

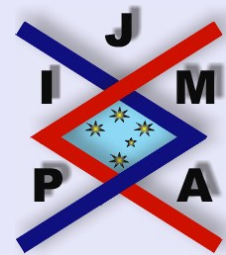
# Journal of Inequalities in Pure and Applied Mathematics

## AN OSTROWSKI TYPE INEQUALITY FOR $p$ -NORMS

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Abstract

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## Abstract

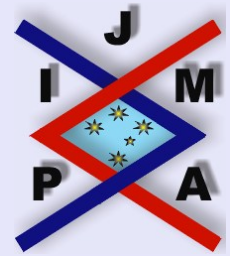
In this paper, we establish general form of an inequality of Ostrowski type for twice differentiable mappings in terms of  $L_p$ -norm, with first derivative absolutely continuous. The integral inequality of similar type already pointed out in literature is a special case of ours. The already established inequality contains a mistake and as a result incorrect consequences and applications. The corrected version of the inequality is pointed out and the inequality is also applied to special means and numerical integration.

*2000 Mathematics Subject Classification:* 26D15.

*Key words:* Ostrowski inequality, Numerical integration, Special means.

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# 1. Introduction

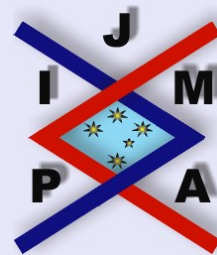
We establish here the general form of an inequality of Ostrowski type, different to that of Cerone, Dragomir and Roumeliotis [1], for twice differentiable mappings in terms of  $L_p$ -norm. The integral inequality of similar type already pointed out by N.S. Barnett, P. Cerone, S.S. Dragomir, J. Roumeliotis and A. Sofo [2], contains a mistake which has already been reported by N.A. Mir and A. Rafiq in their research work [3]. The same mistake has been carried out in their other research article, namely Theorem 20 of [2] and as a result incorrect consequences and applications of this theorem. The corrected form of the theorem is as follows:

**Theorem 1.1.** *Let  $g : [a, b] \rightarrow \mathbb{R}$  be a mapping whose first derivative is absolutely continuous on  $[a, b]$ . If we assume that the second derivative  $g'' \in L_p(a, b)$ ,  $1 < p < \infty$ , then we have the inequality*

$$(1.1) \quad \left| \int_a^b g(t) dt - \frac{1}{2} \left[ g(x) + \frac{g(a) + g(b)}{2} \right] (b - a) + \frac{1}{2} (b - a) \left( x - \frac{a + b}{2} \right) g'(x) \right| \leq \frac{1}{2} \left( \frac{b - a}{2} \right)^{2 + \frac{1}{q}} \|g''\|_p$$

$$\times \begin{cases} [B(q + 1, q + 1) + B_{x_1}(q + 1, q + 1) + \Psi_{x_2}(q + 1, q + 1)]^{\frac{1}{q}} \text{ for } x \in [a, \frac{a+b}{2}], \\ [B(q + 1, q + 1) + B_{x_3}(q + 1, q + 1) + B_{x_4}(q + 1, q + 1)]^{\frac{1}{q}} \text{ for } x \in (\frac{a+b}{2}, b], \end{cases}$$

where  $\frac{1}{p} + \frac{1}{q} = 1$ ,  $p > 1$ ,  $q > 1$ , and  $B(\cdot, \cdot)$  is the Beta function of Euler given



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by

$$B(l, s) = \int_0^1 t^{l-1}(1-t)^{s-1} dt, \quad l, s > 0.$$

Further

$$B_r(l, s) = \int_0^r t^{l-1}(1-t)^{s-1} dt$$

is the incomplete Beta function,

$$\Psi_r(l, s) = \int_0^r t^{l-1}(1+t)^{s-1} dt$$

is the real positive valued integral,

$$x_1 = \frac{2(x-a)}{b-a}, \quad x_2 = 1 - x_1, \quad x_3 = x_1 - 1, \quad x_4 = 2 - x_1$$

and

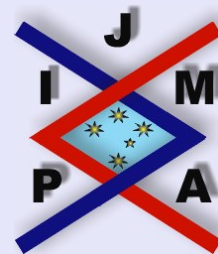
$$\|g''\|_p := \left( \int_a^b |g''(t)|^p dt \right)^{\frac{1}{p}}.$$

If we assume that  $g'' \in L_1(a, b)$ , then we have

$$(1.2) \quad \left| \int_a^b g(t) dt - \frac{1}{2} \left[ g(x) + \frac{g(a) + g(b)}{2} \right] (b-a) + \frac{1}{2} (b-a) \left( x - \frac{a+b}{2} \right) g'(x) \right| \leq \frac{\|g''\|_1}{8} (b-a)^2,$$

where

$$\|g''\|_1 := \int_a^b |g''(t)| dt.$$



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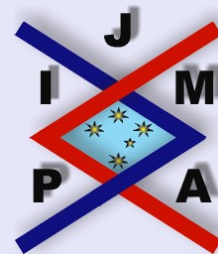
## 2. Main Results

The following theorem is now proved and subsequently applied to numerical integration and special means.

**Theorem 2.1.** *Let  $g : [a, b] \rightarrow \mathbb{R}$  be a mapping whose first derivative is absolutely continuous on  $[a, b]$ . If we assume that the second derivative  $g'' \in L_p(a, b)$ ,  $1 < p < \infty$ , then we have the inequality*

$$(2.1) \quad \left| \frac{1}{\alpha + \beta} \left( \frac{\alpha}{x-a} \int_a^x g(t) dt + \frac{\beta}{b-x} \int_x^b g(t) dt \right) - \frac{1}{2} g(x) - \frac{1}{2(\alpha + \beta)} \left[ \left( x - \frac{a+b}{2} \right) g(x) \left( \frac{\alpha}{x-a} - \frac{\beta}{b-x} \right) + \frac{(b-a)}{2} \left( \frac{\alpha}{x-a} g(a) + \frac{\beta}{b-x} g(b) \right) - (\alpha + \beta) \left( x - \frac{a+b}{2} \right) g'(x) \right] \right| \leq \left( \frac{b-a}{2} \right)^{2+\frac{1}{q}} \|g''\|_p$$

$$\times \begin{cases} \left[ \left( \frac{\beta}{\alpha+\beta} \frac{1}{b-x} \right)^q B(q+1, q+1) + \left( \frac{\alpha}{\alpha+\beta} \frac{1}{x-a} \right)^q B_{x_1}(q+1, q+1) + \left( \frac{\beta}{\alpha+\beta} \frac{1}{b-x} \right)^q \Psi_{x_2}(q+1, q+1) \right]^{\frac{1}{q}} \text{ for } x \in \left[ a, \frac{a+b}{2} \right], \\ \left[ \left( \frac{\alpha}{\alpha+\beta} \frac{1}{x-a} \right)^q B(q+1, q+1) + \left( \frac{\alpha}{\alpha+\beta} \frac{1}{x-a} \right)^q B_{x_3}(q+1, q+1) + \left( \frac{\beta}{\alpha+\beta} \frac{1}{b-x} \right)^q B_{x_4}(q+1, q+1) \right]^{\frac{1}{q}} \text{ for } x \in \left( \frac{a+b}{2}, b \right], \end{cases}$$



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where  $\frac{1}{p} + \frac{1}{q} = 1$ ,  $p > 1$ ,  $q > 1$ , and  $B(\cdot, \cdot)$  is the Beta function of Euler given by

$$B(l, s) = \int_0^1 t^{l-1}(1-t)^{s-1} dt, l, s > 0.$$

Further,

$$B_r(l, s) = \int_0^r t^{l-1}(1-t)^{s-1} dt$$

is the incomplete Beta function,

$$\Psi_r(l, s) = \int_0^r t^{l-1}(1+t)^{s-1} dt$$

is a real positive valued integral,

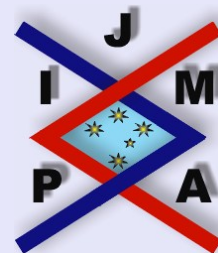
$$x_1 = \frac{2(x-a)}{b-a}, \quad x_2 = 1 - x_1, \quad x_3 = x_1 - 1, \quad x_4 = 2 - x_1$$

and

$$\|g''\|_p := \left( \int_a^b |g''(t)|^p dt \right)^{\frac{1}{p}}.$$

If we assume that  $g'' \in L_1(a, b)$ , then we have

$$(2.2) \quad \left| \frac{1}{\alpha + \beta} \left( \frac{\alpha}{x-a} \int_a^x g(t) dt + \frac{\beta}{b-x} \int_x^b g(t) dt \right) - \frac{1}{2} g(x) \right. \\ \left. - \frac{1}{2(\alpha + \beta)} \left[ \left( x - \frac{a+b}{2} \right) g(x) \left( \frac{\alpha}{x-a} - \frac{\beta}{b-x} \right) \right] \right|$$



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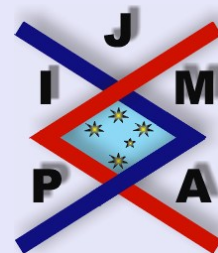


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$$\begin{aligned}
 & + \frac{(b-a)}{2} \left( \frac{\alpha}{x-a} g(a) + \frac{\beta}{b-x} g(b) \right) - (\alpha + \beta) \left( x - \frac{a+b}{2} \right) g'(x) \Bigg| \\
 & \leq \frac{1}{2} \|g''\|_1 \|K(x,t)\|_\infty,
 \end{aligned}$$

where

$$\|g''\|_1 = \int_a^b |g''(t)| dt,$$

and

$$\|K(x,t)\|_\infty = \frac{1}{\alpha + \beta} \max \left( \frac{\alpha}{x-a}, \frac{\beta}{b-x} \right) \frac{(b-a)^2}{4} \quad \text{for } x \in [a, b].$$

*Proof.* We begin by recalling the following integral equality proved by N.A. Mir and A. Rafiq [3] which is generalization of an integral equality proved by Dragomir and Wang [4].

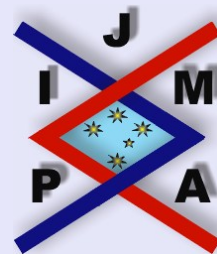
$$\begin{aligned}
 (2.3) \quad & \left| \frac{1}{\alpha + \beta} \left( \frac{\alpha}{x-a} \int_a^x g(t) dt + \frac{\beta}{b-x} \int_x^b g(t) dt \right) - \frac{1}{2} g(x) \right. \\
 & - \frac{1}{2(\alpha + \beta)} \left[ \left( x - \frac{a+b}{2} \right) g(x) \left( \frac{\alpha}{x-a} - \frac{\beta}{b-x} \right) \right. \\
 & \left. \left. + \frac{(b-a)}{2} \left( \frac{\alpha}{x-a} g(a) + \frac{\beta}{b-x} g(b) \right) - (\alpha + \beta) \left( x - \frac{a+b}{2} \right) g'(x) \right] \right| \\
 & = \frac{1}{2} \left| \int_a^b p(x,t) \left( t - \frac{a+b}{2} \right) g''(t) dt \right|
 \end{aligned}$$

whose left hand side is equivalent to that of (2.1). From the right hand side of (2.3) we have, by Hölder's inequality, that

$$\begin{aligned} & \left| \int_a^b p(x, t) \left( t - \frac{a+b}{2} \right) g''(t) dt \right| \\ & \leq \left( \int_a^b |g''(t)|^p dt \right)^{\frac{1}{p}} \left( \int_a^b |p(x, t)|^q \left| t - \frac{a+b}{2} \right|^q dt \right)^{\frac{1}{q}} \\ & = \|g''\|_p \left( \int_a^b |p(x, t)|^q \left| t - \frac{a+b}{2} \right|^q dt \right)^{\frac{1}{q}}, \end{aligned}$$

and from (2.3) we obtain the inequality

$$\begin{aligned} (2.4) \quad & \left| \frac{1}{\alpha + \beta} \left( \frac{\alpha}{x-a} \int_a^x g(t) dt + \frac{\beta}{b-x} \int_x^b g(t) dt \right) \right. \\ & \quad \left. - \frac{1}{2} g(x) - \frac{1}{2(\alpha + \beta)} \left[ \left( x - \frac{a+b}{2} \right) g(x) \left( \frac{\alpha}{x-a} - \frac{\beta}{b-x} \right) \right. \right. \\ & \quad \left. \left. + \frac{(b-a)}{2} \left( \frac{\alpha}{x-a} g(a) + \frac{\beta}{b-x} g(b) \right) - (\alpha + \beta) \left( x - \frac{a+b}{2} \right) g'(x) \right] \right| \\ & \leq \frac{1}{2} \|g''\|_p \left( \int_a^b |p(x, t)|^q \left| t - \frac{a+b}{2} \right|^q dt \right)^{\frac{1}{q}}. \end{aligned}$$



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From the right hand side of (2.4) we may define

$$\begin{aligned}
 I &:= \int_a^b |p(x, t)|^q \left| t - \frac{a+b}{2} \right|^q dt \\
 &= \left( \frac{\alpha}{\alpha + \beta} \cdot \frac{1}{x-a} \right)^q \int_a^x (t-a)^q \left| t - \frac{a+b}{2} \right|^q dt \\
 (2.5) \quad &+ \left( \frac{\beta}{\alpha + \beta} \cdot \frac{1}{b-x} \right)^q \int_x^b |t-b|^q \left| t - \frac{a+b}{2} \right|^q dt
 \end{aligned}$$

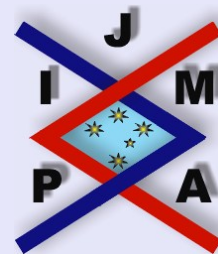
such that we can identify two distinct cases.

(a) For  $x \in [a, \frac{a+b}{2}]$

$$\begin{aligned}
 I_A &= \left( \frac{\alpha}{\alpha + \beta} \frac{1}{x-a} \right)^q \int_a^x (t-a)^q \left( \frac{a+b}{2} - t \right)^q dt \\
 &+ \left( \frac{\beta}{\alpha + \beta} \frac{1}{b-x} \right)^q \int_x^{\frac{a+b}{2}} (b-t)^q \left( \frac{a+b}{2} - t \right)^q dt \\
 &+ \left( \frac{\beta}{\alpha + \beta} \frac{1}{b-x} \right)^q \int_{\frac{a+b}{2}}^b (b-t)^q \left( t - \frac{a+b}{2} \right)^q dt.
 \end{aligned}$$

Investigating the three separate integrals, we may evaluate as follows:

$$I_1 = \int_a^x (t-a)^q \left( \frac{a+b}{2} - t \right)^q dt,$$



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making the change of variable  $t = a + \left(\frac{b-a}{2}\right) w$ , we arrive at

$$\begin{aligned} I_1 &= \left(\frac{b-a}{2}\right)^{2q+1} \int_0^{x_1} w^q(1-w)^q dw, \\ &= \left(\frac{b-a}{2}\right)^{2q+1} B_{x_1}(q+1, q+1), \end{aligned}$$

where  $B_{x_1}(\cdot, \cdot)$  is the incomplete Beta function and  $x_1 = \frac{2(x-a)}{b-a}$ .

$$I_2 = \int_x^{\frac{a+b}{2}} (b-t)^q \left(\frac{a+b}{2} - t\right)^q dt,$$

making the change of variable  $t = \frac{a+b}{2} - \left(\frac{b-a}{2}\right) w$ , we obtain

$$I_2 = \left(\frac{b-a}{2}\right)^{2q+1} \int_0^{x_2} w^q(1+w)^q dw = \left(\frac{b-a}{2}\right)^{2q+1} \Psi_{x_2}(q+1, q+1),$$

where

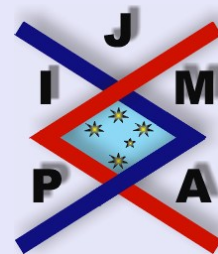
$$\Psi_{x_2} := \int_0^{x_2} w^q(1+w)^q dw$$

and  $x_2 = \frac{a+b-2x}{b-a} = 1 - x_1$ .

$$I_3 = \int_{\frac{a+b}{2}}^b (b-t)^q \left(t - \frac{a+b}{2}\right)^q dt,$$

making the change of variable  $t = \frac{a+b}{2} + \left(\frac{b-a}{2}\right) w$ , we get

$$I_3 = \left(\frac{b-a}{2}\right)^{2q+1} \int_0^1 w^q(1-w)^q dw = \left(\frac{b-a}{2}\right)^{2q+1} B(q+1, q+1),$$



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where  $B(\cdot, \cdot)$  is the Beta function.

We may now write

$$\begin{aligned} I_A &= I_1 + I_2 + I_3 \\ &= \left(\frac{b-a}{2}\right)^{2q+1} \left[ \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q Bx_1(q+1, q+1) \right. \\ &\quad + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q \Psi_{x_2}(q+1, q+1) \\ &\quad \left. + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q B(q+1, q+1) \right] \end{aligned}$$

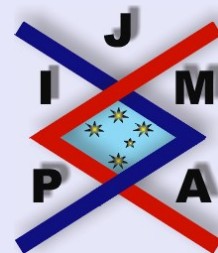
for  $x \in [a, \frac{a+b}{2}]$ .

(b) For  $x \in (a, \frac{a+b}{2}]$

$$\begin{aligned} I_B &= \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q \int_a^{\frac{a+b}{2}} (t-a)^q \left(\frac{a+b}{2} - t\right)^q dt \\ &\quad + \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q \int_{\frac{a+b}{2}}^x (t-a)^q \left(t - \frac{a+b}{2}\right)^q dt \\ &\quad + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q \int_x^b (b-t)^q \left(t - \frac{a+b}{2}\right)^q dt. \end{aligned}$$

In a similar fashion to the previous case, we have

$$I_4 = \int_a^{\frac{a+b}{2}} (t-a)^q \left(\frac{a+b}{2} - t\right)^q dt.$$



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Letting  $t = a + \left(\frac{b-a}{2}\right) w$ , we obtain

$$I_4 = \left(\frac{b-a}{2}\right)^{2q+1} \int_0^1 w^q(1-w)^q dw = \left(\frac{b-a}{2}\right)^{2q+1} B(q+1, q+1),$$

where  $B(\cdot, \cdot)$  is the Beta function.

$$I_5 = \int_{\frac{a+b}{2}}^x (t-a)^q \left(t - \frac{a+b}{2}\right)^q dt,$$

making the change of variable  $t = \frac{a+b}{2} + \left(\frac{b-a}{2}\right) w$ , we arrive at

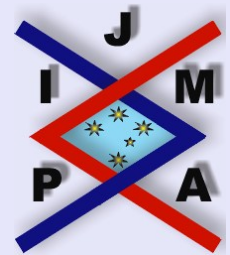
$$I_5 = \left(\frac{b-a}{2}\right)^{2q+1} \int_0^{x_3} w^q(1-w)^q dw = \left(\frac{b-a}{2}\right)^{2q+1} B_{x_3}(q+1, q+1),$$

where  $B_{x_3}(\cdot, \cdot)$  is the incomplete Beta function and  $x_3 = x_1 - 1$ .

$$I_6 = \int_x^b (b-t)^q \left(t - \frac{a+b}{2}\right)^q dt,$$

making the change of variable  $t = b - \left(\frac{b-a}{2}\right) w$ , we get

$$I_6 = \left(\frac{b-a}{2}\right)^{2q+1} \int_0^{x_4} w^q(1-w)^q dw = \left(\frac{b-a}{2}\right)^{2q+1} B_{x_4}(q+1, q+1),$$




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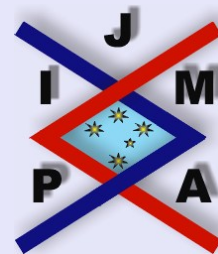
where  $B_{x_4}(\cdot, \cdot)$  is the incomplete Beta function and  $x_4 = 2 - x_1$ .

$$\begin{aligned}
 I_B &= I_4 + I_5 + I_6 \\
 &= \left(\frac{b-a}{2}\right)^{2q+1} \left[ \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q B(q+1, q+1) \right. \\
 &\quad + \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q B_{x_3}(q+1, q+1) \\
 &\quad \left. + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q B_{x_4}(q+1, q+1) \right]
 \end{aligned}$$

for  $x \in (\frac{a+b}{2}, b]$ .

Also from (2.5)

$$\begin{aligned}
 I &= I_A + I_B \\
 &= \left(\frac{b-a}{2}\right)^{2q+1} \left\{ \begin{array}{l} \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q B_{x_1}(q+1, q+1) \\ \quad + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q \Psi_{x_2}(q+1, q+1) \\ \quad + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q B(q+1, q+1) \text{ for } x \in [a, \frac{a+b}{2}], \\ \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q B(q+1, q+1) \\ \quad + \left(\frac{\alpha}{\alpha+\beta} \frac{1}{x-a}\right)^q B_{x_3}(q+1, q+1) \\ \quad + \left(\frac{\beta}{\alpha+\beta} \frac{1}{b-x}\right)^q B_{x_4}(q+1, q+1) \text{ for } x \in [\frac{a+b}{2}, b]. \end{array} \right.
 \end{aligned}$$



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Using (2.4), we obtain the result (2.1). Using the inequality (2.3), we can also state that

$$\begin{aligned} & \left| \frac{1}{\alpha + \beta} \left( \frac{\alpha}{x - a} \int_a^x g(t) dt + \frac{\beta}{b - x} \int_x^b g(t) dt \right) \right. \\ & \quad - \frac{1}{2} g(x) - \frac{1}{2(\alpha + \beta)} \left[ \left( x - \frac{a + b}{2} \right) g(x) \left( \frac{\alpha}{x - a} - \frac{\beta}{b - x} \right) \right. \\ & \quad \left. \left. + \frac{(b - a)}{2} \left( \frac{\alpha}{x - a} g(a) + \frac{\beta}{b - x} g(b) \right) - (\alpha + \beta) \left( x - \frac{a + b}{2} \right) g'(x) \right] \right| \\ & \leq \frac{1}{2} \|g''\|_1 \|K(x, t)\|_\infty, \end{aligned}$$

where

$$\|K(x, t)\|_\infty = p(x, t) \left( t - \frac{a + b}{2} \right).$$

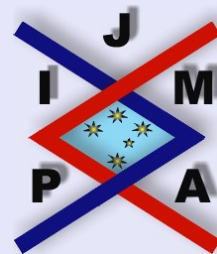
As it is easy to see that

$$\|K(x, t)\|_\infty = \frac{1}{\alpha + \beta} \cdot \max \left( \frac{\alpha}{x - a}, \frac{\beta}{b - x} \right) \cdot \frac{(b - a)^2}{4} \quad \text{for } x \in [a, b],$$

we deduce (2.2).  $\square$

**Remark 1.** Putting  $\alpha = x - a$  and  $\beta = b - x$  in (2.1) and (2.2), we get the inequalities (1.1) and (1.2).

**Remark 2.** Simple manipulation of (2.1) will allow for the corrected result of (1.1) and (1.2), owing to a missing factor of  $\frac{1}{2}$  in the third term of the original result (1.1) of the Barnett, Cerone, Dragomir, Roumeliotis and Sofo, this will not be done here.



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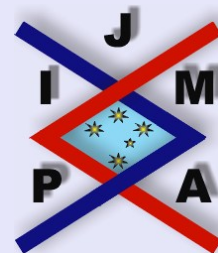
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