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## INEQUALITIES FOR WALSH POLYNOMIALS WITH SEMI-MONOTONE AND SEMI-CONVEX COEFFICIENTS

ŽIVORAD TOMOVSKI

University "St. Cyril and Methodius"  
Faculty of Natural Sciences and Mathematics  
Institute of Mathematics  
PO Box 162, 91000 Skopje  
Republic of Macedonia.

*E*Mail: [tomovski@iunona.pmf.ukim.edu.mk](mailto:tomovski@iunona.pmf.ukim.edu.mk)

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## Abstract

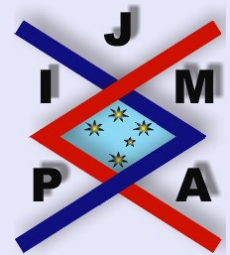
Using the concept of majorant sequences (see [4, ch. XXI], [5], [7], [8]) some new inequalities for Walsh polynomials with complex semi-monotone, complex semi-convex, complex monotone and complex convex coefficients are given.

*2000 Mathematics Subject Classification:* 26D05, 42C10.

*Key words:* Petrovic inequality, Walsh polynomial, Complex semi-convex coefficients, Complex convex coefficients, Complex semi-monotone coefficients, Complex monotone coefficients, Fine inequality.

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# 1. Introduction and Preliminaries

We consider the Walsh orthonormal system  $\{w_n(x)\}_{n=0}^{\infty}$  defined on  $[0, 1)$  in the Paley enumeration. Thus  $w_0(x) \equiv 1$  and for each positive integer with dyadic development

$$n = \sum_{i=1}^p 2^{\nu_i}, \quad \nu_1 > \nu_2 > \dots > \nu_p \geq 0,$$

we have

$$w_n(x) = \prod_{i=1}^p r_{\nu_i}(x),$$

where  $\{r_n(x)\}_{n=0}^{\infty}$  denotes the Rademacher system of functions defined by (see, e.g. [1, p. 60], [3, p. 9-10])

$$r_{\nu}(x) = \text{sign} \sin 2^{\nu} \pi(x) \quad (\nu = 0, 1, 2, \dots; 0 \leq x < 1).$$

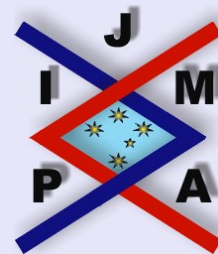
In this paper we shall consider the Walsh polynomials  $\sum_{k=n}^m \lambda_k w_k(x)$  with complex-valued coefficients  $\{\lambda_k\}$ .

Let  $\Delta \lambda_n = \lambda_n - \lambda_{n+1}$  and  $\Delta^2 \lambda_n = \Delta(\Delta \lambda_n) = \Delta \lambda_n - \Delta \lambda_{n+1} = \lambda_n - 2\lambda_{n+1} + \lambda_{n+2}$ , for all  $n = 1, 2, 3, \dots$

Petrović [6] proved the following complementary triangle inequality for a sequence of complex numbers  $\{z_1, z_2, \dots, z_n\}$ .

**Theorem A.** *Let  $\alpha$  be a real number and  $0 < \theta < \frac{\pi}{2}$ . If  $\{z_1, z_2, \dots, z_n\}$  are complex numbers such that  $\alpha - \theta \leq \arg z_{\nu} \leq \alpha + \theta$ ,  $\nu = 1, 2, \dots, n$ , then*

$$\left| \sum_{\nu=1}^n z_{\nu} \right| \geq (\cos \theta) \sum_{\nu=1}^n |z_{\nu}|.$$



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For  $0 < \theta < \frac{\pi}{2}$  denote by  $K(\theta)$  the cone  $K(\theta) = \{z : |\arg z| \leq \theta\}$ .

Let  $\{b_k\}$  be a positive nondecreasing sequence. The following definitions are given in [7] and [8]. The sequence of complex numbers  $\{u_k\}$  is said to be **complex semi-monotone** if there exists a cone  $K(\theta)$  such that  $\Delta \left( \frac{u_k}{b_k} \right) \in K(\theta)$  or  $\Delta(u_k b_k) \in K(\theta)$ . For  $b_k = 1$ , the sequence  $\{u_k\}$  shall be called a **complex monotone** sequence. On the other hand, the sequence  $\{u_k\}$  is said to be **complex semi-convex** if there exists a cone  $K(\theta)$ , such that  $\Delta^2 \left( \frac{u_k}{b_k} \right) \in K(\theta)$  or  $\Delta^2(u_k b_k) \in K(\theta)$ . For  $b_k = 1$ , the sequence  $\{u_k\}$  shall be called a **complex convex sequence**.

The following two Theorems were proved by Tomovski in [7] and [8].

**Theorem B ([7]).** Let  $\{z_k\}$  be a sequence such that  $|\sum_{k=n}^m z_k| \leq A$ , ( $\forall n, m \in \mathbb{N}, m > n$ ), where  $A$  is a positive number.

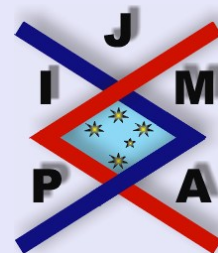
(i) If  $\Delta \left( \frac{u_k}{b_k} \right) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k z_k \right| \leq A \left[ \left( 1 + \frac{1}{\cos \theta} \right) |u_m| + \frac{1}{\cos \theta} \frac{b_m}{b_n} |u_n| \right], \quad (\forall n, m \in \mathbb{N}, m > n).$$

(ii) If  $\Delta(u_k b_k) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k z_k \right| \leq A \left[ \left( 1 + \frac{1}{\cos \theta} \right) |u_n| + \frac{1}{\cos \theta} \frac{b_m}{b_n} |u_m| \right], \quad (\forall n, m \in \mathbb{N}, m > n).$$

**Theorem C ([8]).** Let  $A = \max_{n \leq p \leq q \leq m} \left| \sum_{j=p}^q \sum_{k=i}^j z_k \right|$ .



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(i) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta^2\left(\frac{u_k}{b_k}\right) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k z_k \right| \leq A \left[ |u_m| + b_m \left( 1 + \frac{1}{\cos \theta} \right) \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| + \frac{b_m}{\cos \theta} \left| \Delta \left( \frac{u_n}{b_n} \right) \right| \right],$$

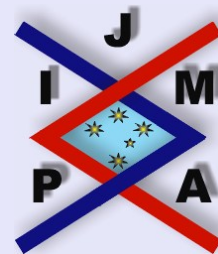
$$(\forall n, m \in \mathbb{N}, m > n).$$

(ii) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta^2(u_k b_k) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k z_k \right| \leq A \left[ |u_n| + b_n^{-1} \left( 1 + \frac{1}{\cos \theta} \right) (|\Delta(u_n b_n)| + |\Delta(u_{m-1} b_{m-1})|) \right],$$

$$(\forall n, m \in \mathbb{N}, m > n).$$

Using the concept of majorant sequences we shall give some estimates for Walsh polynomials with complex semi-monotone, complex monotone, complex semi-convex and complex convex coefficients.



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## 2. Main Results

For the main results we require the following Lemma.

**Lemma 2.1.** For all  $p, q, r \in \mathbb{N}$ ,  $p < q$  the following inequalities hold:

$$(i) \quad \left| \sum_{k=p}^q \omega_k(x) \right| \leq \frac{2}{x}, 0 < x < 1.$$

$$(ii) \quad \left| \sum_{j=p}^q \sum_{k=l}^j \omega_k(x) \right| \leq \begin{cases} \frac{2(q-p+1)}{x} = C_1(p, q, x) : 0 < x < 1 \\ \frac{8}{x(x-2^{-r})} + \frac{8}{x^2} + \frac{2(q-p+1)}{x} + 1 = C_2(p, q, r, x) : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

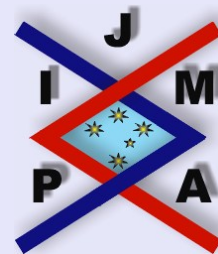
*Proof.* (i) Let  $D_q(x) = \sum_{i=0}^{q-1} w_i(x)$  be the Dirichlet kernel. Then it is known that (see [3, p. 28])  $|D_q(x)| \leq \frac{1}{x}$ ,  $0 < x < 1$ . Hence

$$\left| \sum_{k=p}^q w_k(x) \right| = |D_{q+1}(x) - D_p(x)| \leq |D_{q+1}(x)| + |D_p(x)| \leq \frac{2}{x}.$$

(ii) By (i) we get

$$\left| \sum_{j=p}^q \sum_{k=l}^j w_k(x) \right| \leq \sum_{j=p}^q \left| \sum_{k=l}^j w_k(x) \right| \leq \frac{2(q-p+1)}{x}, \quad 0 < x < 1.$$

Let  $F_n(x) = \frac{1}{n+1} \sum_{k=0}^n D_k(x)$  be the Fejer kernel. Applying Fine's inequality (see [2])



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$$(n+1)F_n(x) < \frac{4}{x(x-2^{-r})} + \frac{4}{x^2}, \quad x \in (2^{-r}, 2^{-r+1}),$$

we get

$$\begin{aligned} \left| \sum_{j=p}^q \sum_{k=l}^j w_k(x) \right| &= \left| \sum_{j=p}^q (D_{j+1}(x) - D_l(x)) \right| \\ &\leq \left| \sum_{j=p}^q D_{j+1}(x) \right| + \frac{q-p+1}{x} \\ &\leq |(q+1)F_q(x)| + |D_{q+1}(x)| + |D_0(x)| \\ &\quad + |pF_{p-1}(x)| + \frac{q-p+1}{x} \\ &< \frac{8}{x(x-2^{-r})} + \frac{8}{x^2} + \frac{2(q-p+1)}{x} + 1, \quad x \in (2^{-r}, 2^{-r+1}). \end{aligned}$$

□

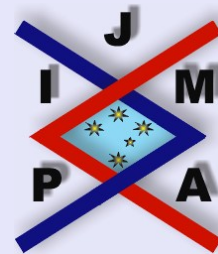
Applying the inequality (i) of the above lemma and Theorem B, we obtain following theorem.

**Theorem 2.2.** Let  $0 < x < 1$ .

(i) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta\left(\frac{u_k}{b_k}\right) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq \frac{2}{x} \left[ \left(1 + \frac{1}{\cos \theta}\right) |u_m| + \frac{1}{\cos \theta} \frac{b_m}{b_n} |u_n| \right],$$

$(\forall n, m \in \mathbb{N}, m > n).$



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(ii) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta(u_k b_k) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq \frac{2}{x} \left[ \left( 1 + \frac{1}{\cos \theta} \right) |u_n| + \frac{1}{\cos \theta} \frac{b_m}{b_n} |u_m| \right],$$

$$(\forall n, m \in \mathbb{N}, m > n).$$

Specially for  $b_k = 1$  we get the following inequalities for Walsh polynomials with complex monotone coefficients.

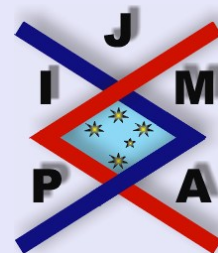
**Corollary 2.3.** Let  $0 < x < 1$ . If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta u_k \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k \omega_k(x) \right| \leq \frac{2}{x} \left[ \left( 1 + \frac{1}{\cos \theta} \right) |u_m| + \frac{1}{\cos \theta} |u_n| \right], \quad (\forall n, m \in \mathbb{N}, m > n).$$

**Corollary 2.4.** Let  $0 < x < 1$ . If  $\{u_k\}$  is a complex monotone sequence such that  $\lim_{k \rightarrow \infty} u_k = 0$ , then

$$\left| \sum_{k=n}^{\infty} u_k \omega_k(x) \right| \leq \frac{2}{x \cos \theta} |u_n|.$$

In [4] (chapter XXI), [5] Mitrinović and Pečarić obtained inequalities for cosine and sine polynomials with monotone nonnegative coefficients. Applying Theorem 2.2, we get analogical results for Walsh polynomials with monotone nonnegative coefficients.



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**Corollary 2.5.** Let  $0 < x < 1$ .

(i) If  $\{a_k\}$  is a nonnegative sequence such that  $\{a_k b_k^{-1}\}$  is a decreasing sequence, then

$$\left| \sum_{k=n}^m a_k w_k(x) \right| \leq \frac{a_n}{x} \left( \frac{b_m}{b_n} \right), \quad (\forall n, m \in \mathbb{N}, m > n).$$

(ii) If  $\{a_k\}$  is a nonnegative sequence such that  $\{a_k b_k\}$  is an increasing sequence, then

$$\left| \sum_{k=n}^m a_k w_k(x) \right| \leq \frac{a_m}{x} \left( \frac{b_m}{b_n} \right), \quad (\forall n, m \in \mathbb{N}, m > n).$$

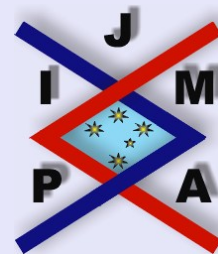
Now, applying the inequality (ii) of Lemma 2.1, we obtain new inequalities for Walsh polynomials with complex semi-convex coefficients.

**Theorem 2.6.**

(i) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta^2 \left( \frac{u_k}{b_k} \right) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq \begin{cases} C_1(m, n, x) \left[ |u_m| + b_{m-1} \left( 1 + \frac{1}{\cos \theta} \right) \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + \frac{b_{m-2}}{\cos \theta} \left| \Delta \left( \frac{u_n}{b_n} \right) \right| \right] : 0 < x < 1 \\ C_2(m, n, r, x) \left[ |u_m| + b_{m-1} \left( 1 + \frac{1}{\cos \theta} \right) \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + \frac{b_{m-2}}{\cos \theta} \left| \Delta \left( \frac{u_n}{b_n} \right) \right| \right] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

for all  $n, m, r \in \mathbb{N}, m > n$ .



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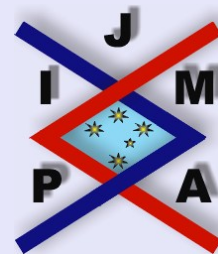
(ii) If  $\{u_k\}$  is a sequence of complex numbers such that  $\Delta^2(u_k b_k) \in K(\theta)$ , then

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq \begin{cases} C_1(m, n, x) \left[ |u_n| + b_n^{-1} \left( 1 + \frac{1}{\cos \theta} \right) \right. \\ \quad \left. \times (|\Delta(u_n b_n)| + |\Delta(u_{m-1} b_{m-1})|) \right] : 0 < x < 1 \\ C_2(m, n, r, x) \left[ |u_n| + b_n^{-1} \left( 1 + \frac{1}{\cos \theta} \right) \right. \\ \quad \left. \times (|\Delta(u_n b_n)| + |\Delta(u_{m-1} b_{m-1})|) \right] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

for all  $n, m, r \in \mathbb{N}, m > n$ .

*Proof.* (i) Applying Abel's transformation twice and the triangle inequality, we get:

$$\begin{aligned} \left| \sum_{k=n}^m \frac{u_k}{b_k} (b_k w_k) \right| &= \left| \frac{u_m}{b_m} \sum_{k=n}^m b_k w_k + \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \sum_{j=n}^{m-1} \sum_{k=n}^j b_k w_k \right. \\ &\quad \left. + \sum_{r=n}^{m-2} \Delta^2 \left( \frac{u_r}{b_r} \right) \sum_{j=n}^r \sum_{k=n}^j b_k w_k \right| \\ &\leq \frac{|u_m|}{b_m} b_m \left| \sum_{k=n}^m w_k \right| + b_{m-1} \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \left| \sum_{j=n}^{m-1} \sum_{k=n}^j w_k \right| \\ &\quad + b_{m-2} \sum_{r=n}^{m-2} \left| \Delta^2 \left( \frac{u_r}{b_r} \right) \right| \left| \sum_{j=n}^r \sum_{k=n}^j w_k \right|. \end{aligned}$$



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Using the Petrović inequality and inequality (ii) of Lemma 2.1, we obtain:

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq |u_m| \left| \sum_{k=n}^m w_k \right| + b_{m-1} \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \left| \sum_{j=n}^{m-1} \sum_{k=n}^j w_k \right|$$

$$+ \frac{b_{m-2}}{\cos \theta} \left| \sum_{r=n}^{m-2} \Delta^2 \left( \frac{u_r}{b_r} \right) \sum_{j=n}^r \sum_{k=n}^j w_k \right|$$

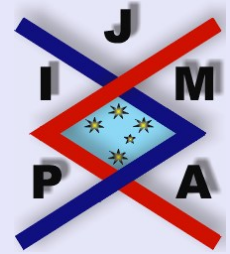
$$\leq \begin{cases} C_1(m, n, x) \left[ |u_m| + b_{m-1} \left( 1 + \frac{1}{\cos \theta} \right) \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + \frac{b_{m-2}}{\cos \theta} \left| \Delta \left( \frac{u_n}{b_n} \right) \right| \right] : 0 < x < 1 \\ C_2(m, n, r, x) \left[ |u_m| + b_{m-1} \left( 1 + \frac{1}{\cos \theta} \right) \left| \Delta \left( \frac{u_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + \frac{b_{m-2}}{\cos \theta} \left| \Delta \left( \frac{u_n}{b_n} \right) \right| \right] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

(ii) Analogously as the proof of (i), we obtain:

$$\left| \sum_{k=n}^m (u_k b_k) b_k^{-1} w_k \right|$$

$$= \left| u_n b_n \sum_{k=n}^m b_k^{-1} w_k - \sum_{j=n+1}^{m-1} \Delta^2(u_{j-1} b_{j-1}) \sum_{r=n}^j \sum_{k=r}^m b_k^{-1} w_k \right.$$

$$\left. + \Delta(u_n b_n) \sum_{k=n}^m b_k^{-1} w_k - \Delta(u_{m-1} b_{m-1}) \sum_{r=n}^m \sum_{k=r}^m b_k^{-1} w_k \right|$$



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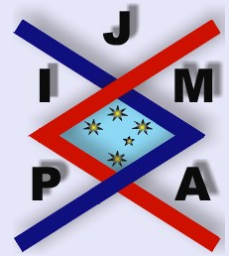


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$$\leq |u_n| b_n b_n^{-1} \left| \sum_{k=n}^m w_k \right| + b_n^{-1} \sum_{j=n+1}^{m-1} |\Delta^2(u_{j-1} b_{j-1})| \left| \sum_{r=n}^j \sum_{k=r}^m w_k \right| \\ + b_n^{-1} |\Delta(u_n b_n)| \left| \sum_{k=n}^m w_k \right| + b_n^{-1} |\Delta(u_{m-1} b_{m-1})| \left| \sum_{r=n}^m \sum_{k=r}^m w_k \right|.$$

Hence,

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq |u_n| \left| \sum_{k=n}^m w_k \right| + \frac{b_n^{-1}}{\cos \theta} \left| \sum_{j=n+1}^{m-1} \Delta^2(u_{j-1} b_{j-1}) \sum_{r=n}^j \sum_{k=r}^m w_k \right| \\ + b_n^{-1} |\Delta(u_n b_n)| \left| \sum_{k=n}^m w_k \right| + b_n^{-1} |\Delta(u_{m-1} b_{m-1})| \left| \sum_{r=n}^m \sum_{k=r}^m w_k \right| \\ \leq \begin{cases} C_1(m, n, x) \left[ |u_n| + b_n^{-1} \left( 1 + \frac{1}{\cos \theta} \right) \right. \\ \quad \left. \times (|\Delta(u_n b_n)| + |\Delta(u_{m-1} b_{m-1})|) \right] : 0 < x < 1, \\ C_2(m, n, r, x) \left[ |u_n| + b_n^{-1} \left( 1 + \frac{1}{\cos \theta} \right) \right. \\ \quad \left. \times (|\Delta(u_n b_n)| + |\Delta(u_{m-1} b_{m-1})|) \right] : x \in (2^{-r}, 2^{-r+1}). \end{cases}$$

□

If  $b_k = 1, k = n, n + 1, \dots, m$  from Theorem 2.6, we obtain the following corollary.

**Corollary 2.7.** *Let  $\{u_k\}$  be a complex-convex sequence. Then,*

$$\left| \sum_{k=n}^m u_k w_k(x) \right| \leq \begin{cases} C_1(m, n, x) \left[ |u_m| + \left(1 + \frac{1}{\cos \theta}\right) \right. \\ \quad \left. \times |\Delta u_{m-1}| + \frac{1}{\cos \theta} |\Delta u_n| \right] : 0 < x < 1 \\ C_2(m, n, r, x) \left[ |u_m| + \left(1 + \frac{1}{\cos \theta}\right) \right. \\ \quad \left. \times |\Delta u_{m-1}| + \frac{1}{\cos \theta} |\Delta u_n| \right] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

for all  $n, m, r \in \mathbb{N}, m > n$ .

**Remark 1.** Similarly, the results of Theorem 2.2, Theorem 2.6, Corollary 2.3, Corollary 2.5 and Corollary 2.7 were given by the author in [7, 8] for trigonometric polynomials with complex valued coefficients.

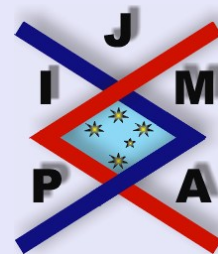
### Corollary 2.8.

(i) If  $\{a_k\}$  is a nonnegative sequence such that  $\{a_k b_k^{-1}\}$  is a convex sequence, then

$$\left| \sum_{k=n}^m a_k w_k(x) \right| \leq \begin{cases} C_1(m, n, x) \left[ |a_m| + 2b_{m-1} \left| \Delta \left( \frac{a_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + b_{m-2} \left| \Delta \left( \frac{a_n}{b_n} \right) \right| \right] : 0 < x < 1 \\ C_2(m, n, r, x) \left[ |a_m| + 2b_{m-1} \left| \Delta \left( \frac{a_{m-1}}{b_{m-1}} \right) \right| \right. \\ \quad \left. + b_{m-2} \left| \Delta \left( \frac{a_n}{b_n} \right) \right| \right] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

for all  $n, m, r \in \mathbb{N}, m > n$ .

(ii) If  $\{a_k\}$  is a nonnegative sequence such that  $\{a_k b_k\}$  is a convex sequence,



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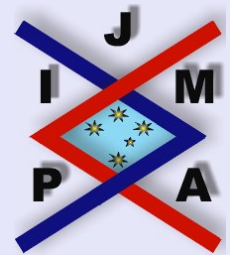
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then

$$\left| \sum_{k=n}^m a_k w_k(x) \right| \leq \begin{cases} C_1(m, n, x) [|a_n| + 2b_n^{-1} |\Delta(a_n b_n)| \\ \quad + |\Delta(a_{m-1} b_{m-1})|] : 0 < x < 1 \\ C_2(m, n, r, x) [|a_n| + 2b_n^{-1} |\Delta(a_n b_n)| \\ \quad + |\Delta(a_{m-1} b_{m-1})|] : x \in (2^{-r}, 2^{-r+1}) \end{cases}$$

for all  $n, m, r \in \mathbb{N}, m > n$ .




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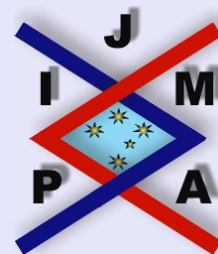
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