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## CONVEX FUNCTIONS IN A HALF-PLANE

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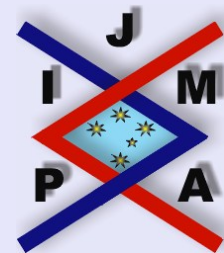
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Abstract

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## Abstract

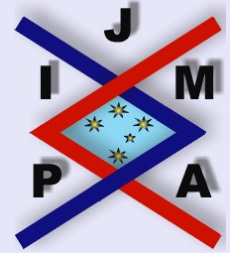
The class of convex hydrodynamically normalized functions in a half-plane was introduced by J. Stankiewicz. In this paper we introduce the general class of convex functions in the upper half-plane  $D$  (not necessarily hydrodynamically normalized) and we obtain necessary and sufficient conditions for an analytic function in  $D$ , to be convex univalent in  $D$ .

*2000 Mathematics Subject Classification:* 30C45.

*Key words:* Univalent function, Convex function, Half-plane.

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# 1. Introduction

We denote by  $D$  the upper half-plane  $\{z \in \mathbb{C} : \text{Im}(z) > 0\}$ , by  $\mathcal{H}$  the class of analytic functions in  $D$ , and by  $\mathcal{H}_1$  the class of functions  $f \in \mathcal{H}$  satisfying:

$$(1.1) \quad \lim_{D \ni z \rightarrow \infty} [f(z) - z] = 0.$$

The normalization (1.1) is known in the literature as hydrodynamic normalization, being related to fluid flows in Mechanics.

The notion of convexity for functions belonging to the class  $\mathcal{H}_1$  was introduced by J. Stankiewicz and Z. Stankiewicz ([4], [5]) as follows:

**Definition 1.1.** *The function  $f \in \mathcal{H}_1$  is said to be convex if  $f$  is univalent in  $D$  and  $f(D)$  is a convex domain.*

We denote by  $C_{\mathcal{H}_1}(D)$  the class of convex functions satisfying the hydrodynamic normalization (1.1).

J. Stankiewicz and Z. Stankiewicz obtained ([4], [5]) the following sufficient conditions for a function  $f \in \mathcal{H}_1$  to be a convex function:

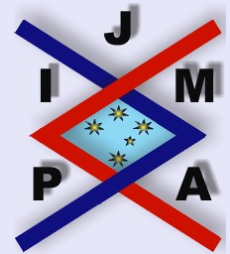
**Theorem 1.1.** *If the function  $f \in \mathcal{H}_1$  satisfies:*

$$f'(z) \neq 0, \text{ for all } z \in D$$

and

$$(1.2) \quad \text{Im} \frac{f''(z)}{f'(z)} > 0, \text{ for all } z \in D,$$

then  $f$  is a convex function.



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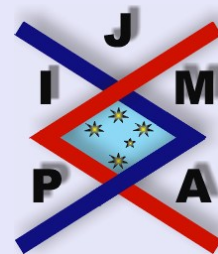
The class of analytic univalent functions in a half-plane has been studied by F.G. Avhadiev [1] starting from the 1970's. He examined the class of convex and univalent functions in a half plane that are not hydrodynamically normalized, obtaining the following theorem:

**Theorem 1.2.** ([1]) *The function  $f : D \rightarrow \mathbb{C}$ , analytic in  $D$ , is convex and univalent in  $D$  if and only if  $f'(i) \neq 0$  and for any  $z \in D$  the following inequality holds:*

$$\operatorname{Im} \left( 2z + (z^2 + 1) \frac{f''(z)}{f'(z)} \right) > 0.$$

Another result that characterizes the convexity property for univalent functions in the half plane that are not hydrodynamically normalized was obtained by the second author in [2].

After 1974, the year when Avhadiev's paper was published, the only classes of univalent functions in the half-plane that had been studied were the univalent functions hydrodynamically normalized. We make the remark that the analytic representation of a geometric property (in this case the convexity property) is not unique.




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## 2. Main Results

The function  $\varphi : U \rightarrow D$  given by

$$\varphi(u) = i \frac{1-u}{1+u}$$

is a conformal mapping of the unit disk  $U$  onto the upper half-plane  $D$ .

For  $0 < r < 1$ , the image of the disk  $U_r = \{z \in \mathbb{C} : |z| < r\}$  under  $\varphi$  is the disk  $D_r = \{z \in \mathbb{C} : |z - z_r| < R_r\}$ , where:

$$(2.1) \quad \begin{cases} z_r = i \frac{1+r^2}{1-r^2}; \\ R_r = \frac{2r}{1-r^2} \end{cases}.$$

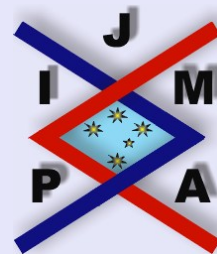
To see this, note that in polar coordinates  $u = re^{it}$ , using the identity:

$$|1 + re^{-it}| = |1 + re^{it}|$$

we obtain:

$$\left| i \frac{1 - re^{it}}{1 + re^{it}} - i \frac{1 + r^2}{1 - r^2} \right| = \left| \frac{2r(1 + re^{-it})}{(1 + re^{it})(1 - r^2)} \right| = \frac{2r}{1 - r^2},$$

for any  $r \in (0, 1)$  and any  $t \in [0, 2\pi)$ , which shows that the image under  $\varphi$  of the boundary of the disk  $U_r$  is the boundary of the disk  $D_r$ . Since  $\varphi(0) = i \in D_r$ , it follows that  $\varphi(U_r) = D_r$ .



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**Lemma 2.1.** ([3]) *The family of domains  $\{D_r\}_{r \in (0,1)}$  has the following properties:*

- i) *for any positive real numbers  $0 < r < s < 1$  we have  $D_r \subset D_s$ ;*
- ii) *for any complex number  $z \in D$  there exists  $r_z \in (0, 1)$  such that  $z \in D_r$ , for any  $r \in (r_z, 1)$ ;*
- iii) *for any  $z \in D$  and  $r \in (r_z, 1)$  arbitrarily fixed, there exists  $u_r \in U$  such that*

$$z = z_r + R_r u_r.$$

*Moreover, we have the following equalities:*

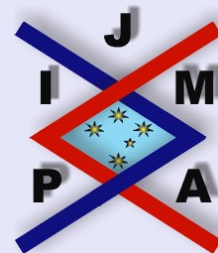
$$\begin{cases} \lim_{r \rightarrow 1} u_r = -i \\ \lim_{r \rightarrow 1} R_r (1 - |u_r|) = \text{Im } z \end{cases}.$$

*Proof.*

- i) For any  $0 < r < s < 1$  we have:

$$D_r = \varphi(U_r) \subset \varphi(U_s) = D_s.$$

- ii) For  $z \in D$  we have  $\varphi^{-1}(z) \in U$ , hence considering  $r_z = |\varphi^{-1}(z)|$  we have  $r_z \in (0, 1)$ , and for any  $r \in (r_z, 1)$  we obtain  $z \in \varphi(U_r) = D_r$ .



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iii) If  $z = X + iY$  is an arbitrarily fixed point in  $D_r$ ,  $r \in (r_z, 1)$ , then the complex number  $u_r = x_r + iy_r$  given by:

$$u_r = \frac{z - z_r}{R_r}$$

has the property that  $|u_r| < 1$ . Using the relations (2.1) we get:

$$X + iY = \frac{2r}{1 - r^2}x_r + i \left( \frac{1 + r^2}{1 - r^2} + \frac{2r}{1 - r^2}y_r \right)$$

and therefore:

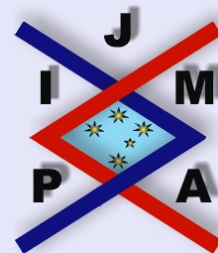
$$\begin{cases} x_r = \frac{1 - r^2}{2r}X, \\ y_r = \frac{(1 - r^2)Y - (1 + r^2)}{2r}, \end{cases}$$

hence it follows:

$$\begin{aligned} \lim_{r \rightarrow 1} u_r &= \lim_{r \rightarrow 1} \frac{(1 - r^2)}{2r}X + i \frac{(1 - r^2)Y - (1 + r^2)}{2r} \\ &= -i, \end{aligned}$$

and

$$\begin{aligned} \lim_{r \rightarrow 1} R_r (1 - |u_r|^2) &= \lim_{r \rightarrow 1} \frac{2r}{1 - r^2} \cdot \frac{4r^2 - |z|^2(1 - r^2)^2 + 2(1 - r^4)Y - (1 + r^2)^2}{4r^2} \\ &= \lim_{r \rightarrow 1} -\frac{|z|^2(1 - r^2)}{2r} + Y(1 + r^2) - \frac{1 - r^2}{2r} \\ &= 2Y = 2 \operatorname{Im} z. \end{aligned}$$



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As  $\lim_{r \rightarrow 1} 1 + |u_r| = 2$ , follows from the previous inequality, the final result follows from the second part of iii), completing the proof. □

The next theorem is obtained as a consequence of “the second coefficient inequality” for univalent functions in the unit disk, due to Bieberbach:

**Theorem 2.2.** *If  $g : U \rightarrow \mathbb{C}$  is analytic and univalent in  $U$ , then for any  $z \in U$  the following inequality holds:*

$$\left| -2|z|^2 + (1 - |z|^2) \frac{zg''(z)}{g'(z)} \right| \leq 4|z|.$$

Using Lemma 2.1 we obtain the following result, which corresponds to the previous theorem in the case of univalent functions in the half-plane:

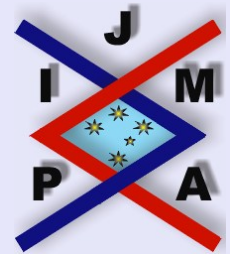
**Theorem 2.3.** ([3]) *If the function  $f : D \rightarrow \mathbb{C}$  is analytic and univalent in the half-plane  $D$ , then for any  $z \in D$  we have the inequality*

$$(2.2) \quad \left| i - \operatorname{Im}(z) \frac{f''(z)}{f'(z)} \right| \leq 2.$$

The equality is satisfied for the function given by

$$f(z) = z^2$$

at the point  $z = i$ .




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We make the observation that a simple function such as  $f : D \rightarrow \mathbb{C}$  defined by

$$f(z) = \sqrt{z},$$

(where we consider a fixed branch of the logarithm for the square root) is univalent in the domain  $D$ ,  $f(D)$  is a convex domain, yet the function  $f$  is not considered to be convex in the sense of Definition 1.1 since it does not belong to the class  $\mathcal{H}_1$  ( $f$  does not satisfy the hydrodynamic normalization (1.1)).

This observation suggested the idea that it is necessary to give up the hydrodynamic normalization condition, a much too restrictive normalization. In this sense we propose a new definition of convexity for analytic functions in  $D$ , to include a larger class of analytic functions in  $D$ , not necessarily hydrodynamically normalized:

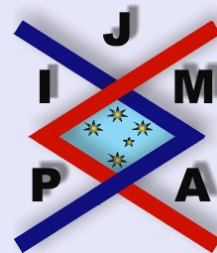
**Definition 2.1.** A function  $f \in \mathcal{H}$  is said to be convex in  $D$  if  $f$  is univalent in  $D$  and  $f(D)$  is a convex domain.

We will denote by  $C(D)$  the class of convex functions (in the sense of Definition 2.1). The next theorem gives necessary and sufficient conditions for a function  $f \in \mathcal{H}$  to belong to the class  $C(D)$ :

**Theorem 2.4.** For an analytic function  $f : D \rightarrow \mathbb{C}$ , the following are equivalent:

- i)  $f \in C(D)$ ;
- ii)  $f'(iy) \neq 0$  for any  $y > 1$ , and for any  $r \in (0, 1)$  and  $z \in D_r$  the following inequality holds:

$$(2.3) \quad \operatorname{Re} \frac{(z - z_r) f''(z)}{f'(z)} + 1 > 0,$$




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where  $D_r$  is the disk  $\{z \in \mathbb{C} : |z - z_r| < R_r\}$  and

$$(2.4) \quad \begin{cases} z_r = i \frac{1+r^2}{1-r^2}, \\ R_r = \frac{2r}{1-r^2}. \end{cases}$$

*Proof.* Given the function  $f \in C(D)$ , denote by  $\Delta$  the convex domain  $f(D)$ . The function  $\varphi : U \rightarrow D$  given by

$$\varphi(u) = i \frac{1-u}{1+u}$$

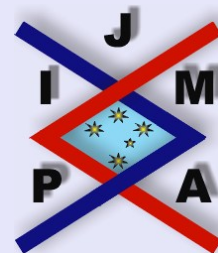
represents conformally the disk  $U$  to the half-plane  $D$ , and for any  $r \in (0, 1)$  we have  $\varphi(U_r) = D_r$ .

The function  $f \circ \varphi : U \rightarrow \mathbb{C}$  represents conformally the unit disk  $U$  onto  $\Delta = f(D)$ . Since the domain  $\Delta$  is convex, it follows that the function  $f \circ \varphi$  is convex and univalent in the unit disk  $U$ , and hence represents conformally any disk  $U_r$  ( $0 < r < 1$ ), onto a convex domain. Since  $\varphi(U_r) = D_r$ , it follows that for any  $r \in (0, 1)$  the domain  $\Delta_r = f(D_r)$  is convex. For  $r \in (0, 1)$  arbitrarily fixed, the function  $g_r : U \rightarrow \mathbb{C}$  given by

$$(2.5) \quad g_r(u) = f(z_r + R_r u),$$

where  $z_r, R_r$  are given by (2.4), represents conformally the disk  $U$  onto the convex domain  $\Delta_r$ . Using the results for convex and univalent functions in the unit disk, it follows that the domain  $\Delta_r$  is convex if and only if

$$(2.6) \quad g_r'(0) = R_r f'(z_r) \neq 0$$



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and for any  $u \in U$  the following inequality holds:

$$(2.7) \quad \operatorname{Re} \frac{zg_r''(u)}{g_r'(u)} + 1 = \operatorname{Re} \frac{zR_rf''(z_r + R_ru)}{f'(z_r + R_ru)} + 1 > 0.$$

Denoting  $z = z_r + R_ru$ , and observing that  $u \in U$  if and only if  $z \in D_r$ , the previous inequality can be written as

$$(2.8) \quad \operatorname{Re} \frac{(z - z_r) f''(z)}{f'(z)} + 1 > 0,$$

for any  $z \in D_r$ , proving the necessity for condition (2.3).

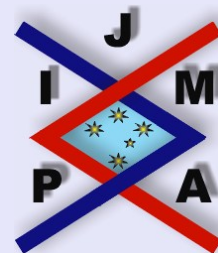
Since  $z_r = i \frac{1+r^2}{1-r^2}$ , for  $r \in (0, 1)$  we have:

$$|z_r| = \frac{1+r^2}{1-r^2} > 1$$

for any  $r \in (0, 1)$  and thus the condition (2.6) is equivalent to  $f'(iy) \neq 0$  for any  $y > 1$ .

Conversely, if ii) holds, then for any arbitrarily fixed  $r \in (0, 1)$  the function  $g_r(u) = f(z_r + R_ru)$  is convex and univalent in the disk  $U$ . It follows that for any  $r \in (0, 1)$  the domain  $\Delta_r = g_r(U)$  is convex, and since  $\Delta_r = f(D_r)$ , it follows that the function  $f$  is convex and univalent in the domain  $D_r$ , for any  $r \in (0, 1)$ . Since  $\bigcup_{r \in (0,1)} D_r = D$ , it follows that the function  $f$  is convex and univalent in the half-plane  $D$ , completing the proof.  $\square$

In the previous proof we obtained the following result:



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**Corollary 2.5.** *If the function  $f : D \rightarrow \mathbb{C}$  is convex and univalent in  $D$ , then  $\Delta_r = f(D_r)$  is a convex domain for any  $r \in (0, 1)$ .*

**Remark 2.1.** *In [2] the second author introduced the subclass  $C_1(D)$  of the class of convex univalent functions as follows:*

**Definition 2.2.** *([2]) We say that the analytic function  $f : D \rightarrow \mathbb{C}$  belongs to the class  $C_1(D)$  if for any  $z \in D$  we have:*

$$(2.9) \quad f'(z) \neq 0$$

and

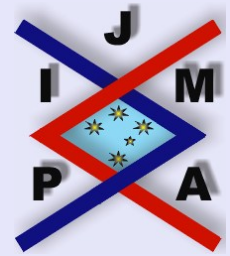
$$(2.10) \quad \begin{cases} \operatorname{Re} \frac{zf''(z)}{f'(z)} + 1 > 0, \\ \operatorname{Im} \frac{f''(z)}{f'(z)} > 0. \end{cases}$$

It is known that if the function  $g : U \rightarrow \mathbb{C}$  is convex and univalent in the unit disk  $U$ , with the Taylor series expansion:

$$g(z) = z + a_2z^2 + \dots,$$

then  $|a_2| \leq 1$ . The class of convex and univalent functions in the unit disk, normalized by  $f(0) = f'(0) - 1 = 0$  is denoted by  $C$ .

The above property for functions belonging to the class  $C$  has the following important consequence:




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**Theorem 2.6.** *If the function  $g : U \rightarrow \mathbb{C}$  belongs to the class  $C$ , then for any  $z \in U$  the following inequality holds:*

$$\left| -2|z|^2 + (1 - |z|^2) \frac{zg''(z)}{g'(z)} \right| \leq 2|z|.$$

Using this result we obtain a differential characterization of the class  $C(D)$  of convex and univalent functions in the half-plane  $D$ :

**Theorem 2.7.** *If the function  $f : D \rightarrow \mathbb{C}$  belongs to the class  $C(D)$ , then for any  $z \in D$  we have the inequality:*

$$(2.11) \quad \left| i - \operatorname{Im}(z) \frac{f''(z)}{f'(z)} \right| \leq 1.$$

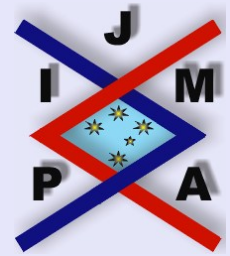
*Proof.* If the function  $f$  belongs to  $C(D)$ , by Corollary 2.5 it follows that  $\Delta_r = f(D_r)$  is a convex domain for any  $r \in (0, 1)$ .

The function  $g_r$  given by formula (2.5) represents conformally the unit disk  $U$  onto  $g_r(U) = \Delta_r$ , and since  $\Delta_r$  is a convex domain, it follows that the function  $g_r$  is convex and univalent. The function

$$\frac{g_r(u) - g_r(0)}{g'_r(0)} = \frac{f(z_r + R_r u) - f(z_r)}{R_r f'(z_r)}$$

is therefore convex and univalent in  $u \in U$ , normalized by  $g_r(0) = g'_r(0) - 1 = 0$  for any  $r \in (0, 1)$ . By Theorem 2.6 it follows that for any  $r \in (0, 1)$  and any  $u \in U$  the following inequality holds:

$$(2.12) \quad \left| -2|u|^2 + (1 - |u|^2) \frac{uR_r f''(z_r + R_r u)}{f'(z_r + R_r u)} \right| \leq 2|u|.$$



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Given  $z \in D$ , by Lemma 2.1 there exists  $r_z \in (0, 1)$  such that for any fixed  $r \in (r_z, 1)$ , there is  $u_r \in U$  such that  $z = z_r + R_r u_r \in D_r$  and

$$\begin{cases} \lim_{r \rightarrow 1} u_r = -i, \\ \lim_{r \rightarrow 1} (1 - |u_r|) R_r = \text{Im } z. \end{cases}$$

Considering  $u = u_r$  in the inequality (2.12) and passing to the limit with  $r \rightarrow 1$ , we obtain:

$$\left| -2 + 2 \text{Im}(z) \frac{-i f''(z)}{f'(z)} \right| \leq 2.$$

Since  $z \in D$  was arbitrarily chosen, we have shown that for any  $z \in D$  the following inequality holds:

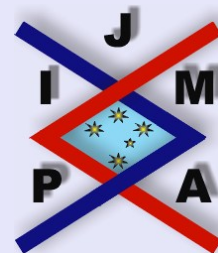
$$\left| i - \text{Im}(z) \frac{f''(z)}{f'(z)} \right| \leq 1,$$

and the theorem is proved.  $\square$

The next result is an important consequence of Theorem 2.7:

**Corollary 2.8.** *If the function  $f : D \rightarrow \mathbb{C}$  is convex and univalent in the half-plane  $D$ , then for any  $z \in D$  we have the inequality:*

$$(2.13) \quad \text{Im} \frac{f''(z)}{f'(z)} > 0.$$




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*Proof.* If the function  $f$  is convex and univalent in the half-plane  $D$ , by the inequality (2.11) given by Theorem 2.7, it follows that for any  $z \in D$ , the point  $w = \operatorname{Im}(z) \frac{f''(z)}{f'(z)}$  belongs to the disk centered at  $i$  with radius 1. Since this disk belongs to the upper half-plane, it follows that for any  $z \in D$  the inequality (2.13) holds.  $\square$

**Remark 2.2.** *The result in the previous corollary was obtained, using different methods, by F.G. Avhadiev [1].*

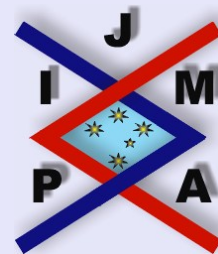
**Example 2.1.** *The function  $f : D \rightarrow \mathbb{C}$  given by*

$$f(z) = z^a,$$

*is convex and univalent for any  $a \in [-1, 0) \cup (0, 1]$ , since the function  $f$  is analytic and univalent in  $D$ , and the domains:  $f(D) = \{z \in \mathbb{C} : \arg(z) \in (0, a\pi)\}$ , for  $a \in (0, 1)$ , and  $f(D) = \{z \in \mathbb{C} : \arg(z) \in (a\pi, 0)\}$ , for  $a \in (-1, 0)$ , are convex.*

The following inequalities hold:

$$\begin{aligned} \operatorname{Re} \frac{(z - z_r) f''(z)}{f'(z)} + 1 &= (a - 1) \operatorname{Re} \frac{z - z_r}{z} + 1 \\ &= \frac{a |z|^2 - |z_r| (a - 1) \operatorname{Im} z}{|z|^2} \\ &= \frac{a}{|z|^2} \left[ (\operatorname{Re} z)^2 + (\operatorname{Im} z)^2 - \frac{a - 1}{a} |z_r| \operatorname{Im} z \right]. \end{aligned}$$



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Let us observe that if  $a \in [-1, 0)$ , then for any  $r \in (0, 1)$ , we have the following inequality:

$$(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2 - \frac{a-1}{a} |z_r| \operatorname{Im} z < 0,$$

for any  $z$  in the disk centered at  $\frac{i(a-1)|z_r|}{2a}$  with radius  $\frac{(a-1)|z_r|}{2a}$ . Since

$$\frac{a-1}{2a} |z_r| \geq |z_r|,$$

this disk is contained in the disk  $D_r$ , and hence by Theorem 2.4 it follows that for  $a \in [-1, 0)$  we have  $f \in C(D)$ .

For  $a \in (0, 1]$  we have:

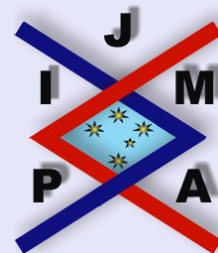
$$\operatorname{Re} \frac{(z - z_r) f''(z)}{f'(z)} + 1 = a - (a-1) |z_r| \frac{\operatorname{Im} z}{|z^2|} > 0$$

for any  $r \in (0, 1)$  and for any  $z \in D$ , and therefore by Theorem 2.4 the function  $f$  belongs to the class  $C(D)$  for  $a \in (0, 1]$  as well.

Applying Theorem 1.2 to the same function  $f$ , we obtain:

$$\begin{aligned} \operatorname{Im} \left[ 2z + \frac{(z^2 + 1) f''(z)}{f'(z)} \right] &= 2y + \operatorname{Im} \frac{(z^2 + 1)(a-1)}{z} \\ &= |z|^{-2} y [(a+1)|z|^2 - (a-1)] > 0 \end{aligned}$$

for any  $z \in D$ , if and only if  $a \in [-1, 1]$ . The condition  $f'(i)$  is satisfied for  $a \neq 0$ , hence it follows that  $f \in C(D)$  for any  $a \in [-1, 0) \cup (0, 1]$ .



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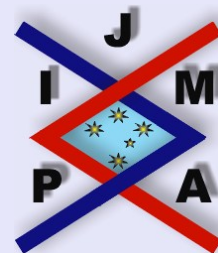
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Trying to apply the result due to J. Stankiewicz, we can see that  $f \notin C_{\mathcal{H}_1}(D)$  for any value of  $a \in [-1, 0) \cup (0, 1]$  since the considered function  $f$  satisfies the hydrodynamic normalization just for  $a = 1$ , but in this case

$$\operatorname{Im} \frac{f''(z)}{f'(z)} = 0,$$

and the condition obtained by J. Stankiewicz is not satisfied. We therefore have the inclusion  $C_{\mathcal{H}_1}(D) \subsetneq C(D)$ .




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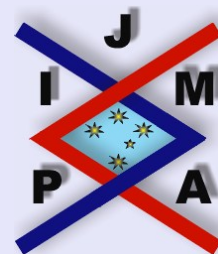
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