

## REGULARITY FOR VECTOR VALUED MINIMIZERS OF SOME ANISOTROPIC INTEGRAL FUNCTIONALS

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[Abstract](#)

[Contents](#)



[Home Page](#)

[Go Back](#)

[Close](#)

[Quit](#)

## Abstract

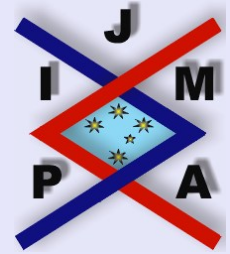
We deal with anisotropic integral functionals  $\int_{\Omega} f(x, Du(x)) dx$  defined on vector valued mappings  $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^N$ . We show that a suitable "monotonicity" inequality, on the density  $f$ , guarantees global pointwise bounds for minimizers  $u$ .

*2000 Mathematics Subject Classification:* 49N60, 35J60.

*Key words:* Anisotropic, Integral, Functional, Regularity, Minimizer.

## Contents

1	Introduction .....	3
2	Proofs .....	7
	References	



### Regularity for Vector Valued Minimizers of Some Anisotropic Integral Functionals

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 2 of 16

# 1. Introduction

We consider the integral functional

$$(1.1) \quad \mathcal{F}(u) = \int_{\Omega} f(x, Du(x)) dx$$

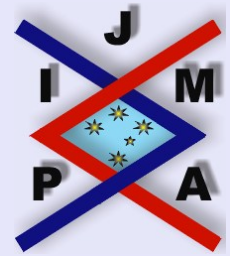
where  $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^N$  and  $\Omega$  is a bounded open set. When  $N = 1$  we are dealing with scalar functions  $u : \Omega \rightarrow \mathbb{R}$ ; on the contrary, vector valued mappings  $u : \Omega \rightarrow \mathbb{R}^N$  appear when  $N \geq 2$ . Local and global pointwise bounds for scalar minimizers of (1.1) have been proved in [2], [7], [5], [4]. A model functional for these results is

$$(1.2) \quad \int_{\Omega} \left( \sum_{i,j=1}^n a_{ij}(x) D_j u(x) D_i u(x) \right)^{\frac{p}{2}} dx,$$

where coefficients  $a_{ij}$  are measurable, bounded and elliptic. Previous results for scalar minimizers are no longer true in the vector valued case  $N \geq 2$  as De Giorgi's counterexample shows, [3]. Some years later, attention has been paid to anisotropic functionals whose model is

$$(1.3) \quad \int_{\Omega} (|D_1 u(x)|^{p_1} + |D_2 u(x)|^{p_2} + \dots + |D_n u(x)|^{p_n}) dx,$$

where each component  $D_i u$  of the gradient  $Du = (D_1 u, D_2 u, \dots, D_n u)$  may have a (possibly) different exponent  $p_i$ : this seems useful when dealing with some reinforced materials, [9]; see also [6, Example 1.7.1, page 169]. In the framework of anisotropic functionals, global pointwise bounds have been



---

**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



Go Back

Close

Quit

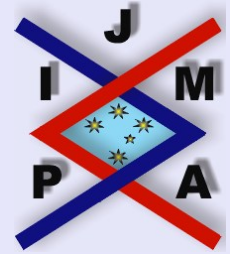
Page 3 of 16

proved for scalar minimizers in [1] and [8]. If no additional conditions are assumed, these bounds are false in the vectorial case, as the above mentioned counterexample shows, [3]. The aim of this paper is to present a “monotonicity” assumption ensuring boundedness of vector valued minimizers. In order to do that, we recall that  $u : \Omega \subset \mathbb{R}^n \rightarrow \mathbb{R}^N$  thus  $Du(x)$  is a matrix with  $N$  rows and  $n$  columns; the density  $f(x, A)$  in (1.1) is assumed to be measurable with respect to  $x$ , continuous with respect to  $A$  and  $f : \Omega \times \mathbb{R}^{N \times n} \rightarrow [0, +\infty)$ . Every matrix  $A = \{A_i^\alpha\} \in \mathbb{R}^{N \times n}$  will have  $N$  rows  $A^1, \dots, A^N$  and  $n$  columns  $A_1, \dots, A_n$ . In this paper we will show that the following “monotonicity” inequality guarantees global pointwise bounds for vector valued minimizers of (1.1):

$$(1.4) \quad f(x, \tilde{A}) + \mu \sum_{i=1}^n \left| \tilde{A}_i - A_i \right|^{p_i} \leq f(x, A) + M(x)$$

for every pair of matrices  $\tilde{A}, A \in \mathbb{R}^{N \times n}$  such that there exists a row  $\beta$  with  $\tilde{A}^\beta = 0$  and for every remaining row  $\alpha \neq \beta$  we have  $\tilde{A}^\alpha = A^\alpha$ . In (1.4)  $\mu, p_1, \dots, p_n$  are positive constants with  $p_i > 1$  and  $M : \Omega \rightarrow [0, +\infty)$  with  $M \in L^r(\Omega)$ ,  $r \geq 1$ . If we keep in mind that  $A = Du(x)$ , then the left hand side of (1.4) shows  $\sum_{i=1}^n |\tilde{A}_i - D_i u(x)|^{p_i}$ , thus each component  $D_i u$  of the gradient  $Du$  may have a possibly different exponent  $p_i$ , so we are in the anisotropic framework:  $u \in W^{1,1}(\Omega, \mathbb{R}^N)$  with  $D_i u \in L^{p_i}(\Omega, \mathbb{R}^N)$ . In this case the harmonic mean  $\bar{p} = \left( \frac{1}{n} \sum_{i=1}^n \frac{1}{p_i} \right)^{-1}$  comes into play. In Section 2 we will prove the following

**Theorem 1.1.** *We consider the functional (1.1) under the “monotonicity” in-*




---

**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



Go Back

Close

Quit

Page 4 of 16

equality (1.4) with

$$(1.5) \quad \frac{\bar{p}^*}{\bar{p}} \left(1 - \frac{1}{r}\right) > 1$$

where  $\bar{p}^*$  is the Sobolev exponent of  $\bar{p} < n$ . We consider  $u = (u^1, \dots, u^N) \in W^{1,1}(\Omega, \mathbb{R}^N)$ , with  $D_i u \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$ , such that

$$\mathcal{F}(u) < +\infty$$

and

$$(1.6) \quad \mathcal{F}(u) \leq \mathcal{F}(v)$$

for every  $v \in u + W_0^{1,1}(\Omega, \mathbb{R}^N)$  with  $D_i v \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$ . Then, for every component  $u^\beta$ , we have

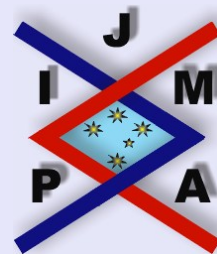
$$(1.7) \quad \inf_{\partial\Omega} u^\beta - c_* \leq u^\beta(x) \leq \sup_{\partial\Omega} u^\beta + c_*$$

for almost every  $x \in \Omega$ , where

$$c_* = c \left( \frac{\|M\|_{L^r(\Omega)}}{\mu} \right)^{\frac{1}{\bar{p}}} |\Omega| \left[ \left(1 - \frac{1}{r}\right)^{\frac{\bar{p}^*}{\bar{p}}} - 1 \right]^{\frac{1}{\bar{p}^*}} 2 \left(1 - \frac{1}{r}\right)^{\frac{\bar{p}^*}{\bar{p}}} \left[ \left(1 - \frac{1}{r}\right)^{\frac{\bar{p}^*}{\bar{p}}} - 1 \right]^{-1},$$

$c = c(n, p_1, \dots, p_n) > 0$  and  $|\Omega|$  is the Lebesgue measure of  $\Omega$ .

A model density  $f$  for the “monotonicity” inequality (1.4) is given in the following.



### Regularity for Vector Valued Minimizers of Some Anisotropic Integral Functionals

Francesco Leonetti and Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

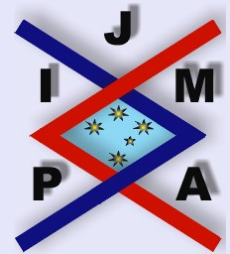
Quit

Page 5 of 16

**Lemma 1.2.** For every  $i = 1, \dots, n$ , let us consider  $p_i \in [2, +\infty)$  and  $a_i \in (0, +\infty)$ ; we take  $m : \Omega \rightarrow [0, +\infty)$  and  $h : \mathbb{R} \rightarrow \mathbb{R}$  with  $-\infty < \inf_{\mathbb{R}} h$ . Let us consider  $f : \Omega \times \mathbb{R}^{n \times n} \rightarrow \mathbb{R}$  defined as follows:

$$(1.8) \quad f(x, A) = \sum_{i=1}^n a_i |A_i|^{p_i} + m(x)h(\det A).$$

Then the “monotonicity” inequality (1.4) holds true with  $\mu = \min_j a_j$  and  $M(x) = m(x)[h(0) - \inf_{\mathbb{R}} h]$ . Moreover, if  $h \geq 0$ , then  $f \geq 0$  too.




---

**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



Go Back

Close

Quit

Page 6 of 16

## 2. Proofs

In order to prove Theorem 1.1, we need the following

**Lemma 2.1.** *Let us consider the functional (1.1) under the “monotonicity” assumption (1.4). Then, for every  $v = (v^1, \dots, v^N) \in W^{1,1}(\Omega, \mathbb{R}^N)$  with  $D_i v \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$ , for any  $\beta \in \{1, \dots, N\}$ , for all  $t \in \mathbb{R}$ , it results that*

$$(2.1) \quad \mathcal{F}(I_{\beta,t}(v)) + \mu \sum_{i=1}^n \int_{\Omega} |D_i(I_{\beta,t}(v(x))) - D_i v(x)|^{p_i} dx \leq \mathcal{F}(v) + \int_{\{v^\beta > t\}} M(x) dx$$

where  $I_{\beta,t} : \mathbb{R}^N \rightarrow \mathbb{R}^N$  is defined as follows:

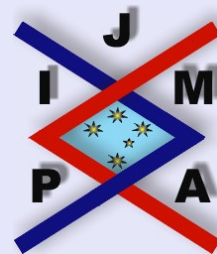
$$\forall y = (y^1, \dots, y^N) \in \mathbb{R}^N, \quad I_{\beta,t}(y) = (I_{\beta,t}^1(y), \dots, I_{\beta,t}^N(y))$$

with

$$(2.2) \quad I_{\beta,t}^\alpha(y) = \begin{cases} y^\alpha & \text{if } \alpha \neq \beta \\ y^\beta \wedge t = \min \{y^\beta, t\} & \text{if } \alpha = \beta. \end{cases}$$

*Proof.* For every  $v \in W^{1,1}(\Omega, \mathbb{R}^N)$ , with  $D_i v \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$ , it results that  $I_{\beta,t}(v) \in W^{1,1}(\Omega, \mathbb{R}^N)$ ; moreover

$$(2.3) \quad D_i(I_{\beta,t}^\alpha(v)) = \begin{cases} D_i v^\alpha & \text{if } \alpha \neq \beta \\ 1_{\{v^\beta \leq t\}} D_i v^\beta & \text{if } \alpha = \beta, \end{cases}$$



Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 7 of 16

where  $1_B$  is the characteristic function of the set  $B$ , that is,  $1_B(x) = 1$  if  $x \in B$  and  $1_B(x) = 0$  if  $x \notin B$ . Therefore  $D_i(I_{\beta,t}(v)) \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$ . On  $\{x \in \Omega : v^\beta(x) > t\}$  we have  $D(I_{\beta,t}^\beta(v)) = 0$  and, for  $\alpha \neq \beta$ ,  $D(I_{\beta,t}^\alpha(v)) = Dv^\alpha$ ; so we can apply (1.4) with  $\tilde{A} = D(I_{\beta,t}(v))$  and  $A = Dv$ ; we obtain

$$(2.4) \quad f(x, D(I_{\beta,t}(v(x)))) + \mu \sum_{i=1}^n |D_i(I_{\beta,t}(v(x))) - D_i v(x)|^{p_i} \leq f(x, Dv(x)) + M(x)$$

for  $x \in \{v^\beta > t\}$ . On  $\{x \in \Omega : v^\beta(x) \leq t\}$   $D(I_{\beta,t}(v)) = Dv$ , thus

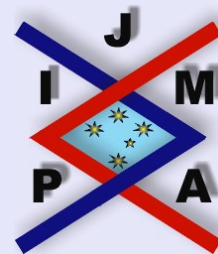
$$(2.5) \quad f(x, D(I_{\beta,t}(v(x)))) + \mu \sum_{i=1}^n |D_i(I_{\beta,t}(v(x))) - D_i v(x)|^{p_i} = f(x, Dv(x))$$

for  $x \in \{v^\beta \leq t\}$ . From (2.4) and (2.5) we have

$$f(x, D(I_{\beta,t}(v(x)))) + \mu \sum_{i=1}^n |D_i(I_{\beta,t}(v(x))) - D_i v(x)|^{p_i} \leq f(x, Dv(x)) + M(x)1_{\{v^\beta > t\}}(x)$$

for  $x \in \Omega$ . If  $x \rightarrow f(x, Dv(x)) \in L^1(\Omega)$ , then  $x \rightarrow f(x, D(I_{\beta,t}(v(x)))) \in L^1(\Omega)$  too and, integrating the last inequality with respect to  $x$ , we get (2.1). When  $x \rightarrow f(x, Dv(x)) \notin L^1(\Omega)$ , we have  $\mathcal{F}(v) = +\infty$  and (2.1) holds true. This ends the proof of Lemma 2.1.  $\square$

Now we are ready to prove Theorem 1.1.



### Regularity for Vector Valued Minimizers of Some Anisotropic Integral Functionals

Francesco Leonetti and Pier Vincenzo Petricca

Title Page

Contents



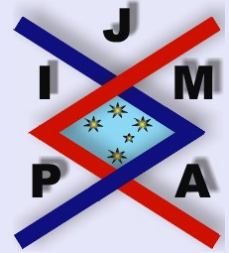
Go Back

Close

Quit

Page 8 of 16





**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 9 of 16

*Proof.* Let us fix  $\beta \in \{1, \dots, N\}$ . If  $\sup_{\partial\Omega} u^\beta = +\infty$  then the right hand side of (1.7) is satisfied. Thus we assume  $\sup_{\partial\Omega} u^\beta < t_0 \leq t < +\infty$  and we note that under this assumption  $I_{\beta,t}(u) \in u + W_0^{1,1}(\Omega, \mathbb{R}^N)$  and  $D_i(I_{\beta,t}(u)) \in L^{p_i}(\Omega, \mathbb{R}^N) \forall i \in \{1, \dots, n\}$  since

$$u^\beta \wedge t = \min \{u^\beta, t\} = u^\beta - [\max \{u^\beta - t, 0\}] = u^\beta - [(u^\beta - t) \vee 0],$$

where  $(u^\beta - t) \vee 0 \in W_0^{1,1}(\Omega)$  and  $D_i((u^\beta - t) \vee 0) = D_i u^\beta 1_{\{u^\beta > t\}} \in L^{p_i}(\Omega) \forall i \in \{1, \dots, n\}$ . From (1.6) and (2.1) it results that

$$\begin{aligned} \mathcal{F}(u) &\leq \mathcal{F}(I_{\beta,t}(u)) \\ &\leq \mathcal{F}(u) - \mu \sum_{i=1}^n \int_{\Omega} |D_i(I_{\beta,t}(u(x))) - D_i u(x)|^{p_i} dx + \int_{\{u^\beta > t\}} M(x) dx, \end{aligned}$$

that is

$$(2.6) \quad \mu \sum_{i=1}^n \int_{\Omega} |D_i(I_{\beta,t}(u(x))) - D_i u(x)|^{p_i} dx \leq \int_{\{u^\beta > t\}} M(x) dx.$$

If we define  $\phi = (u^\beta - t) \vee 0$ , then we can write (2.6) as follows:

$$(2.7) \quad \mu \sum_{i=1}^n \int_{\Omega} |D_i \phi(x)|^{p_i} dx \leq \int_{\{u^\beta > t\}} M(x) dx.$$

If  $r < +\infty$ , we apply Hölder's inequality to  $\int_{\{u^\beta > t\}} M(x) dx$  and we obtain

$$\int_{\{u^\beta > t\}} M(x) dx \leq \|M\|_{L^r(\Omega)} |\{u^\beta > t\}|^{(1-\frac{1}{r})}.$$

If  $r = +\infty$ , then

$$\int_{\{u^\beta > t\}} M(x) dx \leq \|M\|_{L^\infty(\Omega)} |\{u^\beta > t\}| = \|M\|_{L^r(\Omega)} |\{u^\beta > t\}|^{(1-\frac{1}{r})}.$$

In both cases, from (2.7) it results that

$$\sum_{i=1}^n \int_{\Omega} |D_i \phi(x)|^{p_i} dx \leq \frac{\|M\|_{L^r(\Omega)}}{\mu} |\{u^\beta > t\}|^{(1-\frac{1}{r})}$$

in particular,  $\forall i \in \{1, \dots, n\}$

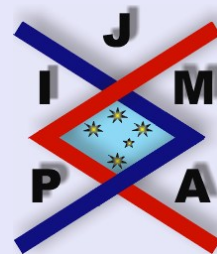
$$\int_{\Omega} |D_i \phi(x)|^{p_i} dx \leq \frac{\|M\|_{L^r(\Omega)}}{\mu} |\{u^\beta > t\}|^{(1-\frac{1}{r})}$$

from which

$$\left( \int_{\Omega} |D_i \phi(x)|^{p_i} dx \right)^{\frac{1}{p_i}} \leq \left[ \frac{\|M\|_{L^r(\Omega)}}{\mu} |\{u^\beta > t\}|^{(1-\frac{1}{r})} \right]^{\frac{1}{p_i}}$$

and finally

$$(2.8) \quad \left[ \prod_{i=1}^n \left( \int_{\Omega} |D_i \phi(x)|^{p_i} dx \right)^{\frac{1}{p_i}} \right]^{\frac{1}{n}} \leq \left[ \frac{\|M\|_{L^r(\Omega)}}{\mu} |\{u^\beta > t\}|^{(1-\frac{1}{r})} \right]^{\frac{1}{p}}.$$



**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 10 of 16

We apply the anisotropic imbedding theorem [10] and we use (2.8):

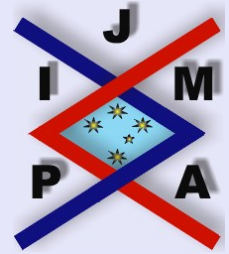
$$\begin{aligned}
 (2.9) \quad 0 &\leq \left( \int_{\{u^\beta > t\}} [u^\beta(x) - t]^{\bar{p}^*} dx \right)^{\frac{1}{\bar{p}^*}} \\
 &= \|\phi\|_{L^{\bar{p}^*}(\Omega)} \\
 &\leq c \left[ \prod_{i=1}^n \left( \int_{\Omega} |D_i \phi(x)|^{p_i} dx \right)^{\frac{1}{p_i}} \right]^{\frac{1}{n}} \\
 &\leq c \left[ \frac{\|M\|_{L^r(\Omega)}}{\mu} |\{u^\beta > t\}|^{(1-\frac{1}{r})} \right]^{\frac{1}{\bar{p}^*}},
 \end{aligned}$$

where  $c = c(n, p_1, \dots, p_n) > 0$ . If  $\|M\|_{L^r(\Omega)} = 0$ , then from (2.9) it results that  $u^\beta \leq t$  almost everywhere in  $\Omega$  and we are done. If  $\|M\|_{L^r(\Omega)} > 0$ , then for  $T > t$  we have

$$\begin{aligned}
 (2.10) \quad (T - t)^{\bar{p}^*} |\{u^\beta > T\}| &= \int_{\{u^\beta > T\}} (T - t)^{\bar{p}^*} dx \\
 &\leq \int_{\{u^\beta > T\}} [u^\beta(x) - t]^{\bar{p}^*} dx \\
 &\leq \int_{\{u^\beta > t\}} [u^\beta(x) - t]^{\bar{p}^*} dx
 \end{aligned}$$

and from (2.9) and (2.10) we get

$$(2.11) \quad |\{u^\beta > T\}| \leq c^{\bar{p}^*} \left( \frac{\|M\|_{L^r(\Omega)}}{\mu} \right)^{\frac{\bar{p}^*}{\bar{p}^*}} \frac{1}{(T - t)^{\bar{p}^*}} |\{u^\beta > t\}|^{(1-\frac{1}{r}) \frac{\bar{p}^*}{\bar{p}^*}}$$



**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 11 of 16

for every  $T, t$  with  $T > t \geq t_0$ . We set  $\chi(t) = |\{u^\beta > t\}|$  and we use [7, Lemma 4.1, p. 93], that we provide below for the convenience of the reader.

**Lemma 2.2.** *Let  $\chi : [t_0, +\infty) \rightarrow [0, +\infty)$  be decreasing. We assume that there exist  $k, a \in (0, +\infty)$  and  $b \in (1, +\infty)$  such that*

$$(2.12) \quad T > t \geq t_0 \implies \chi(T) \leq \frac{k}{(T-t)^a} (\chi(t))^b.$$

Then it results that

$$(2.13) \quad \chi(t_0 + d) = 0 \quad \text{where} \quad d = \left[ k(\chi(t_0))^{b-1} 2^{\frac{ab}{b-1}} \right]^{\frac{1}{a}}.$$

We use the previous Lemma 2.2 and we have

$$(2.14) \quad |\{u^\beta > t_0 + d\}| = 0$$

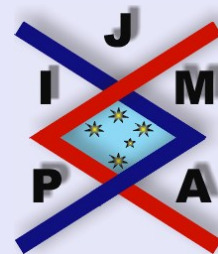
that is

$$(2.15) \quad u^\beta \leq t_0 + d$$

almost everywhere in  $\Omega$ , where

$$d = c \left( \frac{\|M\|_{L^r(\Omega)}}{\mu} \right)^{\frac{1}{p}} |\{u^\beta > t_0\}| \left[ \left(1 - \frac{1}{r}\right)^{\frac{p^*}{p} - 1} \right]^{\frac{1}{p^*}} 2 \left(1 - \frac{1}{r}\right)^{\frac{p^*}{p}} \left[ \left(1 - \frac{1}{r}\right)^{\frac{p^*}{p} - 1} \right]^{-1}.$$

In order to get the right hand side of (1.7), we control  $|\{u^\beta > t_0\}|$  by means of  $|\Omega|$  and we take a sequence  $\{(t_0)_m\}_m$  with  $(t_0)_m \rightarrow \sup_{\partial\Omega} u^\beta$ . Let us show how we obtain the left hand side of (1.7): we apply the right hand side of (1.7) to  $-u$ . This ends the proof of Theorem 1.1.  $\square$



**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

Quit

Page 12 of 16

Now we are going to prove Lemma 1.2.

*Proof.* We assume that  $\tilde{A}, A \in \mathbb{R}^{n \times n}$  with  $\tilde{A}^\beta = 0$  and  $\tilde{A}^\alpha = A^\alpha$  for  $\alpha \neq \beta$ . Then

$$\begin{aligned}
 (2.16) \quad \sum_{\alpha} |A_i^\alpha|^2 &= |A_i^\beta|^2 + \sum_{\alpha \neq \beta} |A_i^\alpha|^2 \\
 &= |A_i^\beta - \tilde{A}_i^\beta|^2 + \sum_{\alpha \neq \beta} |\tilde{A}_i^\alpha|^2 \\
 &= \sum_{\alpha} |A_i^\alpha - \tilde{A}_i^\alpha|^2 + \sum_{\alpha} |\tilde{A}_i^\alpha|^2
 \end{aligned}$$

so

$$(2.17) \quad |A_i|^2 = |A_i - \tilde{A}_i|^2 + |\tilde{A}_i|^2.$$

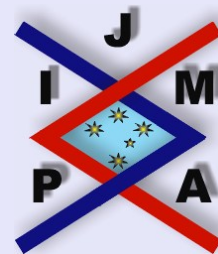
Since  $p_i \geq 2$ , the previous equality gives

$$(2.18) \quad |A_i|^{p_i} \geq |A_i - \tilde{A}_i|^{p_i} + |\tilde{A}_i|^{p_i}.$$

Moreover

$$(2.19) \quad h(\det A) \geq \inf_{\mathbb{R}} h = h(0) - [h(0) - \inf_{\mathbb{R}} h] = h(\det \tilde{A}) - [h(0) - \inf_{\mathbb{R}} h].$$

Now we are able to estimate  $f(x, A)$  and  $f(x, \tilde{A})$  by means of (2.18) and (2.19)



**Regularity for Vector Valued Minimizers of Some Anisotropic Integral Functionals**

Francesco Leonetti and Pier Vincenzo Petricca

Title Page

Contents



Go Back

Close

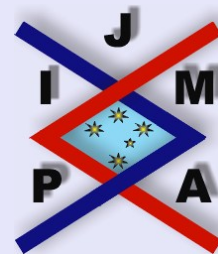
Quit

Page 13 of 16

as follows:

$$\begin{aligned}
 (2.20) \quad f(x, \tilde{A}) + (\min_j a_j) \sum_{i=1}^n |A_i - \tilde{A}_i|^{p_i} \\
 \leq \sum_{i=1}^n a_i |\tilde{A}_i|^{p_i} + m(x)h(\det \tilde{A}) + \sum_{i=1}^n a_i |A_i - \tilde{A}_i|^{p_i} \\
 \leq \sum_{i=1}^n a_i |A_i|^{p_i} + m(x)h(\det A) + m(x)[h(0) - \inf_{\mathbb{R}} h] \\
 = f(x, A) + m(x)[h(0) - \inf_{\mathbb{R}} h]
 \end{aligned}$$

thus the “monotonicity” inequality (1.4) holds true with  $\mu = \min_j a_j$  and  $M(x) = m(x)[h(0) - \inf_{\mathbb{R}} h]$ . This ends the proof of Lemma 1.2.  $\square$




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**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



Go Back

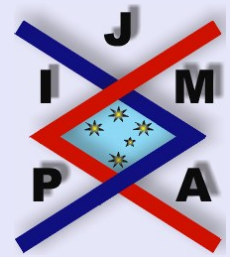
Close

Quit

Page 14 of 16

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Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



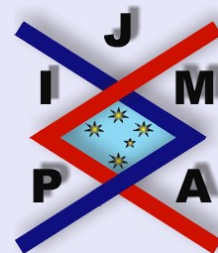
Go Back

Close

Quit

Page 15 of 16

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---

**Regularity for Vector Valued  
Minimizers of Some Anisotropic  
Integral Functionals**

Francesco Leonetti and  
Pier Vincenzo Petricca

---

Title Page

Contents



Go Back

Close

Quit

Page 16 of 16