

LIE GYROVECTOR SPACES

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Abstract. An arbitrary section of the canonical projection of a group onto the cosets modulo a subgroup is associated with a binary operation on the cosets. We provide sufficient conditions for obtaining a left loop, a left gyrogroup or a gyrocommutative gyrogroup in such a way. The non-positively curved sections in Lie groups allow a scalar multiplication, which turns them into quasi left Lie gyrovector spaces. The left invariant metrics on homogeneous spaces turn out to be compatible with the gyro-structure. For instance, their geodesics are gyro-lines; the associated distance to the origin is a gyro-homogeneous norm, satisfying gyro-triangle inequality; etc. The work establishes infinitesimal criteria for a homogeneous space to bear a left Lie gyrovector space or a Lie gyrovector space structure. It characterizes the Cartan gyrovector spaces and works out explicitly the example of the upper half-plane.

1. Introduction

Based on Einstein's velocity addition law and the relativistic Thomas precession, the second named author has developed in a series of articles (e.g., [13], [14], [15], [17], [5], [3], [4], etc.) and the monograph [16] the theory of gyrogroups and gyrovector spaces. It introduces the so called Thomas gyration, which measures the deviation of the addition of the relativistically admissible velocities from being associative. From mathematical point of view, one of the most important results of this theory is the proof of the fact that the gyro-semidirect product of a gyrogroup (\mathcal{L}, \oplus) with a gyroautomorphism group $H \subset \text{Aut}(\mathcal{L}, \oplus)$ is a group G (cf. Theorem 2.23 from [16]). Thus, the Thomas gyrations of a gyrogroup (\mathcal{L}, \oplus) appear to be a sort of "extension cocycles" of \mathcal{L} with values in $\text{Aut}(\mathcal{L}, \oplus)$. Therefore, Thomas gyrations techniques can be applicable for transmitting the classification of the finite simple groups H to finite groups $G \supset H$, in which H are of comparatively small index $[G : H]$. On the other hand, ideas, similar to the gyro-formalism have proved to be quite fruitful for studying affine connections on