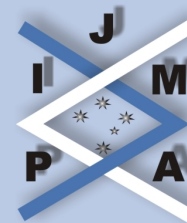


ON AN OPEN QUESTION REGARDING AN INTEGRAL INEQUALITY



Open Question Regarding an
Integral Inequality
K. Boukerrioua and
A. Guezane-Lakoud
vol. 8, iss. 3, art. 77, 2007

[Title Page](#)

[Contents](#)

[◀◀](#) [▶▶](#)

[◀](#) [▶](#)

Page 1 of 7

[Go Back](#)

[Full Screen](#)

[Close](#)

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756

© 2007 Victoria University. All rights reserved.

K. BOUKERRIOUA

Department of Mathematics
University of Guelma
Guelma, Algeria
EMail: khaledV2004@yahoo.fr

A. GUEZANE-LAKOUD

Department of Mathematics
University Badji Mokhtar, Annaba
Annaba, Algeria
EMail: a_guezane@yahoo.fr

Received: 16 January, 2007

Accepted: 14 July, 2007

Communicated by: J.E. Pečarić

2000 AMS Sub. Class.: 26D15.

Key words: Integral inequality, AG inequality.

Abstract: In the paper "Notes on an integral inequality" published in *J. Inequal. Pure & Appl. Math.*, 7(4) (2006), Art. 120, an open question was posed. In this short paper, we give the solution and we generalize the results of the mentioned paper.

Acknowledgements: The authors thank the referee for making several suggestions for improving the presentation of this paper.

Contents

- | | | |
|---|----------------------------------|---|
| 1 | Introduction | 3 |
| 2 | The Answer to the Posed Question | 4 |



Open Question Regarding an

Integral Inequality

K. Boukerrioua and

A. Guezane-Lakoud

vol. 8, iss. 3, art. 77, 2007

Title Page

Contents



Page 2 of 7

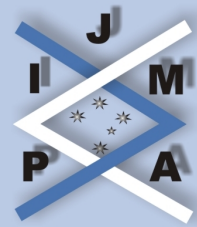
Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756



1. Introduction

The following open question was proposed in the paper [1]:

Under what conditions does the inequality

$$(1.1) \quad \int_0^1 f^{\alpha+\beta}(x) dx \geq \int_0^1 x^\beta f^\alpha(x) dx$$

hold for α and β ?

In the above paper, the authors established some integral inequalities and derived their results using an analytic approach.

In the present paper, we give a solution and further generalization of the integral inequalities presented in [1].

Open Question Regarding an

Integral Inequality

K. Boukerrioua and

A. Guezane-Lakoud

vol. 8, iss. 3, art. 77, 2007

Title Page

Contents



Page 3 of 7

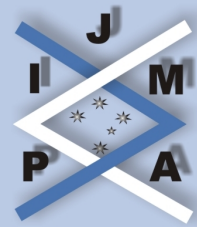
Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756



Title Page

Contents

◀◀ ▶▶

◀ ▶

Page 4 of 7

Go Back

Full Screen

Close

2. The Answer to the Posed Question

Throughout this paper, we suppose that $f(x)$ is a continuous and nonnegative function on $[0, 1]$.

In [1], the following lemma was proved.

Lemma 2.1. *If f satisfies*

$$(2.1) \quad \int_x^1 f(t) dt \geq \frac{1-x^2}{2}, \quad \forall x \in [0, 1],$$

then

$$(2.2) \quad \int_0^1 x^{\alpha+1} f(x) dx \geq \frac{1}{\alpha+3}, \quad \forall \alpha > 0.$$

Theorem 2.2. *If the function f satisfies (2.1), then the inequality*

$$(2.3) \quad \int_0^1 x^\beta f^\alpha(x) dx \geq \frac{1}{\alpha + \beta + 1}$$

holds for every real $\alpha \geq 1$ and $\beta > 0$.

Proof. Applying the AG inequality, we get

$$(2.4) \quad \frac{1}{\alpha} f^\alpha(x) + \frac{\alpha-1}{\alpha} x^\alpha \geq f(x) x^{\alpha-1}.$$

Multiplying both sides of (2.4) by x^β and integrating the resultant inequality from 0 to 1, we obtain

$$(2.5) \quad \int_0^1 x^\beta f^\alpha(x) dx + \frac{\alpha-1}{\alpha + \beta + 1} \geq \alpha \int_0^1 x^{\alpha+\beta-1} f(x) dx.$$



Open Question Regarding an

Integral Inequality

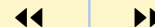
K. Boukerrioua and

A. Guezane-Lakoud

vol. 8, iss. 3, art. 77, 2007

Title Page

Contents



Page 5 of 7

Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756

© 2007 Victoria University. All rights reserved.

Taking into account Lemma 2.1, we have

$$\int_0^1 x^\beta f^\alpha(x) dx + \frac{\alpha - 1}{\alpha + \beta + 1} \geq \frac{\alpha}{\alpha + \beta + 1}.$$

That is,

$$\int_0^1 x^\beta f^\alpha(x) dx \geq \frac{1}{\alpha + \beta + 1}.$$

This completes the proof. □

Theorem 2.3. *If the function f satisfies (2.1), then*

$$(2.6) \quad \int_0^1 f^{\alpha+\beta}(x) dx \geq \int_0^1 x^\beta f^\alpha(x) dx$$

for every real $\alpha \geq 1$ and $\beta > 0$.

Proof. Using the AG inequality, we obtain

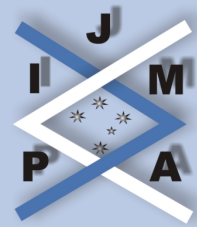
$$(2.7) \quad \frac{\alpha}{\alpha + \beta} f^{\alpha+\beta}(x) + \frac{\beta}{\alpha + \beta} x^{\alpha+\beta} \geq x^\beta f^\alpha(x).$$

Integrating both sides of (2.7), we get

$$(2.8) \quad \frac{\alpha}{\alpha + \beta} \int_0^1 f^{\alpha+\beta}(x) dx + \frac{\beta}{(\alpha + \beta)(\alpha + \beta + 1)} \geq \int_0^1 x^\beta f^\alpha(x) dx.$$

From

$$\int_0^1 x^\beta f^\alpha(x) dx = \frac{\alpha}{\alpha + \beta} \int_0^1 x^\beta f^\alpha(x) dx + \frac{\beta}{\alpha + \beta} \int_0^1 x^\beta f^\alpha(x) dx$$



and by virtue of Theorem 2.3, it follows that

$$(2.9) \quad \int_0^1 x^\beta f^\alpha(x) dx \geq \frac{\alpha}{\alpha + \beta} \int_0^1 x^\beta f^\alpha(x) dx + \frac{\beta}{(\alpha + \beta)(\alpha + \beta + 1)}.$$

From this inequality and using (2.8) we have,

$$\frac{\alpha}{\alpha + \beta} \int_0^1 f^{\alpha + \beta}(x) dx \geq \frac{\alpha}{\alpha + \beta} \int_0^1 x^\beta f^\alpha(x) dx.$$

Thus (2.6) is proved. □

Open Question Regarding an

Integral Inequality

K. Boukerrioua and

A. Guezane-Lakoud

vol. 8, iss. 3, art. 77, 2007

Title Page

Contents



Page 6 of 7

Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756

References

- [1] Q.A. NGÔ, D.D. THANG, T.T. DAT AND D.A. TUAN, Notes On an integral inequality, *J. Inequal. Pure & Appl. Math.*, **7**(4) (2006), Art. 120. [ONLINE: <http://jipam.vu.edu.au/article.php?sid=737>].



Open Question Regarding an

Integral Inequality

K. Boukerrioua and

A. Guezane-Lakoud

vol. 8, iss. 3, art. 77, 2007

Title Page

Contents



Page 7 of 7

Go Back

Full Screen

Close

journal of **inequalities**
in pure and applied
mathematics

issn: 1443-5756