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ON THE CONTI–OPIAL TYPE EXISTENCE AND UNIQUENESS THEOREMS FOR GENERAL NONLINEAR BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF DISCRETE EQUATIONS

Abstract. The general nonlinear boundary value problem for systems of discrete equations is considered. The sufficient, among them effective, conditions for the solvability and unique solvability of this problem are given.

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In the present paper, we consider the problem on the solvability of a system of nonlinear discrete equations

$$\Delta y(l-1) = g(l, y(l), y(l-1)) \ \text{ for } \ l \in \mathbb{N}_{m_0} \tag{1}$$

under the boundary value condition

$$\zeta(y) = 0, \tag{2}$$

where $m_0 \geq 2$ is a fixed natural number, the function $g = (g_i)_{i=1}^n$ belongs to the discrete Carathéodory class $Car(\mathbb{N}_{m_0} \times \mathbb{R}^n, \mathbb{R}^n)$, and $\zeta : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ is a continuous, nonlinear in general, vector-functional.

In the paper, the sufficient, among them effective, conditions are given for the solvability and unique solvability of the general nonlinear discrete boundary value problem (1), (2). We have established the Conti–Opial type theorems for the solvability and unique solvability of this problem. Analogous problems are investigated in [9, 12–14, 17] (see also the references therein) for the general nonlinear boundary value problems for ordinary differential and functional-differential systems.

The results obtained in the paper are analogous to those given in [12–14] for ordinary differential and functional-differential problems.

Quite a number of issues on the theory of systems of difference equations (both linear and nonlinear) have been studied sufficiently well (for a survey of the results see e.g. [1–8, 10, 11, 15, 16, 18–21] and references therein). But the above-mentioned works, as we know, do not contain the results obtained in the present paper.

Throughout the paper, the use will be made of the following notation and definitions.

$\mathbb{N} = \{1, 2, \dots\}$, $\mathbb{N}_0 = \{0, 1, \dots\}$, $\mathbb{Z}$ is the set of all integers.

If $m \in \mathbb{N}$, then $\mathbb{N}_m = \{1, \dots, m\}$, $\widetilde{\mathbb{N}}_m = \{0, 1, \dots, m\}$.

$\mathbb{R} =]-\infty, +\infty[\,$, $\mathbb{R}_+ = [0, +\infty[\,$.

$\mathbb{R}^{n\times m}$ is the space of all real $n \times m$-matrices $X = (x_{ij})_{i,j=1}^{n,m}$ with the norm $\|X\| = \max_{j=1,\dots,m} \sum_{i=1}^{n} |x_{ij}|$; $\quad |X| = (|x_{ij}|)_{i,j}^{n,m}$;

$\mathbb{R}_+^{n\times m} = \big\{(x_{ij})_{i,j=1}^{n,m} :\ x_{ij} \geq 0\ (i = 1, \dots, n;\ j = 1, \dots, m)\big\}$.

$\mathbb{R}^n = \mathbb{R}^{n\times 1}$ is the space of all real column n-vectors $x = (x_i)_{i=1}^n$; $\mathbb{R}_+^n = R_+^{n\times 1}$.

If $X \in \mathbb{R}^{n\times n}$, then X^{-1}, $\det X$ and $r(X)$ are, respectively, the matrix inverse to X, the determinant of X and the spectral radius of X; $I_{n\times n}$ is the identity $n \times n$-matrix.

$E(J, \mathbb{R}^{n\times m})$, where $J \subset \mathbb{Z}$, is the space of all matrix-functions $Y = (y_{ij})_{i,j=1}^{n,m} : J \to \mathbb{R}^{n\times m}$ with the norm

$$\|Y\|_J = \max\big\{\|Y(l)\| :\ l \in J\big\}, \quad |Y|_J = \big(|y_{ij}|_J\big)_{i,j=1}^{n,m}.$$

Δ is the difference operator of the first order, i.e.,

$$\Delta Y(k-1) = Y(k) - Y(k-1) \quad \text{for } Y \in E(\widetilde{\mathbb{N}}_l, \mathbb{R}^{n\times m}),\ \ k \in \mathbb{N}_l.$$

If a function Y is defined on $\mathbb{N}_l$ or $\widetilde{\mathbb{N}}_{l-1}$, then we assume $Y(0) = O_{n\times m}$, or $Y(l) = O_{n\times m}$, respectively, if necessary.

$C(D_1, D_2)$, where $D_1 \subset R^n$ and $D_2 \subset R^{n\times m}$, is the set of all continuous matrix-functions $X : D_1 \to D_2$;

If B_1 and B_2 are the normed spaces, then an operator $g : B_1 \to B_2$ (nonlinear, in general) is said to be positive homogeneous if $g(\lambda x) = \lambda g(x)$ for every $\lambda \in R_+$ and $x \in B_1$; if, in addition, the spaces are partially ordered, then the operator g is called nondecreasing if $g(x) \leq g(y)$ for every $x, y \in B_1$ such that $x \leq y$.

A matrix-function is said to be continuous, nondecreasing, integrable, etc., if each of its components is such.

If $J \subset \mathbb{Z}$, $D_1 \subset \mathbb{R}^n$ and $D_2 \subset \mathbb{R}^{n\times m}$, then $Car(J \times D_1, D_2)$ is the discrete Carathéodory class, i.e., the set of all mappings $F = (f_{kj})_{k,j=1}^{n,m} : J \times D_1 \to D_2$ such that the function $f_{kj}(t, \cdot\,) : D_1 \to D_2$ is continuous for every $i \in \{1, \dots, l\}$, $j \in \{1, \dots, m\}$ and $k \in \{1, \dots, n\}$.

By a solution of the difference problem (1), (2) we understand a vector-function $y \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n)$ satisfying both the system (1) for $i \in \{1, \dots, m_0\}$ and the boundary value condition (2).

Definition. Let $\mathcal{L} : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ be a linear continuous operator, and let $\mathcal{L} : E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n) \to \mathbb{R}_+^n$ be a positive homogeneous operator. We say that a pair (G_1, G_2), consisting of matrix-functions $G_j \in Car(\mathbb{N}_{m_0} \times \mathbb{R}^{2n}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$, satisfies the Opial condition with respect to the pair $(\mathcal{L}, \mathcal{L}_0)$ if:

(a) there exists a matrix-function $\Phi \in E(\mathbb{N}_{m_0}, \mathbb{R}^n_+)$ such that

$$|G_j(l, x, y)| \le \Phi(l) \ \text{ for } \ x, y \in \mathbb{R}^n \ \ (j = 1, 2;\ l = 1, \dots, m_0); \tag{3}$$

(b)

$$\det\big(I_{n\times n} + (-1)^j B_j(l)\big) \ne 0 \ \ (j = 1, 2;\ \ l = 1, \dots, m_0) \tag{4}$$

and the problem

$$\Delta y(l-1) = B_1(l)y(l) + B_2(l)y(l-1) \ \ (l \in \mathbb{N}_{m_0}), \tag{5}$$

$$|\mathcal{L}(y)| \le \mathcal{L}_0(y) \tag{6}$$

has only the trivial solution for every matrix-functions $B_j \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$ for which there exists a sequence $x_k, y_k \in E(\mathbb{N}_{m_0}, \mathbb{R}^{n\times n})$ $(k = 1, 2, \dots)$ such that

$$\lim_{k\to+\infty} G_j(l, x_k(l), y_k(l)) = B_j(l) \ \ (j = 1, 2;\ \ l = 1, \dots, m_0).$$

Remark 1. Note that, due to the condition (3), the condition (4) holds if

$$\|\Phi(l)\| < 1 \ \ (l = 1, \dots, m_0).$$

We assume that $g \in Car(\mathbb{N}_{m_0} \times \mathbb{R}^n, \mathbb{R}^n)$.

Theorem 1. *Let the condition*

$$\big\|g(l, x, y) - G_1(l, x, y)x - G_2(l, x, y)y\big\| \le \\ \le \alpha\big(l, \|x\|, \|y\|\big) \ \textit{ for } \ l \in \mathbb{N}_{m_0}, \ \ x, y \in \mathbb{R}^n \tag{7}$$

and

$$|\zeta(y) - \mathcal{L}(y)| \le \mathcal{L}_0(y) + \ell_1\big(\|y\|_{m_0}\big) \ \textit{ for } \ y \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n) \tag{8}$$

hold, where $\mathcal{L} : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ and $\mathcal{L}_0 : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n_+$ are, respectively, the linear continuous and the positive homogeneous continuous operators, the pair (G_1, G_2) satisfies the Opial condition with respect to the pair $(\mathcal{L}, \mathcal{L}_0)$; and $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$ is a function nondecreasing in the second variable and $\ell_1 \in C(\mathbb{R}, \mathbb{R}^n_+)$ is a nondecreasing vector-function such that

$$\lim_{\rho\to+\infty} \frac{1}{\rho}\bigg(\|\ell_1(\rho)\| + \sum_{l=1}^{m_0} \alpha(l, \rho)\bigg) = 0. \tag{9}$$

Then the problem (1), (2) *is solvable.*

Theorem 2. *Let the conditions* (7), (8) *and*

$$P_{j1}(l) \le G_j(l, x, y) \le P_{j2}(l) \ \textit{ for } \ l \in \mathbb{N}_{m_0}, \ \ x, y \in \mathbb{R}^n \ \ (j = 1, 2)$$

hold, where $P_{j1}, P_{j2} \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)(j = 1, 2)$, $\mathcal{L} : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ and $\mathcal{L}_0 : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n_+$ are, respectively, the linear continuous and the positive homogeneous continuous operators; and $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$

is a function nondecreasing in the second variable and $\ell_1 \in C(\mathbb{R}, \mathbb{R}^n_+)$ *is a nondecreasing vector-function such that the condition* (9) *holds. Let, moreover, the condition* (4) *hold and the problem* (5), (6) *have only the trivial solution for every matrix-functions* B_1 *and* B_2 *from* $E(\mathbb{N}_{m_0}, \mathbb{R}^n)$ *such that*

$$P_{j1}(l) \le B_j(l) \le P_{j2}(l) \ \ for \ \ l \in \mathbb{N}_{m_0} \ \ (j = 1, 2).$$

Then the problem (1), (2) *is solvable.*

Remark 2. Theorem 2 is interesting only in case $G_j(l, \cdot\,, \cdot\,) \notin C(\mathbb{R}^{2n}, \mathbb{R}^{n\times n})$ for some $j \in \{1, 2\}$ and $l \in \{1, \dots, m_0\}$, because the theorem immediately follows from Theorem 1 in case $G_j \in Car(\mathbb{N}_{m_0} \times \mathbb{R}^{2n}, \mathbb{R}^{n\times n})$ $(j = 1, 2)$.

Theorem 3. *Let the conditions* (8),

$$\big|g(l, x, y) - P_1(l)x - P_2(l)y\big| \le$$
$$\le Q_1(l)|x| + Q_2(l)|y| + q\big(l, \|x\| + \|y\|\big) \ \ for \ \ l \in \mathbb{N}_{m_0}, \ \ x, y \in \mathbb{R}^n,$$
$$\det\big(I_{n\times n} + (-1)^j P_j(l)\big) \ne 0 \ \ for \ \ l \in \mathbb{N}_{m_0} \ \ (j = 1, 2), \tag{10}$$

and

$$\big(\|Q_1(l)\| + \|Q_2(l)\|\big) \cdot \Big(1 + \big\|\big(I_{n\times n} + (-1)^j P_j(l)\big)^{-1}\big\| +$$
$$+ \big\|\big(I_{n\times n} + (-1)^j P_{3-j}(l)\big)^{-1}\big\|\Big) < 1 \ \ for \ \ l \in \mathbb{N}_{m_0}, \ \ x, y \in \mathbb{R}^n \ \ (j = 1, 2) \tag{11}$$

hold, where $P_1, P_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*;* $Q_1, Q_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n_+)$*;* $\mathcal{L} : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ *and* $\mathcal{L}_0 : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n_+$ *are, respectively, the linear continuous and the positive homogeneous continuous operators; and* $q \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}^n_+)$ *is a vector-function nondecreasing in the second variable and* $\ell_1 \in C(\mathbb{R}, \mathbb{R}^n_+)$ *is a nondecreasing vector-function such that*

$$\lim_{\rho\to+\infty} \frac{1}{\rho}\left(\|\ell_1(\rho)\| + \sum_{l=1}^{m_0} \|q(l, \rho)\|\right) = 0.$$

Let, moreover, the problem

$$\big|\Delta y(l-1) - P_1(l)y(l) - P_2(l)y(l-1)\big| \le$$
$$\le Q_1(l)|y(l)| + Q_2(l)|y(l-1)| \ \ (l \in \mathbb{N}_{m_0}), \tag{12}$$
$$|\mathcal{L}(y)| \le \mathcal{L}_0(y) \tag{13}$$

have only the trivial solution. Then the problem (1), (2) *is solvable.*

Corollary 1. *Let the conditions*

$$\big\|g(l, x, y) - G_1(l)x - G_2(l)y\big\| \le$$
$$\le \alpha\big(l, \|x\| + \|y\|\big) \ \ for \ \ l \in \mathbb{N}_{m_0}, \ \ x, y \in \mathbb{R}^n, \tag{14}$$
$$\det\big(I_{n\times n} + (-1)^j G_j(l)\big) \ne 0 \ \ for \ \ l \in \mathbb{N}_{m_0} \ \ (j = 1, 2) \tag{15}$$

and

$$\big\|\zeta(y) - \mathcal{L}(y)\big\| \le \beta\big(\|y\|_{m_0}\big) \ \ for \ \ y \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^n) \tag{16}$$

hold, where $G_1, G_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*;* $\mathcal{L} : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ *is the linear continuous operator; and* $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$ *is a function nondecreasing in the second variable and* $\beta \in C(\mathbb{R}, \mathbb{R}^n_+)$ *is a nondecreasing function such that*

$$\lim_{\rho \to +\infty} \frac{1}{\rho} \left(\beta(\rho) + \sum_{l=1}^{m_0} \alpha(l, \rho) \right) = 0. \tag{17}$$

Let, moreover, the problem

$$\Delta y(l-1) = G_1(l) y(l) + G_2(l) y(l-1) \ \ (l \in \mathbb{N}_{m_0}),$$
$$\mathcal{L}(y) = 0$$

have only the trivial solution. Then the problem (1), (2) *is solvable.*

Corollary 2. *Let the conditions* (14)–(16) *and*

$$\mathcal{L}(y) \equiv \sum_{j=1}^{n_0} L_j y(k_j) \tag{18}$$

hold, where n_0 *is a fixed natural number;* $k_j \in \{0, \dots, m_0\}$ *and* $L_j \in \mathbb{R}^{n \times n}$ $(j = 1, \dots, n_0)$*;* $G_1, G_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*; and* $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$ *is a function nondecreasing in the second variable and* $\beta \in C(\mathbb{R}, \mathbb{R}^n_+)$ *is a nondecreasing function such that the condition* (17) *holds. Let, moreover,*

$$\det \left(\sum_{j=1}^{n_0} L_j \big(I_n + G_2(k_j + 1)\big)^{-1} \cdot \prod_{i=0}^{k_j+1} \big(I_n - G_1(i)\big)^{-1} \big(I_n + G_2(i)\big) \right) \neq 0.$$

Then the problem (1), (2) *is solvable.*

On the set $E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n \times n}) \times E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n \times})$ we introduce the operators as follows. If $G_1, G_2 \in E(\widetilde{\mathbb{N}}_{m_0}, \mathbb{R}^{n \times n})$ and, in addition, $\det(I_{n \times n} + G_2(l)) \neq 0$ $(l = 1, \dots, m_0)$, then we assume

$$\begin{gathered} \big[(G_1, G_2)(l)\big]_0 \equiv I_n, \quad \big[(G_1, G_2)(l)\big]_k \equiv - \sum_{i=l+1}^{m_0} \big(G_1(i) + G_2(i+1)\big) \times \\ \times \big(I_n + G_2(i)\big)^{-1} \big[(G_1, G_2)(i)\big]_{k-1} \ \ (k = 1, 2, \dots), \end{gathered} \tag{19}$$

and

$$\begin{aligned} V_1(G_1, G_2)(l) &\equiv \sum_{i=l+1}^{m_0} \big| \big(G_1(i) + G_2(i+1)\big) \big(I_n + G_2(i+1)\big)^{-1} \big|, \\ V_{k+1}(G_1, G_2)(l) &\equiv \sum_{i=l+1}^{m_0} \big| \big(G_1(i) + G_2(i+1)\big) \big(I_n + G_2(i+1)\big)^{-1} \big| \times \\ &\qquad \times V_k(G_1, G_2)(i) \ \ (k = 1, 2, \dots). \end{aligned} \tag{20}$$

Theorem 4. *Let the conditions* (14)–(16) *and* (18) *hold, where* n_0 *is a fixed natural number,* $k_j \in \{1, \dots, m_0\}$ *and* $L_j \in \mathbb{R}^{n \times n}$ $(j = 1, \dots, n_0)$*;* $G_1, G_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*; and* $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$ *is a function nondecreasing in* *the second variable and* $\beta \in C(\mathbb{R}, \mathbb{R}_+^n)$ *is a nondecreasing function such that the condition* (17) *holds. Let, moreover, there exist natural numbers* k *and* m *such that*

$$\det(M_k) \neq 0$$

and

$$r(M_{k,m}) < 1,$$

where

$$M_k = \sum_{j=1}^{n_0} \sum_{i=0}^{k-1} L_j \big(I_n + G_2(k_j+1)\big)^{-1} \big[(G_1, G_2)(k_j)\big]_i,$$

$$M_{k,m} = V_m(G_1, G_2)(0) +$$

$$+ \sum_{i=0}^{m-1} \big| \big[(G_1, G_2)(\,\cdot\,)\big]_i \big|_{\widetilde{\mathbb{N}}_{m_0}} \cdot \sum_{j=1}^{n_0} |M_k^{-1} L_j| \big(I_n + G_2(k_j+1)\big)^{-1} V_k(G_1, G_2)(k_j),$$

and the matrix-functions $[(G_1, G_2)(l)]_i$ *and* $V_i(G_1, G_2)(l)$ *are defined by* (19) *and* (20), *respectively. Then the problem* (1), (18) *is solvable.*

Corollary 3. *Let the conditions* (14)–(16) *and* (18) *hold,* n_0 *is a fixed natural number, where* $t_j \in \{1, \dots, m_0\}$ *and* $L_j \in \mathbb{R}^{n \times n}$ $(j = 1, \dots, n_0)$*;* $G_1, G_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*; and* $\alpha \in Car(\widetilde{E}_{m_0} \times \mathbb{R}_+, \mathbb{R}_+)$ *is a function nondecreasing in the second variable and* $\beta \in C(\mathbb{R}, \mathbb{R}_+^n)$ *is a nondecreasing function such that the condition* (17) *holds. Let, moreover,*

$$\det\Big(\sum_{j=1}^{n_0} L_j \big(I_n + G_2(k_j+1)\big)^{-1}\Big) \neq 0$$

and

$$r(L_0 M_0) < 1,$$

where

$$L_0 = I_n + \Big|\Big(\sum_{j=1}^{n_0} L_{1j} \big(I_n + G_2(k_j+1)\big)^{-1}\Big)^{-1}\Big| \cdot \sum_{j=1}^{n_0} \big|L_j \big(I_n + G_1(k_j)\big)^{-1}\big|$$

and

$$M_0 = \sum_{i=1}^{m_0} \big|\big(G_1(i) + G_2(i+1)\big)\big(I_n + G_2(i+1)\big)^{-1}\big|.$$

Then the problem (1), (2) *is solvable.*

Theorem 5. *Let the conditions* (10), (11),

$$\big|g(l,x,u) - g(l,y,v) - P_1(l)(x-y) - P_2(l)(u-v)\big| \leq$$
$$\leq Q_1(l)|x-y| + Q_2(l)|u-v| \ \text{ for } \ l \in \mathbb{N}_{m_0}, \ \ x, y, u, v \in \mathbb{R}^n,$$

$$\big|I_l(x) - I_l(y) - J_{0l} \cdot (x-y)\big| \leq$$
$$\leq H_k \cdot |x-y| \ \text{ for } \ x, y \in \mathbb{R}^n \ \ (k = l, \dots, m_0)$$

and

$$\left|\zeta(x)-\zeta(y)-\mathcal{L}(x-y)\right| \le \mathcal{L}_0(x-y) \ \ \textit{for} \ \ y\in E(\widetilde{\mathbb{N}}_{m_0},\mathbb{R}^n)$$

hold, where $P_1, P_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n)$*;* $Q_1, Q_2 \in E(\mathbb{N}_{m_0}, \mathbb{R}^n_+)$*;* $\mathcal{L} : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n$ *and* $\mathcal{L}_0 : E(\widetilde{E}_{m_0}, \mathbb{R}^n) \to \mathbb{R}^n_+$ *are, respectively, the linear continuous and the positive homogeneous continuous operators. Let, moreover, the problem* (12), (13) *has only the trivial solution. Then the problem* (1), (2) *is uniquely solvable.*

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