{3}{128}$	$\frac{1}{256}$	0	0	$-\frac{101}{728}$	$-\frac{1}{273}$	$\frac{1}{312}$
59	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{3}{64}$	$\frac{1}{64}$	0	0	0	$-\frac{10}{91}$	$-\frac{1}{273}$	0
60	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{3}{128}$	$-\frac{1}{32}$	$-\frac{39}{1024}$	$-\frac{15}{1024}$	$-\frac{1}{512}$	$\frac{9}{91}$	$\frac{1}{728}$	$-\frac{5}{104}$
61	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{1}{64}$	$-\frac{3}{64}$	$-\frac{1}{32}$	0	0	$\frac{11}{91}$	$-\frac{1}{273}$	$-\frac{1}{39}$
62	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$-\frac{3}{256}$	$-\frac{13}{128}$	$-\frac{3}{256}$	$\frac{3}{64}$	$\frac{1}{64}$	$\frac{243}{728}$	$-\frac{1}{91}$	$-\frac{77}{104}$
63	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$-\frac{1}{64}$	$-\frac{7}{64}$	0	$\frac{1}{16}$	0	$\frac{8}{91}$	$-\frac{3}{91}$	$\frac{10}{13}$
64	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$-\frac{3}{64}$	$-\frac{11}{64}$	$\frac{3}{32}$	$\frac{3}{16}$	$-\frac{1}{8}$	$-\frac{171}{91}$	$-\frac{19}{91}$	$\frac{167}{13}$
65	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{3}{512}$	$\frac{1}{512}$	$\frac{1}{4096}$	$-\frac{197}{5824}$	$-\frac{1}{1456}$	$\frac{7}{832}$
66	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{5}{1024}$	$\frac{1}{1024}$	0	$-\frac{3}{91}$	$-\frac{1}{2184}$	$\frac{5}{312}$
67	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$\frac{1}{256}$	0	0	$-\frac{3}{104}$	0	$\frac{1}{312}$
68	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$-\frac{3}{1024}$	$-\frac{7}{1024}$	$-\frac{1}{512}$	$-\frac{19}{728}$	$\frac{1}{728}$	$\frac{1}{13}$
69	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$-\frac{1}{256}$	$-\frac{1}{128}$	0	$\frac{3}{728}$	$\frac{1}{273}$	$-\frac{31}{312}$
70	0	0	0	0	0	0	0	0	$\frac{1}{256}$	$\frac{1}{128}$	$-\frac{3}{256}$	$-\frac{1}{64}$	$\frac{1}{64}$	$\frac{179}{728}$	$\frac{2}{91}$	$-\frac{157}{104}$
71	0	0	0	0	0	0	0	0	0	0	$\frac{1}{1024}$	$\frac{1}{1024}$	$\frac{1}{4096}$	$-\frac{5}{5824}$	$-\frac{1}{4368}$	$-\frac{19}{2496}$
72	0	0	0	0	0	0	0	0	0	0	$\frac{1}{1024}$	$\frac{1}{1024}$	0	$-\frac{3}{728}$	$-\frac{1}{2184}$	$\frac{1}{78}$
73	0	0	0	0	0	0	0	0	0	0	$\frac{1}{1024}$	$\frac{1}{1024}$	$-\frac{1}{512}$	$-\frac{11}{364}$	$-\frac{5}{2184}$	$\frac{55}{312}$
74	0	0	0	0	0	0	0	0	0	0	$\frac{1}{4096}$	$\frac{19}{5824}$	$\frac{1}{4368}$	$-\frac{17}{832}$	0	0

Table 2/A

No	b_1	b_2	b_3	b_4	b_5	a_1	a_2	a_3	a_4	a_6	a_{12}	c_1	c_2	c_3	c_4	c_6	c_{12}
1	0	0	0	3	0	0	0	-24	0	60	-24	0	0	-8	0	0	512
2	0	0	0	9	0	-24	60	0	-24	0	0	-8	0	0	512	0	0
3	0	1	3	2	0	0	0	-8	0	36	-16	0	0	0	0	-8	512
4	0	2	6	1	0	0	0	8	0	12	-8	0	0	0	0	-8	512
5	0	3	1	6	0	-8	36	0	-16	0	0	0	-8	0	512	0	0
6	0	3	9	0	0	0	0	24	0	-12	0	0	0	8	0	-528	1024
7	0	6	2	3	0	8	12	0	-8	0	0	0	-8	0	512	0	0
8	0	9	3	0	0	24	-12	0	0	0	0	8	-528	0	1024	0	0
9	1	0	0	0	11	0	-16	0	32	4	-8	$-\frac{5}{704}$	$\frac{15}{64}$	$\frac{9}{2816}$	$-\frac{5}{22}$	$-\frac{27}{256}$	$\frac{9}{88}$
10	1	0	0	6	3	-16	36	0	-8	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
11	1	0	1	0	9	0	-15	0	27	9	-9	$-\frac{9}{2816}$	$\frac{27}{256}$	$\frac{9}{2816}$	$-\frac{9}{88}$	$-\frac{27}{256}$	$\frac{9}{88}$
12	1	0	2	0	7	0	-14	0	22	14	-10	$-\frac{1}{1056}$	$\frac{1}{32}$	$\frac{1}{1056}$	$-\frac{1}{33}$	$-\frac{1}{32}$	$\frac{1}{33}$
13	1	1	0	1	9	0	-7	0	23	1	-5	$\frac{79}{2816}$	$-\frac{237}{256}$	$\frac{9}{2816}$	$\frac{79}{88}$	$-\frac{27}{256}$	$\frac{9}{88}$
14	1	1	1	1	7	0	-6	0	18	6	-6	$\frac{3}{176}$	$-\frac{9}{16}$	$-\frac{3}{176}$	$\frac{6}{11}$	$\frac{9}{16}$	$-\frac{6}{11}$
15	1	1	2	1	5	0	-5	0	13	11	-7	$\frac{1}{132}$	$-\frac{1}{4}$	$-\frac{1}{132}$	$\frac{8}{33}$	$\frac{1}{4}$	$-\frac{8}{33}$

16	1	2	0	2	7	0	2	0	14	-2	-2	$-\frac{25}{352}$	$\frac{75}{32}$	$-\frac{63}{352}$	$-\frac{25}{11}$	$\frac{189}{32}$	$-\frac{63}{11}$
17	1	2	1	2	5	0	3	0	9	3	-3	$-\frac{3}{44}$	$\frac{9}{4}$	$\frac{3}{44}$	$-\frac{24}{11}$	$-\frac{9}{4}$	$\frac{24}{11}$
18	1	2	2	2	3	0	4	0	4	8	-4	$-\frac{2}{33}$	2	$\frac{2}{33}$	$-\frac{64}{33}$	-2	$\frac{64}{33}$
19	1	3	0	3	5	0	11	0	5	-5	1	$-\frac{1}{22}$	$\frac{3}{2}$	$\frac{45}{22}$	$-\frac{16}{11}$	$-\frac{135}{2}$	$\frac{720}{11}$
20	1	3	2	3	1	0	13	0	-5	5	-1	$\frac{16}{33}$	-16	$-\frac{16}{33}$	$\frac{512}{33}$	16	$-\frac{512}{33}$
21	1	4	0	4	3	0	20	0	-4	-8	4	$\frac{4}{11}$	-12	$-\frac{180}{11}$	$\frac{128}{11}$	540	$-\frac{5760}{11}$
22	1	4	1	4	1	0	21	0	-9	-3	3	$\frac{48}{11}$	-144	$-\frac{48}{11}$	$\frac{1536}{11}$	144	$-\frac{1536}{11}$
23	1	5	0	5	1	0	29	0	-13	-11	7	$\frac{400}{11}$	-1200	$\frac{1008}{11}$	$\frac{12800}{11}$	-3024	$\frac{32256}{11}$
24	1	6	2	0	3	16	-12	0	8	0	0	$-\frac{1}{2}$	$\frac{65}{2}$	0	-32	0	0
25	2	0	0	3	6	-8	12	0	8	0	0	$-\frac{1}{32}$	$\frac{1}{32}$	0	0	0	0
26	2	3	1	0	6	8	-12	0	16	0	0	$\frac{1}{32}$	$-\frac{65}{32}$	0	2	0	0
27	3	0	0	0	9	0	-12	0	24	0	0	$-\frac{1}{256}$	$\frac{33}{256}$	0	$-\frac{1}{8}$	0	0
28	3	0	0	2	1	0	0	-16	0	36	-8	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
29	3	0	1	0	7	0	-11	0	19	5	-1	$-\frac{19}{8448}$	$\frac{19}{256}$	$\frac{19}{8448}$	$-\frac{19}{264}$	$-\frac{19}{256}$	$\frac{19}{264}$
30	3	0	2	0	5	0	-10	0	14	10	-2	$-\frac{1}{1056}$	$\frac{1}{32}$	$\frac{1}{1056}$	$-\frac{1}{33}$	$-\frac{1}{32}$	$\frac{1}{33}$
31	3	1	0	1	7	0	-3	0	15	-3	3	$\frac{31}{2816}$	$-\frac{93}{256}$	$\frac{57}{2816}$	$\frac{31}{88}$	$-\frac{171}{256}$	$\frac{57}{88}$
32	3	1	1	1	5	0	-2	0	10	2	2	$\frac{5}{528}$	$-\frac{5}{16}$	$-\frac{5}{528}$	$\frac{10}{33}$	$\frac{5}{16}$	$-\frac{10}{33}$
33	3	1	2	1	3	0	-1	0	5	7	1	$\frac{1}{132}$	$-\frac{1}{4}$	$-\frac{1}{132}$	$\frac{8}{33}$	$\frac{1}{4}$	$-\frac{8}{33}$
34	3	2	0	2	5	0	6	0	6	-6	6	$-\frac{1}{352}$	$\frac{3}{32}$	$-\frac{87}{352}$	$-\frac{1}{11}$	$\frac{261}{32}$	$-\frac{87}{11}$
35	3	2	1	2	3	0	7	0	1	-1	5	$-\frac{1}{132}$	$\frac{1}{4}$	$\frac{1}{132}$	$-\frac{8}{33}$	$-\frac{1}{4}$	$\frac{8}{33}$
36	3	2	2	2	1	0	8	0	-4	4	4	$-\frac{2}{33}$	2	$\frac{2}{33}$	$-\frac{64}{33}$	-2	$\frac{64}{33}$
37	3	2	6	0	1	0	0	16	0	-12	8	0	0	$-\frac{1}{2}$	0	$\frac{65}{2}$	-32
38	3	3	0	3	3	0	15	0	-3	-9	9	$-\frac{1}{22}$	$\frac{3}{2}$	$\frac{45}{22}$	$-\frac{16}{11}$	$-\frac{135}{2}$	$\frac{720}{11}$
39	3	3	1	3	1	0	16	0	-8	-4	8	$-\frac{16}{33}$	16	$\frac{16}{33}$	$-\frac{512}{33}$	-16	$\frac{512}{33}$
40	3	4	0	4	1	0	24	0	-12	-12	12	-4	132	-12	-128	-396	-384
41	5	0	0	0	7	0	-8	0	16	-4	8	$-\frac{7}{4224}$	$\frac{7}{128}$	$-\frac{19}{8448}$	$-\frac{7}{132}$	$\frac{19}{256}$	$-\frac{19}{264}$
42	5	0	1	0	5	0	-7	0	11	1	7	$-\frac{1}{768}$	$\frac{11}{256}$	$\frac{1}{768}$	$-\frac{1}{24}$	$-\frac{11}{256}$	$\frac{1}{24}$
43	5	0	2	0	3	0	-6	0	6	6	6	$-\frac{1}{1056}$	$\frac{1}{32}$	$\frac{1}{1056}$	$-\frac{1}{33}$	$-\frac{1}{32}$	$\frac{1}{33}$
44	5	1	0	1	5	0	1	0	7	-7	11	$\frac{13}{8448}$	$-\frac{13}{256}$	$\frac{251}{8448}$	$\frac{13}{264}$	$-\frac{251}{256}$	$\frac{251}{264}$
45	5	1	1	1	3	0	2	0	2	-2	10	$\frac{1}{528}$	$-\frac{1}{16}$	$-\frac{1}{528}$	$\frac{2}{33}$	$\frac{1}{16}$	$-\frac{2}{33}$
46	5	1	2	1	1	0	3	0	-3	3	9	$\frac{1}{132}$	$-\frac{1}{4}$	$-\frac{1}{132}$	$\frac{8}{33}$	$\frac{1}{4}$	$-\frac{8}{33}$
47	5	2	0	2	3	0	10	0	-2	-10	14	$\frac{5}{1056}$	$-\frac{5}{32}$	$-\frac{269}{1056}$	$\frac{5}{33}$	$\frac{269}{32}$	$-\frac{269}{33}$
48	5	2	1	2	1	0	11	0	-7	-5	13	$\frac{7}{132}$	$-\frac{7}{4}$	$-\frac{7}{132}$	$\frac{56}{33}$	$\frac{7}{4}$	$-\frac{56}{33}$
49	5	3	0	3	1	0	19	0	-11	-13	17	$\frac{29}{66}$	$-\frac{29}{2}$	$\frac{103}{66}$	$\frac{464}{33}$	$-\frac{103}{2}$	$\frac{1648}{33}$
50	6	0	0	1	2	0	0	-8	0	12	8	0	0	$-\frac{1}{32}$	0	$\frac{1}{32}$	0
51	7	0	0	0	5	0	-4	0	8	-8	16	$-\frac{1}{2816}$	$\frac{3}{256}$	$-\frac{5}{1408}$	$-\frac{1}{88}$	$\frac{15}{128}$	$-\frac{5}{44}$
52	7	0	1	0	3	0	-3	0	3	-3	15	$-\frac{1}{2816}$	$\frac{3}{256}$	$\frac{1}{2816}$	$-\frac{1}{88}$	$-\frac{3}{256}$	$\frac{1}{88}$
53	7	0	2	0	1	0	-2	0	-2	2	14	$-\frac{1}{1506}$	$\frac{1}{32}$	$\frac{1}{1506}$	$-\frac{1}{33}$	$-\frac{1}{32}$	$\frac{1}{33}$
54	7	1	0	1	3	0	5	0	-1	-11	19	$-\frac{1}{2816}$	$\frac{3}{256}$	$\frac{89}{2816}$	$-\frac{1}{88}$	$-\frac{267}{256}$	$\frac{89}{88}$
55	7	1	1	1	1	0	6	0	-6	-6	18	$-\frac{1}{176}$	$\frac{3}{16}$	$\frac{1}{176}$	$-\frac{2}{11}$	$-\frac{3}{16}$	$\frac{2}{11}$
56	7	2	0	2	1	0	14	0	-10	-14	22	$-\frac{17}{352}$	$\frac{51}{32}$	$-\frac{71}{352}$	$-\frac{17}{11}$	$\frac{213}{32}$	$\frac{71}{11}$
57	9	0	0	0	3	0	0	0	0	-12	24	0	0	$-\frac{1}{256}$	0	$\frac{33}{256}$	$-\frac{1}{8}$
58	9	0	1	0	1	0	1	0	-5	-7	23	$\frac{5}{8448}$	$-\frac{5}{256}$	$-\frac{5}{8448}$	$\frac{5}{264}$	$\frac{5}{256}$	$-\frac{5}{264}$
59	9	1	0	1	1	0	9	0	-9	-15	27	$\frac{15}{2816}$	$-\frac{45}{256}$	$\frac{73}{2816}$	$\frac{15}{88}$	$-\frac{219}{256}$	$\frac{73}{88}$
60	11	0	0	0	1	0	4	0	-8	-16	32	$-\frac{5}{8448}$	$\frac{5}{256}$	$-\frac{7}{2112}$	$-\frac{5}{264}$	$\frac{7}{64}$	$-\frac{7}{66}$

Table 2/B

No	k_0	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	r_4	r_5	r_6	r_7
1	1	6	12	8	0	0	0	0	0	0	0	0	0	$\frac{16}{9}$	0	$-\frac{16}{9}$	0
2	1	18	144	672	2016	4032	5376	4608	2304	512	0	0	0	0	16	0	0
3	1	6	12	6	-9	-12	-4	0	0	0	0	0	0	$\frac{8}{9}$	0	$-\frac{8}{9}$	0
4	1	6	12	4	-18	-24	-4	12	9	2	0	0	0	$-\frac{8}{9}$	0	$\frac{8}{9}$	0
5	1	10	36	42	-57	-180	-60	192	144	-64	-64	0	0	0	8	0	0
6	1	6	12	2	-27	-36	0	36	27	-2	-12	-6	-1	$-\frac{16}{9}$	0	$\frac{16}{9}$	0
7	1	2	-8	-12	30	24	-60	-12	57	-10	-20	8	0	0	-8	0	0
8	1	-6	12	-2	-27	36	0	-36	27	2	-12	6	-1	0	-16	0	0
9	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{55}{8}$	$\frac{165}{16}$	$\frac{165}{16}$	$\frac{231}{32}$	$\frac{231}{64}$	$\frac{165}{128}$	$\frac{165}{512}$	$\frac{55}{1024}$	$\frac{11}{2048}$	$\frac{1}{4096}$	$-\frac{791}{2816}$	$-\frac{3}{704}$	$-\frac{1}{2816}$	$\frac{225}{176}$
10	0	$\frac{1}{2}$	$\frac{27}{4}$	$\frac{315}{8}$	$\frac{2073}{16}$	$\frac{1053}{4}$	$\frac{1359}{4}$	276	135	36	4	0	0	0	$\frac{1}{2}$	0	0
11	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{27}{4}$	$\frac{39}{4}$	$\frac{147}{16}$	$\frac{189}{32}$	$\frac{21}{8}$	$\frac{51}{64}$	$\frac{81}{512}$	$\frac{19}{1024}$	$\frac{1}{1024}$	0	$-\frac{791}{2816}$	$-\frac{1}{2816}$	$-\frac{1}{2816}$	$\frac{225}{176}$
12	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{53}{8}$	$\frac{147}{16}$	$\frac{259}{32}$	$\frac{301}{64}$	$\frac{231}{128}$	$\frac{113}{256}$	$\frac{1}{16}$	$\frac{1}{256}$	0	$-\frac{4255}{19008}$	$\frac{1}{2112}$	$-\frac{1}{9504}$	$\frac{1291}{1056}$	
13	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{23}{4}$	$\frac{21}{4}$	$\frac{3}{16}$	$-\frac{147}{32}$	$-\frac{21}{4}$	$-\frac{201}{64}$	$-\frac{591}{512}$	$-\frac{269}{1024}$	$-\frac{35}{1024}$	$-\frac{1}{512}$	$-\frac{791}{2816}$	$\frac{87}{2816}$	$-\frac{1}{2816}$	$\frac{225}{176}$
14	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{45}{8}$	$\frac{75}{16}$	$-\frac{21}{32}$	$-\frac{315}{64}$	$-\frac{609}{128}$	$-\frac{615}{256}$	$-\frac{45}{64}$	$-\frac{29}{256}$	$-\frac{1}{128}$	0	$\frac{491}{2112}$	$\frac{5}{704}$	$\frac{1}{528}$	$\frac{273}{352}$
15	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{11}{2}$	$\frac{33}{8}$	$-\frac{47}{32}$	$-\frac{329}{64}$	$-\frac{267}{64}$	$-\frac{109}{64}$	$-\frac{23}{64}$	$-\frac{1}{32}$	0	0	$\frac{791}{1188}$	$-\frac{1}{264}$	$\frac{1}{1188}$	$\frac{91}{264}$
16	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{37}{8}$	$\frac{3}{16}$	$-\frac{237}{32}$	$-\frac{483}{64}$	$-\frac{105}{128}$	$\frac{897}{256}$	$\frac{93}{32}$	$\frac{277}{256}$	$\frac{13}{64}$	$\frac{1}{64}$	$\frac{3055}{704}$	$-\frac{129}{704}$	$\frac{7}{352}$	$-\frac{1143}{352}$
17	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{9}{2}$	$-\frac{3}{8}$	$-\frac{255}{32}$	$-\frac{441}{64}$	$\frac{33}{64}$	$\frac{249}{64}$	$\frac{153}{64}$	$\frac{5}{8}$	$\frac{1}{16}$	0	$\frac{1091}{264}$	$-\frac{1}{11}$	$-\frac{1}{132}$	$-\frac{273}{88}$
18	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{35}{8}$	$-\frac{15}{16}$	$-\frac{17}{2}$	$-\frac{49}{8}$	$\frac{15}{8}$	$\frac{65}{16}$	$\frac{7}{4}$	$\frac{1}{4}$	0	0	$\frac{1091}{297}$	$\frac{1}{33}$	$-\frac{2}{297}$	$-\frac{91}{33}$
19	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{7}{2}$	$-\frac{39}{8}$	$-\frac{399}{32}$	$-\frac{105}{64}$	$\frac{717}{64}$	$\frac{447}{64}$	$-\frac{111}{64}$	$-\frac{97}{32}$	$-\frac{17}{16}$	$-\frac{1}{8}$	$\frac{109}{44}$	$\frac{57}{88}$	$-\frac{5}{22}$	$-\frac{171}{88}$
20	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{13}{4}$	-6	-13	$\frac{7}{4}$	$\frac{57}{4}$	$\frac{7}{2}$	-5	-2	0	0	$-\frac{8728}{297}$	$-\frac{8}{13}$	$\frac{2689}{297}$	$\frac{728}{33}$
21	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{19}{8}$	$-\frac{159}{16}$	-15	$\frac{105}{8}$	$\frac{201}{8}$	$-\frac{111}{16}$	$-\frac{33}{2}$	$-\frac{1}{2}$	4	1	$-\frac{218}{11}$	$\frac{42}{11}$	$\frac{20}{11}$	$\frac{171}{11}$
22	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{9}{4}$	$-\frac{21}{2}$	-15	$\frac{63}{4}$	$\frac{105}{4}$	-12	-18	4	4	0	$-\frac{8728}{33}$	$-\frac{112}{11}$	$\frac{2689}{33}$	$\frac{2184}{11}$
23	0	$\frac{1}{2}$	$\frac{11}{4}$	$\frac{5}{4}$	-15	-15	$\frac{147}{4}$	$\frac{141}{4}$	$-\frac{105}{2}$	-30	40	4	-8	$-\frac{24440}{11}$	$-\frac{1344}{11}$	$\frac{7907}{11}$	$\frac{18288}{11}$
24	0	$\frac{1}{2}$	$-\frac{5}{4}$	$-\frac{5}{8}$	$\frac{57}{16}$	$-\frac{3}{4}$	$-\frac{15}{4}$	$\frac{3}{2}$	$\frac{15}{8}$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{16}$	0	$\frac{1}{2}$	0	0
25	0	0	$\frac{1}{4}$	$\frac{9}{4}$	$\frac{135}{16}$	$\frac{69}{4}$	$\frac{1359}{64}$	$\frac{1053}{64}$	$\frac{2073}{256}$	$\frac{315}{128}$	$\frac{27}{64}$	$\frac{1}{32}$	0	0	$-\frac{1}{32}$	0	0
26	0	0	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{9}{16}$	$-\frac{3}{4}$	$\frac{15}{64}$	$\frac{45}{64}$	$\frac{57}{256}$	$-\frac{21}{128}$	$-\frac{9}{64}$	$-\frac{5}{128}$	$-\frac{1}{256}$	0	$\frac{1}{32}$	0	0
27	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{9}{8}$	$\frac{21}{16}$	$\frac{63}{64}$	$\frac{63}{128}$	$\frac{21}{128}$	$\frac{9}{256}$	$\frac{9}{2048}$	$\frac{1}{4096}$	0	$-\frac{1}{256}$	0	0
28	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	0	0	0	0	$\frac{1}{18}$	0	$-\frac{1}{18}$	0
29	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{35}{32}$	$\frac{77}{64}$	$\frac{105}{128}$	$\frac{91}{256}$	$\frac{49}{512}$	$\frac{15}{1024}$	$\frac{1}{1024}$	0	$-\frac{4337}{76032}$	$-\frac{7}{8448}$	$-\frac{19}{76032}$	$\frac{59}{1056}$
30	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{17}{16}$	$\frac{35}{32}$	$\frac{85}{128}$	$\frac{61}{256}$	$\frac{3}{64}$	$\frac{1}{256}$	0	0	$-\frac{1879}{19008}$	$\frac{1}{2112}$	$-\frac{1}{9504}$	$\frac{103}{1056}$
31	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{27}{32}$	$\frac{21}{64}$	$-\frac{63}{128}$	$-\frac{189}{256}$	$-\frac{231}{512}$	$-\frac{153}{1024}$	$-\frac{27}{1024}$	$-\frac{1}{512}$	$-\frac{4337}{8448}$	$\frac{67}{2816}$	$-\frac{19}{8448}$	$\frac{177}{352}$
32	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{13}{16}$	$\frac{7}{32}$	$-\frac{75}{128}$	$-\frac{179}{256}$	$-\frac{11}{32}$	$-\frac{21}{256}$	$-\frac{1}{128}$	0	$-\frac{8237}{19008}$	$\frac{23}{2112}$	$\frac{5}{4752}$	$\frac{455}{1056}$
33	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{25}{32}$	$\frac{7}{64}$	$-\frac{43}{64}$	$-\frac{41}{64}$	$-\frac{15}{64}$	$-\frac{1}{32}$	0	0	$-\frac{397}{1188}$	$-\frac{1}{264}$	$\frac{1}{1188}$	$\frac{91}{264}$
34	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{9}{16}$	$-\frac{21}{32}$	$-\frac{171}{128}$	$-\frac{99}{256}$	$\frac{33}{64}$	$\frac{117}{256}$	$\frac{9}{64}$	$\frac{1}{64}$	$\frac{437}{2112}$	$-\frac{65}{704}$	$\frac{29}{1056}$	$-\frac{51}{352}$
35	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{17}{32}$	$-\frac{49}{64}$	$-\frac{87}{64}$	$-\frac{11}{64}$	$\frac{41}{64}$	$\frac{3}{8}$	$\frac{1}{16}$	$-\frac{1}{512}$	$\frac{1091}{2376}$	$-\frac{4}{33}$	$\frac{1}{1188}$	$-\frac{91}{264}$
36	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{1}{2}$	$-\frac{7}{8}$	$-\frac{11}{8}$	$\frac{1}{16}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{1091}{297}$	$\frac{1}{33}$	$-\frac{299}{297}$	$-\frac{91}{33}$
37	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{3}{2}$	$-\frac{9}{8}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{16}$	$\frac{1}{18}$	0	$-\frac{1}{18}$	0
38	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{9}{32}$	$-\frac{105}{64}$	$-\frac{99}{64}$	$\frac{99}{64}$	$\frac{105}{64}$	$-\frac{9}{32}$	$-\frac{9}{16}$	$-\frac{1}{8}$	$\frac{109}{44}$	$-\frac{31}{88}$	$-\frac{5}{22}$	$-\frac{171}{88}$
39	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	$\frac{1}{4}$	$-\frac{7}{4}$	$-\frac{3}{2}$	$\frac{31}{16}$	$\frac{13}{8}$	$-\frac{3}{4}$	$-\frac{1}{2}$	0	$\frac{8728}{297}$	$\frac{41}{33}$	$-\frac{2689}{297}$	$-\frac{728}{33}$
40	0	0	0	$\frac{1}{8}$	$\frac{9}{16}$	0	$-\frac{21}{8}$	$-\frac{9}{8}$	$\frac{81}{16}$	$\frac{3}{2}$	$-\frac{9}{2}$	0	1	$\frac{734}{3}$	14	$-\frac{239}{3}$	-183
41	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{21}{128}$	$\frac{35}{256}$	$\frac{35}{512}$	$\frac{21}{1024}$	$\frac{7}{2048}$	$\frac{1}{4096}$	$\frac{4337}{76032}$	$-\frac{13}{4224}$	$\frac{19}{76032}$	$-\frac{59}{1056}$
42	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{5}{32}$	$\frac{15}{128}$	$\frac{25}{512}$	$\frac{11}{1024}$	$\frac{1}{1024}$	0	$\frac{289}{6912}$	$-\frac{1}{768}$	$-\frac{1}{6912}$	$-\frac{1}{24}$
43	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{19}{128}$	$\frac{25}{256}$	$\frac{1}{32}$	$\frac{1}{256}$	0	0	$\frac{497}{19008}$	$\frac{1}{2112}$	$-\frac{1}{9504}$	$-\frac{29}{1056}$
44	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{3}{32}$	$-\frac{5}{128}$	$-\frac{55}{512}$	$-\frac{69}{1024}$	$-\frac{19}{1024}$	$-\frac{1}{512}$	$-\frac{6085}{76032}$	$\frac{109}{8448}$	$-\frac{251}{76032}$	$\frac{19}{264}$
45	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{11}{128}$	$-\frac{15}{256}$	$-\frac{7}{64}$	$-\frac{13}{256}$	$-\frac{1}{128}$	0	$-\frac{1885}{19008}$	$\frac{31}{2112}$	$\frac{1}{4752}$	$\frac{91}{1056}$
46	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{5}{64}$	$-\frac{5}{64}$	$-\frac{7}{64}$	$-\frac{1}{32}$	0	0	$-\frac{529}{1188}$	$-\frac{1}{264}$	$\frac{133}{1188}$	$\frac{91}{264}$

47	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{3}{128}$	$-\frac{55}{256}$	$-\frac{1}{8}$	$\frac{21}{256}$	$\frac{5}{64}$	$\frac{1}{64}$	$-\frac{4795}{19008}$	$\frac{61}{2112}$	$\frac{269}{9504}$	$\frac{211}{1056}$
48	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$\frac{1}{64}$	$-\frac{15}{64}$	$-\frac{7}{64}$	$\frac{1}{8}$	$\frac{1}{16}$	0	$-\frac{7637}{2376}$	$-\frac{5}{33}$	$\frac{1195}{1188}$	$\frac{637}{264}$
49	0	0	0	0	0	$\frac{1}{32}$	$\frac{7}{64}$	$-\frac{3}{64}$	$-\frac{25}{64}$	$\frac{1}{64}$	$\frac{15}{32}$	$-\frac{1}{16}$	$-\frac{1}{8}$	$-\frac{31969}{1188}$	$-\frac{421}{264}$	$\frac{5243}{594}$	$\frac{5311}{264}$
50	0	0	0	0	0	0	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{9}{256}$	$\frac{1}{128}$	$-\frac{1}{288}$	0	$\frac{1}{288}$	0			
51	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	$\frac{5}{256}$	$\frac{5}{512}$	$\frac{5}{2048}$	$\frac{1}{4096}$	$\frac{193}{12672}$	$-\frac{5}{2816}$	$\frac{5}{12672}$	$-\frac{5}{352}$
52	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	$\frac{9}{512}$	$\frac{7}{1024}$	$\frac{1}{1024}$	0	$\frac{397}{25344}$	$-\frac{5}{2816}$	$-\frac{1}{25344}$	$-\frac{5}{352}$
53	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	$\frac{1}{64}$	$\frac{1}{256}$	0	0	$\frac{2987}{57024}$	$\frac{1}{2112}$	$-\frac{355}{28512}$	$-\frac{131}{3168}$
54	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	$\frac{1}{512}$	$-\frac{17}{1024}$	$-\frac{11}{1024}$	$-\frac{1}{512}$	$\frac{485}{25344}$	$-\frac{5}{2816}$	$-\frac{89}{25344}$	$-\frac{5}{352}$
55	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	0	$-\frac{5}{256}$	$-\frac{1}{128}$	0	$\frac{731}{2112}$	$\frac{13}{704}$	$-\frac{59}{528}$	$-\frac{91}{352}$
56	0	0	0	0	0	0	0	$\frac{1}{128}$	$\frac{5}{256}$	$-\frac{1}{64}$	$-\frac{11}{256}$	$\frac{1}{64}$	$\frac{1}{64}$	$\frac{18767}{6336}$	$\frac{127}{704}$	$-\frac{3097}{3168}$	$-\frac{779}{352}$
57	0	0	0	0	0	0	0	0	0	$\frac{1}{512}$	$\frac{3}{1024}$	$\frac{3}{2048}$	$\frac{1}{4096}$	$-\frac{1}{2304}$	0	$\frac{1}{2304}$	0
58	0	0	0	0	0	0	0	0	0	$\frac{1}{512}$	$\frac{3}{1024}$	$\frac{1}{1024}$	0	$-\frac{8375}{228096}$	$-\frac{19}{8448}$	$\frac{2831}{228096}$	$\frac{43}{1584}$
59	0	0	0	0	0	0	0	0	0	$\frac{1}{512}$	$\frac{3}{1024}$	$-\frac{3}{1024}$	$-\frac{1}{512}$	$-\frac{8287}{25344}$	$-\frac{57}{2816}$	$\frac{2743}{25344}$	$\frac{43}{176}$
60	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2048}$	$\frac{1}{4096}$	$\frac{2069}{57024}$	$\frac{19}{8448}$	$-\frac{683}{57024}$	$-\frac{43}{1584}$