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# CORRECTION TO THE PAPER “BOUNDED LINEAR OPERATOR IN PROBABILISTIC NORMED SPACES"

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## Abstract

We show that Theorem 2.4 of a recent paper by I.H. Jebril and R.I.M. Ali is incorrect.

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*Key words:* Probabilistic normed spaces; Bounded linear operator; Counterexample.

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The purpose of this note is to show, by means of an appropriate counterexample, that Theorem 2.4 of the recent paper [2] is incorrect.

In [2], a linear operator $T$ from the $PN$ space $(V, \nu, \tau, \tau^*)$ to the $PN$ space $(V, \mu, \sigma, \sigma^*)$ is said to be strongly $B-$bounded if there exists a constant $h > 0$ such that, for every $p \in V$ and for every $x > 0$,

$$\mu_{Tp}(hx) \geq \nu_p(x)$$

and, similarly, $T$ is said to be strongly $C-$bounded if there exists a constant $h \in (0, 1)$ such that, for every $p \in V$ and for every $x > 0$,

$$\nu_p(x) > 1 - x \Longrightarrow \mu_{Tp}(hx) > 1 - hx.$$

Theorem 2.4 of [2] asserts that if $T$ is strongly $B-$bounded and $\mu_{Tp}$ is strictly increasing on $[0, 1]$, then $T$ is strongly $C-$bounded. To show that this is not so, consider the simple $PN$ space generated by the real line $\mathbb{R}$ with its usual norm and the distribution function $G$ given by $G(x) = x/(1+x)$, so that for any $p$ in $\mathbb{R}$ and any $x \geq 0$, $\nu_p(x) = x/(x+|p|)$. This space is a Menger space under **M** and therefore a $PN$ space in the sense of Šerstnev [1]. Now let $T : \mathbb{R} \to \mathbb{R}$ be the linear map defined by $Tp = 2p$ and note that $\nu_{2p}$ is strictly increasing on $[0,1]$. Then if $h > 2$,

$$\nu_{Tp}(hx) = \frac{hx}{hx+2|p|} \geq \frac{hx}{hx+h|p|} = \nu_p(x),$$

whence $T$ is strongly $B-$bounded. (Note that this holds in any simple $PN$ space.) But for $x = 1/2$ and $p = 1/4$, we have $\nu_p(x) = 2/3 > 1/2 = 1-x$, whereas, for any $h$ in $(0,1)$, $\nu_{2p}(hx) = h/(1+h) < 1-h/2 = 1-hx$, so that $T$ is not strongly $C-$bounded.

# References


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[2] I.H. JEBRIL AND R.M. ALI, Bounded linear operator in probabilistic normed spaces, *J. Inequal. Pure Appl. Math.*, **4**(1) (2003), Art. 8. [ONLINE: http://jipam.vu.edu.au/article.php?sid=244]