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Abstract

In this paper, we study the Baker's superstability for the following functional equation

$$(E(K)) \quad \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y)k^{-1})d\omega_K(k) = |\Phi|f(x)f(y), \quad x, y \in G$$

where G is a locally compact group, K is a compact subgroup of G , ω_K is the normalized Haar measure of K , Φ is a finite group of K -invariant morphisms of G and f is a continuous complex-valued function on G satisfying the Kannappan type condition, for all $x, y, z \in G$

$$(*) \quad \int_K \int_K f(zkxk^{-1}hyh^{-1})d\omega_K(k)d\omega_K(h) = \int_K \int_K f(zkyk^{-1}hxx^{-1})d\omega_K(k)d\omega_K(h).$$

We treat examples and give some applications.

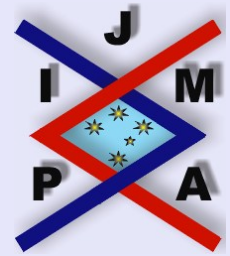
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1. Introduction, Notations and Preliminaries

Let G be a locally compact group. Let K be a compact subgroup of G . Let ω_K be the normalized Haar measure of K . A mapping $\varphi : G \rightarrow G$ is a morphism of G if φ is a homeomorphism of G onto itself which is either a group-homomorphism, i.e. $(\varphi(xy) = \varphi(x)\varphi(y), x, y \in G)$, or a group-antihomomorphism, i.e. $(\varphi(xy) = \varphi(y)\varphi(x), x, y \in G)$. We denote by $Mor(G)$ the group of morphisms of G and Φ a finite subgroup of $Mor(G)$ of a K -invariant morphisms of G (i.e. $\varphi(K) \subset K$). The number of elements of a finite group Φ will be designated by $|\Phi|$. The Banach algebra of bounded measures on G with complex values is denoted by $M(G)$ and the Banach space of all complex measurable and essentially bounded functions on G by $L_\infty(G)$. $\mathcal{C}(G)$ designates the Banach space of all continuous complex valued functions on G . We say that a function f is a K -central function on G if

$$(1.1) \quad f(kx) = f(xk), \quad x \in G, k \in K.$$

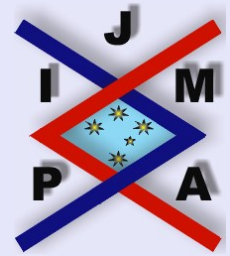
In the case where $G = K$, a function f is central if

$$(1.2) \quad f(xy) = f(yx) \quad x, y \in G.$$

See [2] for more information.

In this note, we are going to generalize the results obtained by J.A. Baker in [8] and [9]. As applications, we discuss the following cases:

- a) $K \subset Z(G)$, ($Z(G)$ is the center of G).
- b) $f(hxk) = f(x)$, $h, k \in K$, $x \in G$ (i.e. f is bi- K -invariant (see [3] and [6])).



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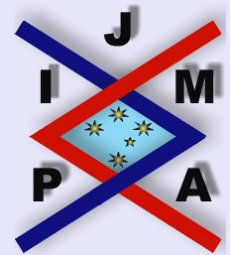
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- c) $f(hxk) = \chi(k)f(x)\chi(h)$, $x \in G$, $k, h \in K$ (χ is a unitary character of K) (see [11]).
- d) (G, K) is a Gelfand pair (see [3], [6] and [11]).
- e) $G = K$ (see [2]).

In the next section, we note some results for later use.



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2. General Properties

In what follows, we study general properties. Let G, K and Φ be given as above.

Proposition 2.1. For an arbitrary fixed $\tau \in \Phi$, the mapping

$$\begin{aligned}\Phi &\longrightarrow \Phi, \\ \varphi &\longrightarrow \varphi \circ \tau\end{aligned}$$

is a bijection.

Proof. Follows from the fact that Φ is a finite group. □

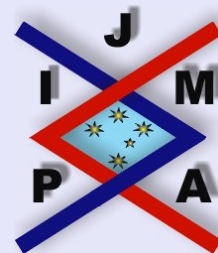
Proposition 2.2. Let $\varphi \in \Phi$ and $f \in \mathcal{C}(G)$, then we have:

$$i) \int_K f(xk\varphi(hy)k^{-1})d\omega_K(k) = \int_K f(xk\varphi(yh)k^{-1})d\omega_K(k), \quad x, y \in G, h \in K.$$

ii) If f satisfy (*), the for all $z, y, x \in G$, we have

$$\begin{aligned}\int_K \int_K f(zh\varphi(ykxk^{-1})h^{-1})d\omega_K(h)d\omega_K(k) \\ = \int_K \int_K f(zh\varphi(xkyk^{-1})h^{-1})d\omega_K(h)d\omega_K(k).\end{aligned}$$

Proof. i) Let $\varphi \in \Phi$ and let $x, y \in G, h \in K$, then we have



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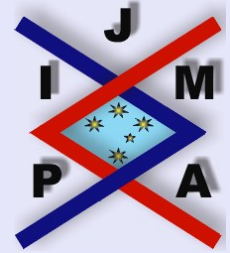


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Case 1: If φ is a group-homomorphism, we obtain, by replacing k by $k\varphi(h)^{-1}$

$$\begin{aligned} \int_K f(xk\varphi(hy)k^{-1})d\omega_K(k) &= \int_K f(xk\varphi(h)\varphi(y)k^{-1})d\omega_K(k) \\ &= \int_K f(xk\varphi(y)\varphi(h)k^{-1})d\omega_K(k) \\ &= \int_K f(xk\varphi(yh)k^{-1})d\omega_K(k). \end{aligned}$$

Case 2: if φ is a group-antihomomorphism, we have, by replacing k by $k\varphi(h)$

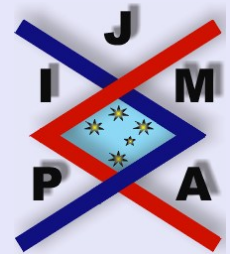
$$\begin{aligned} \int_K f(xk\varphi(hy)k^{-1})d\omega_K(k) &= \int_K f(xk\varphi(y)\varphi(h)k^{-1})d\omega_K(k) \\ &= \int_K f(xk\varphi(h)\varphi(y)k^{-1})d\omega_K(k) \\ &= \int_K f(xk\varphi(yh)k^{-1})d\omega_K(k). \end{aligned}$$

ii) Follows by simple computation. □

Proposition 2.3. For each $\tau \in \Phi$ and $x, y \in G$, we have

$$(2.1) \quad \sum_{\varphi \in \Phi} \int_K f(xk\varphi(\tau(y))k^{-1})d\omega_K(k) = \sum_{\psi \in \Phi} \int_K f(xk\psi(y)k^{-1})d\omega_K(k).$$

Proof. By applying Proposition 2.1, we get that when φ is iterated over Φ , the morphism of the form $\varphi \circ \tau$ annihilates all the elements of Φ . \square



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3. The Main Results

Theorem 3.1. Let G be a locally compact group; let K be a compact subgroup of G with the normalized Haar measure ω_K and let Φ given as above.

Let $\delta > 0$ and let $f \in \mathcal{C}(G)$ such that f satisfies the condition (*) and the functional inequality

$$(3.1) \quad \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y)k^{-1})d\omega_K(k) - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$

Then one of the assertions is satisfied:

(a) If f is bounded, then

$$(3.2) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}.$$

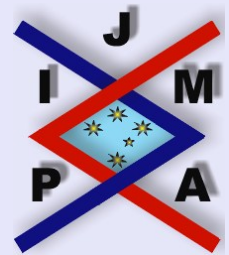
(b) If f is unbounded, then

- i) f is K -central,
- ii) $f \circ \tau = f$, for all $\tau \in \Phi$,
- iii) $\int_K f(xkyk^{-1})d\omega_K(k) = \int_K f(ykxk^{-1})d\omega_K(k), \quad x, y \in G.$

Proof.

a) Let $X = \sup |f|$, then we get for all $x \in G$

$$|\Phi||f(x)f(x)| \leq |\Phi|X + \delta,$$



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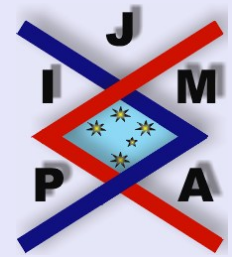


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from which we obtain that

$$|\Phi|X^2 - |\Phi|X - \delta \leq 0,$$

such that

$$X \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}.$$

b) i) Let $x, y \in G$, $h \in K$, then by using Proposition 2.2, we find

$$\begin{aligned} & |\Phi||f(x)||f(hy) - f(yh)| \\ &= ||\Phi|f(x)f(hy) - |\Phi|f(x)f(yh)| \\ &\leq \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(hy)k^{-1})d\omega_K(k) - |\Phi|f(x)f(hy) \right| \\ &\quad + \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(yh)k^{-1})d\omega_K(k) - |\Phi|f(x)f(yh) \right| \\ &\leq 2\delta. \end{aligned}$$

Since f is unbounded it follows that $f(yh) = f(hy)$, for all $h \in K, y \in G$.

ii) Let $\tau \in \Phi$, by using Proposition 2.3, we get for all $x, y \in G$

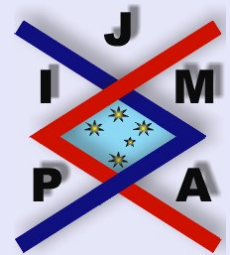
$$\begin{aligned} & |\Phi||f(x)||f \circ \tau(y) - f(y)| \\ &= ||\Phi|f(x)f(\tau(y)) - |\Phi|f(x)f(y)| \end{aligned}$$

$$\begin{aligned} &\leq \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(\tau(y))k^{-1})d\omega_K(k) - |\Phi|f(x)f(\tau(y)) \right| \\ &\quad + \left| \sum_{\psi \in \Phi} \int_K f(xk\psi(y)k^{-1})d\omega_K(k) - |\Phi|f(x)f(y) \right| \\ &\leq 2\delta. \end{aligned}$$

Since f is unbounded it follows that $f \circ \tau = f$, for all $\tau \in \Phi$.

iii) Let f be an unbounded solution of the functional inequality (3.1), such that f satisfies the condition (*), then, for all $x, y \in G$, we obtain, by using Part i) of Proposition 2.2:

$$\begin{aligned} &|\Phi||f(z)| \left| \int_K f(xkyk^{-1})d\omega_K(k) - \int_K f(ykxk^{-1})d\omega_K(k) \right| \\ &= \left| |\Phi| \int_K f(z)f(xkyk^{-1})d\omega_K(k) \right. \\ &\quad \left. - |\Phi| \int_K f(z)f(ykxk^{-1})d\omega_K(k) \right| \\ &\leq \left| \int_K \sum_{\varphi \in \Phi} \int_K f(zh\varphi(xkyk^{-1})h^{-1})d\omega_K(h)d\omega_K(k) \right. \\ &\quad \left. - |\Phi| \int_K f(z)f(xkyk^{-1})d\omega_K(k) \right| \end{aligned}$$



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$$\begin{aligned}
& + \left| \int_K \sum_{\varphi \in \Phi} \int_K f(zh\varphi(ykxk^{-1})h^{-1})d\omega_K(h)d\omega_K(k) \right. \\
& \left. - |\Phi| \int_K f(z)f(ykxk^{-1})d\omega_K(k) \right| \\
& \leq 2\delta.
\end{aligned}$$

Since f is unbounded we get

$$\int_K f(xkyk^{-1})d\omega_K(k) = \int_K f(ykxk^{-1})d\omega_K(k), \quad x, y \in G.$$

□

The main result is the following theorem.

Theorem 3.2. *Let $\delta > 0$ and let $f \in \mathcal{C}(G)$ such that f satisfies the condition (*) and the functional inequality*

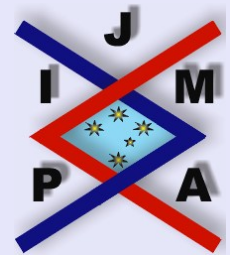
$$(3.3) \quad \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y)k^{-1})d\omega_K(k) - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$

Then either

$$(3.4) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}, \quad x \in G,$$

or

$$(E(K)) \quad \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y)k^{-1})d\omega_K(k) = |\Phi|f(x)f(y), \quad x, y \in G.$$



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Proof. The idea is inspired by the paper [1].

If f is bounded, by using Theorem 3.1, we obtain the first case of the theorem.

Now let f be an unbounded solution of the functional inequality (3.3), then there exists a sequence $(z_n)_{n \in \mathbb{N}}$ in G such that $f(z_n) \neq 0$ and $\lim_n |f(z_n)| = +\infty$.

For the second case we will use the following lemma.

Lemma 3.3. *Let f be an unbounded solution of the functional inequality (3.3) satisfying the condition (*) and let $(z_n)_{n \in \mathbb{N}}$ be a sequence in G such that $f(z_n) \neq 0$ and $\lim_n |f(z_n)| = +\infty$. It follows that the convergence of the sequences of functions:*

i)

$$(3.5) \quad x \mapsto \frac{\sum_{\varphi \in \Phi} \int_K f(z_n k \varphi(x) k^{-1}) d\omega_K(k)}{f(z_n)}, \quad n \in \mathbb{N},$$

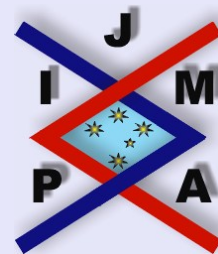
to the function

$$x \mapsto |\Phi| f(x).$$

ii)

$$(3.6) \quad x \mapsto \frac{\sum_{\varphi \in \Phi} \int_K f(z_n h \varphi(x k \varphi(\tau(y)) k^{-1}) h^{-1}) d\omega_K(h)}{f(z_n)},$$

$$n \in \mathbb{N}, \tau \in \Phi, k \in K, y \in G$$



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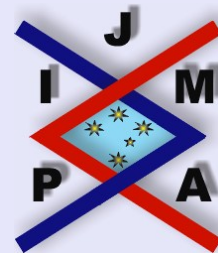


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to the function

$$x \mapsto |\Phi|f(xk\tau(y)k^{-1}) \quad \tau \in \Phi, k \in K, y \in G,$$

is uniform.

By inequality (3.1), we have

$$\left| \frac{\sum_{\varphi \in \Phi} \int_K f(z_n k \varphi(y) k^{-1}) d\omega_K(k)}{f(z_n)} - |\Phi|f(y) \right| \leq \frac{\delta}{|f(z_n)|},$$

then we have, by letting $n \mapsto +\infty$, that

$$\lim_n \frac{\sum_{\varphi \in \Phi} \int_K f(z_n k \varphi(y) k^{-1}) d\omega_K(k)}{f(z_n)} = |\Phi|f(y),$$

and

$$\lim_n \frac{\sum_{\varphi \in \Phi} \int_K f(z_n h \varphi(xk\varphi(\tau(y))k^{-1})h^{-1}) d\omega_K(h)}{f(z_n)} = |\Phi|f(xk\tau(y)k^{-1}).$$

Since by Proposition 2.3, we have

$$\begin{aligned} & \sum_{\tau \in \Phi} \int_K \frac{\sum_{\varphi \in \Phi} \int_K f(z_n h \varphi(x)k\varphi(\tau(y))k^{-1}h^{-1}) d\omega_K(h)}{f(z_n)} d\omega_K(k) \\ &= \sum_{\psi \in \Phi} \int_K \frac{\sum_{\varphi \in \Phi} \int_K f(z_n h \varphi(x)k\psi(y)k^{-1}h^{-1}) d\omega_K(h)}{f(z_n)} d\omega_K(k), \end{aligned}$$

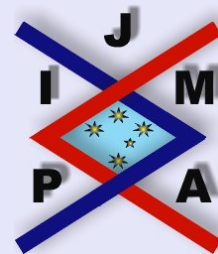
combining this and the fact that f satisfies the condition (*), we obtain

$$\left| \sum_{\tau \in \Phi} \int_K \frac{\sum_{\varphi \in \Phi} \int_K f(z_n h \varphi(x) k \varphi(\tau(y)) k^{-1} h^{-1}) d\omega_K(h)}{f(z_n)} d\omega_K(k) - |\Phi| f(x) \frac{\sum_{\psi \in \Phi} \int_K f(z_n k \psi(y) k^{-1}) d\omega_K(k)}{f(z_n)} \right| \leq \frac{\delta}{|f(z_n)|}.$$

Since the convergence is uniform, we have

$$\left| |\Phi| \sum_{\varphi \in \Phi} \int_K f(x k \varphi(y) k^{-1}) d\omega_K(k) - |\Phi|^2 f(x) f(y) \right| \leq 0,$$

thus $(E(K))$ holds and the proof is complete. □



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4. Applications

If $K \subset Z(G)$, we obtain the following corollary.

Corollary 4.1. *Let $\delta > 0$ and let f be a complex-valued function on G satisfying the Kannappan condition (see [10])*

$$(*) \quad f(zxy) = f(zyx), \quad x, y \in G,$$

and the functional inequality

$$(4.1) \quad \left| \sum_{\varphi \in \Phi} f(x\varphi(y)) - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$

Then either

$$(4.2) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}, \quad x \in G,$$

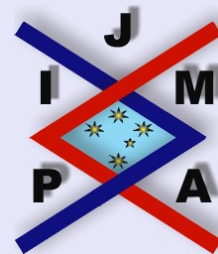
or

$$(4.3) \quad \sum_{\varphi \in \Phi} f(x\varphi(y)) = |\Phi|f(x)f(y), \quad x, y \in G.$$

If G is abelian, then the condition $(*)$ holds and we have the following:

If $\Phi = \{i\}$ (resp. $\Phi = \{i, \sigma\}$), where $i(x) = x$ and $\sigma(x) = -x$, we find the Baker's stability see [8] (resp. [9]).

If $f(kxh) = \chi(k)f(x)\chi(h)$, $k, h \in K$ and $x \in G$, where χ is a character of K (see [11]), then we have the following corollary.



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Corollary 4.2. Let $\delta > 0$ and let $f \in \mathcal{C}(G)$ such that $f(kxh) = \chi(k)f(x)\chi(h)$, $k, h \in K$, $x \in G$,

$$(*) \quad \int_K \int_K f(zkxhy)\bar{\chi}(k)\bar{\chi}(h)d\omega_K(k)d\omega_K(h) \\ = \int_K \int_K f(zkyhx)\bar{\chi}(k)\bar{\chi}(h)d\omega_K(k)d\omega_K(h)$$

and

$$(4.4) \quad \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y))\bar{\chi}(k)d\omega_K(k) - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$

Then either

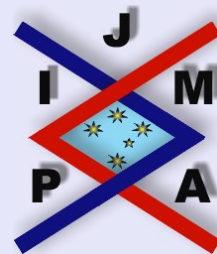
$$(4.5) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}, \quad x \in G,$$

or

$$(4.6) \quad \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y))\bar{\chi}(k)d\omega_K(k) = |\Phi|f(x)f(y), \quad x, y \in G.$$

Proposition 4.3. If the algebra $\bar{\chi}\omega_K \star M(G) \star \bar{\chi}\omega_K$ is commutative then the condition $(*)$ holds.

Proof. Since $f(kxh) = \chi(k)f(x)\chi(h)$, $k, h \in K$, $x \in G$, then we have $\chi\omega_K \star f \star \chi\omega_K = f$. Suppose that the algebra $\bar{\chi}\omega_K \star M(G) \star \bar{\chi}\omega_K$ is commutative,



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then we get:

$$\begin{aligned}
 & \int_K \int_K f(xkyk^{-1}hzh^{-1})d\omega_K(k)d\omega_K(h) \\
 &= \int_K \int_K f(xkyhzh^{-1}k^{-1})d\omega_K(k)d\omega_K(h) \\
 &= \langle \delta_z \star \bar{\chi}\omega_K \star \delta_y \star \bar{\chi}\omega_K \star \delta_x, f \rangle \\
 &= \langle \delta_z \star \bar{\chi}\omega_K \star \delta_y \star \bar{\chi}\omega_K \star \delta_x, \chi\omega_K \star f \star \chi\omega_K \rangle \\
 &= \langle \bar{\chi}\omega_K \star \delta_z \star \bar{\chi}\omega_K \star \delta_y \star \bar{\chi}\omega_K \star \delta_x \star \bar{\chi}\omega_K, f \rangle \\
 &= \langle \bar{\chi}\omega_K \star \delta_z \star \bar{\chi}\omega_K \star \delta_x \star \bar{\chi}\omega_K \star \delta_y \star \bar{\chi}\omega_K, f \rangle \\
 &= \int_K \int_K f(ykxk^{-1}hzh^{-1})d\omega_K(k)d\omega_K(h).
 \end{aligned}$$

□

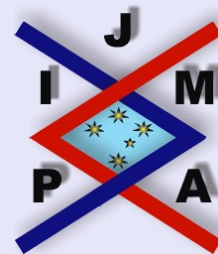
Let f be bi- K -invariant (i.e $f(hxk) = f(x), h, k \in K, x \in G$), then we have:

Corollary 4.4. Let $\delta > 0$ and let $f \in \mathcal{C}(G)$ be bi- K -invariant such that for all $x, y, z \in G$,

$$(*) \quad \int_K \int_K f(zkxhy)d\omega_K(k)d\omega_K(h) = \int_K \int_K f(zkyhx)d\omega_K(k)d\omega_K(h),$$

and

$$(4.7) \quad \left| \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y))d\omega_K(k) - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$



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Then either

$$(4.8) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}, \quad x \in G,$$

or

$$(4.9) \quad \sum_{\varphi \in \Phi} \int_K f(xk\varphi(y)) d\omega_K(k) = |\Phi|f(x)f(y), \quad x, y \in G.$$

Proposition 4.5. *If the pair (G, K) is a Gelfand pair (i.e $\omega_K \star M(G) \star \omega_K$ is commutative), then the condition (*) holds.*

Proof. We take $\chi = 1$ (unit character of K) in Proposition 4.3 (see [3] and [6]). □

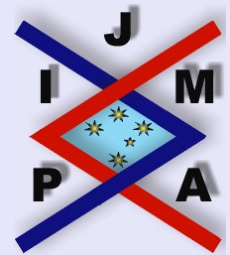
In the next corollary, we assume that $G = K$ is a compact group.

Lemma 4.6. *If f is central, then f satisfies the condition (*). Consequently, we have*

$$(4.10) \quad \int_G f(xtyt^{-1})dt = \int_G f(ytx t^{-1})dt, \quad x, y \in G.$$

Corollary 4.7. *Let $\delta > 0$ and let f be a complex measurable and essentially bounded function on G such that*

$$(4.11) \quad \left| \sum_{\varphi \in \Phi} \int_G f(xt\varphi(y)t^{-1})dt - |\Phi|f(x)f(y) \right| \leq \delta, \quad x, y \in G.$$



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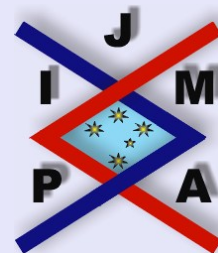
Then

$$(4.12) \quad |f(x)| \leq \frac{|\Phi| + \sqrt{|\Phi|^2 + 4\delta|\Phi|}}{2|\Phi|}, \quad x \in G.$$

Proof. Let $f \in L_\infty(G)$ be a solution of the inequality (4.11), then f is bounded, if not, then f satisfies the second case of Theorem 3.2 which implies that f is central (i.e the condition (*) holds) and f is a solution of the following functional equation

$$(4.13) \quad \sum_{\varphi \in \Phi} \int_G f(xt\varphi(y)t^{-1})dt = |\Phi|f(x)f(y), \quad x, y \in G.$$

In view of the proposition in [5], we have that f is continuous. Since G is compact, then the proof is accomplished. \square



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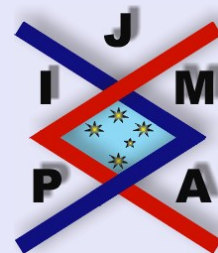
Close

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References

- [1] R. BADORA, On Hyers-Ulam stability of Wilson's functional equation, *Aequationes Math.*, **60** (2000), 211–218.
- [2] J.L. CLERC, Les représentations des groupes compacts, *Analyse Harmoniques*, les Cours du CIMPA, Université de Nancy I, 1980.
- [3] J. FARAUT, Analyse harmonique sur les paires de Guelfand et les espaces hyperboliques, *les Cours du CIMPA*, Université de Nancy I, 1980.
- [4] W. FORG-ROB AND J. SCHWAIGER, The stability of some functional equations for generalized hyperbolic functions and for the generalized hyperbolic functions and for the generalized cosine equation, *Results in Math.*, **26** (1994), 247–280.
- [5] Z. GAJDA, On functional equations associated with characters of unitary representations of groups, *Aequationes Math.*, **44** (1992), 109–121.
- [6] S. HELGASON, *Groups and Geometric Analysis*, Academic Press, New York-London, 1984.
- [7] E. HEWITT AND K.A. ROSS, *Abstract Harmonic Analysis*, Vol. I and II., Springer-Verlag, Berlin-Göttingen-Heidelberg, 1963.
- [8] J. BAKER, J. LAWRENCE AND F. ZORZITTO, The stability of the equation $f(x + y) = f(x)f(y)$, *Proc. Amer. Math. Soc.*, **74** (1979), 242–246.
- [9] J. BAKER, The stability of the cosine equation, *Proc. Amer. Math. Soc.*, **80**(3) (1980), 411–416.



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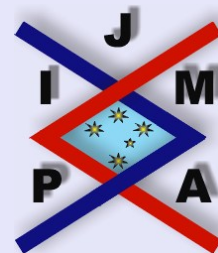
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- [10] P.I. KANNAPPAN, The functional equation $f(xy) + f(xy^{-1}) = 2f(x)f(y)$, for groups, *Proc. Amer. Math. Soc.*, **19** (1968), 69–74.
- [11] R. TAKAHASHI, $SL(2, \mathbb{R})$, *Analyse Harmoniques*, les Cours du CIMPA, Université de Nancy I, 1980.



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