

# SOME INEQUALITIES REGARDING A GENERALIZATION OF EULER'S CONSTANT

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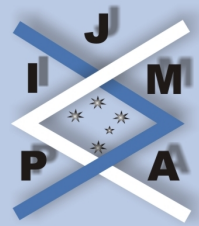
*Key words:* Sequence, Convergence, Euler's constant, Approximation, Estimate, Series.

*Abstract:* The purpose of this paper is to evaluate the limit  $\gamma(a)$  of the sequence

$$\left( \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a} \right)_{n \in \mathbb{N}},$$

where  $a \in (0, +\infty)$ . We give some lower and upper estimates for

$$\frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a} - \gamma(a), \quad n \in \mathbb{N}.$$



Generalization of  
Euler's Constant

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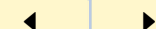
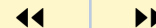
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## 1. Introduction

Let  $(D_n)_{n \in \mathbb{N}}$  be the sequence defined by  $D_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n$ , for each  $n \in \mathbb{N}$ . It is well-known that the sequence  $(D_n)_{n \in \mathbb{N}}$  is convergent and its limit, usually denoted by  $\gamma$ , is called Euler's constant.

For  $D_n - \gamma$ ,  $n \in \mathbb{N}$ , many lower and upper estimates have been obtained in the literature. We recall some of them:

- $\frac{1}{2(n+1)} < D_n - \gamma < \frac{1}{2(n-1)}$ , for each  $n \in \mathbb{N} \setminus \{1\}$  ([14]);
- $\frac{1}{2(n+1)} < D_n - \gamma < \frac{1}{2n}$ , for each  $n \in \mathbb{N}$  ([8], [19]);
- $\frac{1}{2n+1} < D_n - \gamma < \frac{1}{2n}$ , for each  $n \in \mathbb{N}$  ([17]);
- $\frac{1}{2n+\frac{2}{5}} < D_n - \gamma < \frac{1}{2n+\frac{1}{3}}$ , for each  $n \in \mathbb{N}$  ([15], [16]);
- $\frac{1}{2n+\frac{2\gamma-1}{1-\gamma}} \leq D_n - \gamma < \frac{1}{2n+\frac{1}{3}}$ , for each  $n \in \mathbb{N}$  ([16, Editorial comment], [2], [3]).

In Section 2 we present a generalization of Euler's constant as the limit of the sequence

$$\left( \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a} \right)_{n \in \mathbb{N}}, \quad a \in (0, +\infty),$$

and we denote this limit by  $\gamma(a)$ .

In Section 3 we give some lower and upper estimates for

$$\frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a} - \gamma(a), \quad n \in \mathbb{N}.$$



## 2. The Number $\gamma(a)$

It is known that the sequence

$$\left( \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a} \right)_{n \in \mathbb{N}}, \quad a \in (0, +\infty),$$

is convergent (see for example [5, p. 453], [7], where problems in this sense were proposed; [6]; [13]).

The results contained in the following theorem were given in [10].

**Theorem 2.1.** *Let  $a \in (0, +\infty)$ . We consider the sequences  $(x_n)_{n \in \mathbb{N}}$  and  $(y_n)_{n \in \mathbb{N}}$  defined by*

$$x_n = \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n}{a}$$

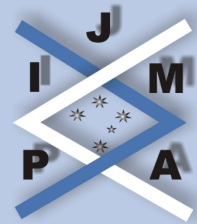
and

$$y_n = \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - \ln \frac{a+n-1}{a},$$

for each  $n \in \mathbb{N}$ .

Then:

- (i) *the sequences  $(x_n)_{n \in \mathbb{N}}$  and  $(y_n)_{n \in \mathbb{N}}$  are convergent to the same number, which we denote by  $\gamma(a)$ , and satisfy the inequalities  $x_n < x_{n+1} < \gamma(a) < y_{n+1} < y_n$ , for each  $n \in \mathbb{N}$ ;*
- (ii)  $0 < \frac{1}{a} - \ln \left(1 + \frac{1}{a}\right) < \gamma(a) < \frac{1}{a}$ ;
- (iii)  $\lim_{n \rightarrow \infty} n(\gamma(a) - x_n) = \frac{1}{2}$  and  $\lim_{n \rightarrow \infty} n(y_n - \gamma(a)) = \frac{1}{2}$ .



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*Remark 1.* The sequence  $(y_n)_{n \in \mathbb{N}}$  from Theorem 2.1, for  $a = 1$ , becomes the sequence  $(D_n)_{n \in \mathbb{N}}$ , so  $\gamma(1) = \gamma$ .

The following theorem was given by the author in [12, Theorem 2.3].

**Theorem 2.2.** *Let  $a \in (0, +\infty)$ . We consider the sequence  $(u_n)_{n \in \mathbb{N}}$  defined by  $u_n = y_n - \frac{1}{2(a+n-1)+\frac{1}{3}}$ , for each  $n \in \mathbb{N}$ , where  $(y_n)_{n \in \mathbb{N}}$  is the sequence from the statement of Theorem 2.1. Also, we specify that  $\gamma(a)$  is the limit of the sequence  $(y_n)_{n \in \mathbb{N}}$ .*

*Then:*

$$(i) \quad u_n < u_{n+1} < \gamma(a), \text{ for each } n \in \mathbb{N} \setminus \{1\}, \text{ and } \lim_{n \rightarrow \infty} n^3(\gamma(a) - u_n) = \frac{1}{72};$$

$$(ii) \quad \frac{1}{2(a+n-1)+\frac{11}{28}} < y_n - \gamma(a) < \frac{1}{2(a+n-1)+\frac{1}{3}}, \text{ for each } n \in \mathbb{N} \setminus \{1\}.$$

*Remark 2.* The lower estimate from part (ii) of Theorem 2.2 holds for  $n = 1$  as well.

*Remark 3.* The second limit from part (iii) of Theorem 2.1 also follows from part (ii) of Theorem 2.2.



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### 3. Proving Some Estimates for $y_n - \gamma(a)$ using the Logarithmic Derivative of the Gamma Function

As we already mentioned in Section 1, it is known that ([16, Editorial comment], [2, Theorem 3], [3, Theorem 1.1])

$$\frac{1}{2n + \frac{2\gamma-1}{1-\gamma}} \leq D_n - \gamma < \frac{1}{2n + \frac{1}{3}},$$

for each  $n \in \mathbb{N}$ , the constants  $\frac{2\gamma-1}{1-\gamma}$  and  $\frac{1}{3}$  being the best possible with this property.

Let  $a \in (0, +\infty)$ . In a similar way as in the proof given by H. Alzer in [2, Theorem 3], we shall obtain lower and upper estimates for  $y_n - \gamma(a)$  ( $n \in \mathbb{N}$ ), where  $(y_n)_{n \in \mathbb{N}}$  is the sequence from the statement of Theorem 2.1, the limit of which we denoted by  $\gamma(a)$ . In order to do this we shall prove, in a similar way as in [3, Lemma 2.1], some finer inequalities than those used by H. Alzer in [2, Theorem 3].

**Lemma 3.1.** *We have:*

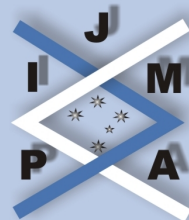
$$(i) \quad \psi(x+1) - \ln x > \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6}, \text{ for each } x \in (0, +\infty);$$

$$(ii) \quad \frac{1}{x} - \psi'(x+1) < \frac{1}{2x^2} - \frac{1}{6x^3} + \frac{1}{30x^5} - \frac{1}{42x^7} + \frac{1}{30x^9}, \text{ for each } x \in (0, +\infty).$$

We specify that the function  $\psi$  is the logarithmic derivative of the gamma function, i.e.  $\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$ , for each  $x \in (0, +\infty)$ .

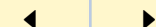
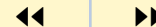
*Proof.* (i) It is known (see, for example, [18, p. 116]) that  $\ln x = \int_0^\infty \frac{e^{-t} - e^{-xt}}{t} dt$ , for each  $x \in (0, +\infty)$ . Also, we shall need the formula

$$\psi(x) = \int_0^\infty \left( \frac{e^{-t}}{t} - \frac{e^{-xt}}{1 - e^{-t}} \right) dt,$$



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which holds for each  $x \in (0, +\infty)$ , known as Gauss' expression of  $\psi(x)$  as an infinite integral (see, for example, [18, p. 247]). Having in view the above relations, we are able to write that

$$\psi(x+1) - \ln x = \int_0^{\infty} \left( \frac{1}{t} - \frac{1}{e^t - 1} \right) e^{-xt} dt,$$

for each  $x \in (0, +\infty)$ .

It is not difficult to verify that

$$\int_0^{\infty} t^n e^{-xt} dt = \frac{n!}{x^{n+1}},$$

for each  $n \in \mathbb{N} \cup \{0\}$ , any  $x \in (0, +\infty)$ .

Then we have

$$\begin{aligned} & \psi(x+1) - \ln x - \frac{1}{2x} + \frac{1}{12x^2} - \frac{1}{120x^4} + \frac{1}{252x^6} \\ &= \int_0^{\infty} \left( \frac{1}{t} - \frac{1}{e^t - 1} - \frac{1}{2} + \frac{t}{12} - \frac{t^3}{720} + \frac{t^5}{30240} \right) e^{-xt} dt \\ &= \int_0^{\infty} \frac{1}{30240t(e^t - 1)} [30240(e^t - 1) - 30240t - 15120t(e^t - 1) + 2520t^2(e^t - 1) \\ & \quad - 42t^4(e^t - 1) + t^6(e^t - 1)] e^{-xt} dt \\ &= \int_0^{\infty} \frac{1}{30240t(e^t - 1)} \left[ 30240 \sum_{n=2}^{\infty} \frac{t^n}{n!} - 15120 \sum_{n=1}^{\infty} \frac{t^{n+1}}{n!} + 2520 \sum_{n=1}^{\infty} \frac{t^{n+2}}{n!} \right. \\ & \quad \left. - 42 \sum_{n=1}^{\infty} \frac{t^{n+4}}{n!} + \sum_{n=1}^{\infty} \frac{t^{n+6}}{n!} \right] e^{-xt} dt \end{aligned}$$



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$$= \int_0^{\infty} \frac{\sum_{n=9}^{\infty} \frac{(n-3)(n-5)(n-7)(n-8)(n^2+8n+36)t^n}{n!}}{30240t(e^t - 1)} \cdot e^{-xt} dt > 0,$$

for each  $x \in (0, +\infty)$ .

(ii) In part (i) we obtained that

$$\ln x - \psi(x+1) = \int_0^{\infty} \left( \frac{1}{e^t - 1} - \frac{1}{t} \right) e^{-xt} dt,$$

for each  $x \in (0, +\infty)$ . Differentiating here we get that

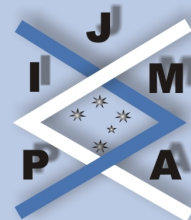
$$\frac{1}{x} - \psi'(x+1) = \int_0^{\infty} \left( 1 - \frac{t}{e^t - 1} \right) e^{-xt} dt,$$

for each  $x \in (0, +\infty)$ .

Then we have

$$\begin{aligned} & \frac{1}{x} - \psi'(x+1) - \frac{1}{2x^2} + \frac{1}{6x^3} - \frac{1}{30x^5} + \frac{1}{42x^7} - \frac{1}{30x^9} \\ &= \int_0^{\infty} \left( 1 - \frac{t}{e^t - 1} - \frac{t}{2} + \frac{t^2}{12} - \frac{t^4}{720} + \frac{t^6}{30240} - \frac{t^8}{1209600} \right) e^{-xt} dt \\ &= \int_0^{\infty} \frac{1}{1209600(e^t - 1)} [1209600(e^t - 1) - 1209600t - 604800t(e^t - 1) \\ & \quad + 100800t^2(e^t - 1) - 1680t^4(e^t - 1) + 40t^6(e^t - 1) - t^8(e^t - 1)] e^{-xt} dt \end{aligned}$$





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$$\begin{aligned}
 &= \int_0^{\infty} \frac{1}{1209600(e^t - 1)} \left[ 1209600 \sum_{n=2}^{\infty} \frac{t^n}{n!} - 604800 \sum_{n=1}^{\infty} \frac{t^{n+1}}{n!} \right. \\
 &\quad \left. + 100800 \sum_{n=1}^{\infty} \frac{t^{n+2}}{n!} - 1680 \sum_{n=1}^{\infty} \frac{t^{n+4}}{n!} + 40 \sum_{n=1}^{\infty} \frac{t^{n+6}}{n!} - \sum_{n=1}^{\infty} \frac{t^{n+8}}{n!} \right] e^{-xt} dt \\
 &= - \int_0^{\infty} \frac{\sum_{n=11}^{\infty} \frac{(n-3)(n-5)(n-7)(n-9)(n-10)(n+4)(n^2+2n+32)t^n}{n!}}{1209600(e^t - 1)} \cdot e^{-xt} dt < 0,
 \end{aligned}$$

for each  $x \in (0, +\infty)$ . □

*Remark 4.* In fact, these inequalities from Lemma 3.1 come from the asymptotic formulae (see, for example, [1, pp. 259, 260])

$$\begin{aligned}
 \psi(x) &\sim \ln x - \frac{1}{2x} - \sum_{n=1}^{\infty} \frac{B_{2n}}{2nx^{2n}} \\
 &= \ln x - \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6} + \dots
 \end{aligned}$$

and

$$\begin{aligned}
 \psi'(x) &\sim \frac{1}{x} + \frac{1}{2x^2} + \sum_{n=1}^{\infty} \frac{B_{2n}}{x^{2n+1}} \\
 &= \frac{1}{x} + \frac{1}{2x^2} + \frac{1}{6x^3} - \frac{1}{30x^5} + \frac{1}{42x^7} - \frac{1}{30x^9} + \dots,
 \end{aligned}$$

where  $B_{2n}$  is the Bernoulli number of index  $2n$ .

**Theorem 3.2.** Let  $a \in (0, +\infty)$ . We consider the sequence  $(y_n)_{n \in \mathbb{N}}$  from the statement of Theorem 2.1, the limit of which we denoted by  $\gamma(a)$ .



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Then

$$\frac{1}{2(a+n-1)+\alpha} \leq y_n - \gamma(a) < \frac{1}{2(a+n-1)+\beta},$$

for each  $n \in \mathbb{N} \setminus \{1, 2\}$ , with  $\alpha = \frac{1}{y_3 - \gamma(a)} - 2(a+2)$  and  $\beta = \frac{1}{3}$ .

Moreover, the constants  $\alpha$  and  $\beta$  are the best possible with this property.

*Proof.* The inequalities from the statement of the theorem can be rewritten in the form

$$\beta < \frac{1}{y_n - \gamma(a)} - 2(a+n-1) \leq \alpha,$$

for each  $n \in \mathbb{N} \setminus \{1, 2\}$ .

Taking into account that  $\psi(x+1) = \psi(x) + \frac{1}{x}$ , for each  $x \in (0, +\infty)$ , we can write that

$$\psi(a+n) - \psi(a) = \frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1},$$

for each  $n \in \mathbb{N}$  (see, for example, [1, p. 258]).

It is known that we have the series expansion (see, for example, [9, p. 336])

$$\psi(x) = \ln x - \sum_{k=0}^{\infty} \left[ \frac{1}{x+k} - \ln \left( 1 + \frac{1}{x+k} \right) \right],$$

for each  $x \in (0, +\infty)$ . So, we are able to write the following relation between  $\gamma(a)$  and the logarithmic derivative of the gamma function:

$$\gamma(a) = \ln a - \psi(a)$$

(see [6, Theorem 7], [11, Theorem 4.1, Remark 4.2]).



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Then

$$\begin{aligned}y_n - \gamma(a) &= \psi(a+n) - \psi(a) - \ln \frac{a+n-1}{a} - [\ln a - \psi(a)] \\ &= \psi(a+n) - \ln(a+n-1),\end{aligned}$$

for each  $n \in \mathbb{N}$ . It means that, in fact, we have to prove that

$$\beta < \frac{1}{\psi(a+n) - \ln(a+n-1)} - 2(a+n-1) \leq \alpha,$$

for each  $n \in \mathbb{N} \setminus \{1, 2\}$ , and that the constants  $\alpha$  and  $\beta$  are the best possible with this property.

We consider the function  $f : (0, +\infty) \rightarrow \mathbb{R}$ , defined by

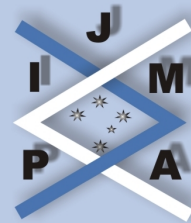
$$f(x) = \frac{1}{\psi(x+1) - \ln x} - 2x,$$

for each  $x \in (0, +\infty)$ . Differentiating, we get that

$$f'(x) = \frac{\frac{1}{x} - \psi'(x+1) - 2[\psi(x+1) - \ln x]^2}{[\psi(x+1) - \ln x]^2},$$

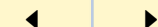
for each  $x \in (0, +\infty)$ . Using the inequalities from Lemma 3.1, we are able to write that

$$\begin{aligned}& \frac{1}{x} - \psi'(x+1) - 2[\psi(x+1) - \ln x]^2 \\ & < \frac{1}{2x^2} - \frac{1}{6x^3} + \frac{1}{30x^5} - \frac{1}{42x^7} + \frac{1}{30x^9} - 2 \left( \frac{1}{2x} - \frac{1}{12x^2} + \frac{1}{120x^4} - \frac{1}{252x^6} \right)^2 \\ & = -\frac{1}{72x^4} + \frac{1}{60x^5} + \frac{1}{360x^6} - \frac{1}{63x^7} - \frac{221}{151200x^8} + \frac{1}{30x^9} + \frac{1}{7560x^{10}} - \frac{1}{31752x^{12}} \\ & =: g(x),\end{aligned}$$



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for each  $x \in (0, +\infty)$ . It is not difficult to verify that  $g(x) < 0$ , for each  $x \in [\frac{3}{2}, +\infty)$  ( $\frac{3}{2}$  not being the best lower value possible with this property). It follows that  $f'(x) < 0$ , for each  $x \in [\frac{3}{2}, +\infty)$ . So, the function  $f$  is strictly decreasing on  $[\frac{3}{2}, +\infty)$ . This means that the sequence  $(f(a + n - 1))_{n \geq 3}$  is strictly decreasing. Therefore

$$\begin{aligned} \lim_{k \rightarrow \infty} f(a + k - 1) &< f(a + n - 1) \\ &\leq f(a + 2) \\ &= \frac{1}{y_3 - \gamma(a)} - 2(a + 2), \end{aligned}$$

for each  $n \in \mathbb{N} \setminus \{1, 2\}$ .

The asymptotic formula for the function  $\psi$ , mentioned in Remark 4, permits us to write that

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{\frac{1}{6} + O\left(\frac{1}{x^2}\right)}{\frac{1}{2} + O\left(\frac{1}{x}\right)} = \frac{1}{3}.$$

□

**Theorem 3.3.** Let  $a \in [\frac{1}{2}, +\infty)$ . We consider the sequence  $(y_n)_{n \in \mathbb{N}}$  from the statement of Theorem 2.1, the limit of which we denoted by  $\gamma(a)$ .

Then

$$\frac{1}{2(a + n - 1) + \alpha} \leq y_n - \gamma(a) < \frac{1}{2(a + n - 1) + \beta},$$

for each  $n \in \mathbb{N} \setminus \{1\}$ , with  $\alpha = \frac{1}{y_2 - \gamma(a)} - 2(a + 1)$  and  $\beta = \frac{1}{3}$ .

Moreover, the constants  $\alpha$  and  $\beta$  are the best possible with this property.

*Proof.* Since  $a \in [\frac{1}{2}, +\infty)$ , it follows that the sequence  $(f(a + n - 1))_{n \geq 2}$  is strictly decreasing, where  $f$  is the function defined in the proof of Theorem 3.2. □



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**Theorem 3.4.** Let  $a \in [\frac{3}{2}, +\infty)$ . We consider the sequence  $(y_n)_{n \in \mathbb{N}}$  from the statement of Theorem 2.1, the limit of which we denoted by  $\gamma(a)$ .

Then

$$\frac{1}{2(a+n-1)+\alpha} \leq y_n - \gamma(a) < \frac{1}{2(a+n-1)+\beta},$$

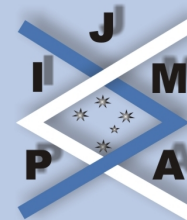
for each  $n \in \mathbb{N}$ , with  $\alpha = \frac{1}{y_1 - \gamma(a)} - 2a = \frac{a[2a\gamma(a)-1]}{1-a\gamma(a)}$  and  $\beta = \frac{1}{3}$ .

Moreover, the constants  $\alpha$  and  $\beta$  are the best possible with this property.

*Proof.* Since  $a \in [\frac{3}{2}, +\infty)$ , it follows that the sequence  $(f(a+n-1))_{n \in \mathbb{N}}$  is strictly decreasing, where  $f$  is the function defined in the proof of Theorem 3.2.  $\square$

## References

- [1] M. ABRAMOWITZ AND I.A. STEGUN, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards Applied Mathematics Series **55**, Washington, 1964.
- [2] H. ALZER, Inequalities for the gamma and polygamma functions, *Abh. Math. Sem. Univ. Hamburg*, **68** (1998), 363–372.
- [3] C.-P. CHEN AND F. QI, The best lower and upper bounds of harmonic sequence, *RGMIA* **6**(2) (2003), 303–308.
- [4] D.W. DeTEMPLE, A quicker convergence to Euler’s constant, *Amer. Math. Monthly*, **100**(5) (1993), 468–470.
- [5] K. KNOPP, *Theory and Application of Infinite Series*, Blackie & Son Limited, London and Glasgow, 1951.
- [6] D.H. LEHMER, Euler constants for arithmetical progressions, *Acta Arith.*, **27** (1975), 125–142.
- [7] I. NEDELICU, Problem 21753, *Gazeta Matematică, Seria B*, **94**(4) (1989), 136.
- [8] P.J. RIPPON, Convergence with pictures, *Amer. Math. Monthly*, **93**(6) (1986), 476–478.
- [9] I.M. RÎJIC AND I.S. GRADȘTEIN, *Tabele de integrale. Sume, serii și produse (Tables of Integrals. Sums, Series and Products)*, Editura Tehnică, București, 1955.
- [10] A. SÎNTĂMĂRIAN, Approximations for a generalization of Euler’s constant (submitted).



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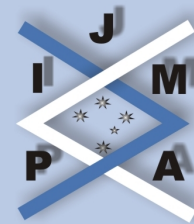
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- [11] A. SÎNTĂMĂRIAN, About a generalization of Euler's constant, *Aut. Comp. Appl. Math.*, **16**(1) (2007), 153–163.
- [12] A. SÎNTĂMĂRIAN, A generalization of Euler's constant, *Numer. Algorithms*, **46**(2) (2007), 141–151.
- [13] T. TASAKA, Note on the generalized Euler constants, *Math. J. Okayama Univ.*, **36** (1994), 29–34.
- [14] S.R. TIMS AND J.A. TYRRELL, Approximate evaluation of Euler's constant, *Math. Gaz.*, **55**(391) (1971), 65–67.
- [15] L. TÓTH, Problem E3432, *Amer. Math. Monthly*, **98**(3) (1991), 264.
- [16] L. TÓTH, Problem E3432 (Solution), *Amer. Math. Monthly*, **99**(7) (1992), 684–685.
- [17] A. VERNESCU, Ordinul de convergență al șirului de definiție al constantei lui Euler (The convergence order of the definition sequence of Euler's constant), *Gazeta Matematică, Seria B*, **88**(10-11) (1983), 380–381.
- [18] E.T. WHITTAKER AND G.N. WATSON, *A Course of Modern Analysis*, Cambridge University Press, Cambridge, 1996.
- [19] R.M. YOUNG, Euler's constant, *Math. Gaz.*, **75**(472) (1991), 187–190.

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