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IA-AUTOMORPHISMS OF p-GROUPS, FINITE POLYCYCLIC GROUPS AND OTHER RESULTS

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Abstract. In this paper, the group $IA(G)$ of all IA-automorphisms of a group G is studied. We prove some results regarding non-triviality, polycyclicity and commutativity of $IA(G)$ in addition to proving some basic results. We also prove some results analogous to a result by Schur and a weak form of its converse in the context of IA-automorphisms.

1. Introduction

For a group G, we denote the group of all automorphisms on G by $Aut\ (G)$. Following Bachmuth [3], we call an automorphism α on G an IA-automorphism iff it preserves all cosets of G', i.e., $x^{-1}\alpha(x) \in G'$, $\forall\ x \in G$; here G' is the derived subgroup of G. The set of all IA-automorphisms of G is denoted by $IA(G)$, and it is a normal subgroup of $Aut(G)$. It is obvious that every inner automorphism is an IA-automorphism, and indeed $I_G \trianglelefteq IA(G)$ (I_G denotes the group of all inner automorphisms of G). An automorphism α of G is known as a central automorphism if it preserves cosets of the centre $Z(G)$. It is clear that every central automorphism commutes with every inner automorphism, and also every central automorphism preserves the derived group G' element wise. We shall denote the group of all central automorphisms of G by $Aut_c(G)$.

For a group G with $[G : G'] = 2$, obviously $IA(G) = Aut\ (G)$. Further, if G is of nilpotency class 2, then $G' \leq Z(G)$ and thus $IA(G) \leq Aut_c(G)$. The inclusion may be proper. For example, suppose G is the central product of a dihedral group D_8 and cyclic group C of order 8. Here the centre of D_8 is identified with subgroup of order 2 of C. G' has order 2 and $Z(G) = C$. Consider the automorphism ϕ of G that acts trivially on D_8 and inverts the elements of C. Then ϕ acts non-trivially on G/G' but acts trivially on G/C. Thus, while ϕ is in $Aut_c(G)$, it is not in $IA(G)$.

It is also clear that every class preserving automorphism is an IA-automorphism. However, for semidihedral group D_{16}, the class preserving automorphisms form a proper subgroup of IA-automorphisms.

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In this paper, we prove some basic results on IA-automorphisms, and also some results related with a result by Schur and its converse. We state below a standard lemma which we shall be using subsequently.

DEFINITION 1.1. Let G be any group and let $G_k \trianglelefteq G_{k-1} \trianglelefteq \ldots \trianglelefteq G_1 \trianglelefteq G_0 = G$ be the derived series. We say that α of $Aut(G)$ stabilizes the series if $\alpha(xG_i) = xG_i$ for $i = 1, 2, \ldots, k$, and for all $x \in G_{i-1}$.

LEMMA 1.2. [9] *If a group G acts faithfully on a group γ and stabilizes a series of normal subgroups of γ of length n, then G is nilpotent of class at most $n-1$.*

2. Some basic results on *IA*-automorphisms

PROPOSITION 2.1. *Let γ_k be the set of all IA-automorphisms of G which stabilize the derived series $G_k \trianglelefteq G_{k-1} \trianglelefteq \ldots \trianglelefteq G_1 \trianglelefteq G_0 = G$, then γ_k is a normal subgroup of $IA(G)$.*

Proof. It is obvious that γ_k is a subgroup of $IA(G)$. Let $\sigma \in IA(G)$ and $f \in \gamma_k$. By the definition it follows that $x^{-1}f(x) \in G_m$, $\forall\ x \in G_{m-1}$, $m \in \{1, 2, \ldots, k\}$. The result will follow if we show that $\sigma^{-1}f\sigma(G_{m-1}/G_m) = G_{m-1}/G_m$. This is equivalent to show that $x^{-1}\sigma^{-1}f\sigma(x) \in G_m$, $\forall\ x \in G_{m-1}$. Thus, let $x \in G_{m-1}$ and denote $\sigma(x)$ by x'. As G_{m-1} is a characteristic subgroup, $\sigma(x) = x' \in G_{m-1}$. So $x^{-1}\sigma^{-1}f\sigma(x) = x^{-1}\sigma^{-1}(x'x'^{-1}f(x'))$. But $x'^{-1}f(x') \in G_m$. Let us denote $x'^{-1}f(x')$ by g. So $g \in G_m$. Thus $x^{-1}\sigma^{-1}f\sigma(x) = x^{-1}\sigma^{-1}(x'g) = x^{-1}\sigma^{-1}(\sigma(x)g)$ $= x^{-1}x\sigma^{-1}(g) = \sigma^{-1}(g) \in G_m$ ($\because G_m$ is the characteristic subgroup). ■

DEFINITION 2.2. A group G is said to be polycyclic if there exists a subnormal series $1 = H_0 \trianglelefteq H_1 \trianglelefteq \ \ldots \ \trianglelefteq H_n = G$ with each factor H_{i+1}/H_i cyclic.

Hall [5] has shown that if G is nilpotent of nilpotency class n, then $IA(G)$ is nilpotent of class $n-1$. The similar result for solvability is not true. For example, consider the solvable group G constructed as the semidirect product of an elementary abelian 3-group E by a cyclic group C of order 2, where the nonidentity element of C acts to invert the elements of E. Then G/G' has order 2, so every automorphism of G is an IA. But if $|E| > 9$, the automorphism group of G is not solvable. This follows from the following argument.

Consider the semidirect product B of the elementary abelian p-group E of order p^n (for $p > 2$ and $n > 1$) acted on naturally by $SL(n, p)$. Since $SL(2, p)$ contains a unique subgroup T of order 2, the subgroup $G = ET$ is normal in B. Also, G has trivial centralizer in B, and thus B is isomorphically embedded in $Aut(G)$. Now G is the semidirect product of E by the group T of order 2, where the non-identity element of T inverts the elements of E. Also, $SL(n, p)$ is nonsolvable unless $n = 2$ and $p = 3$ and so B is nonsolvable. Thus $Aut(G)$ is nonsolvable.

However, the following result holds.

THEOREM 2.3. *If G is a finite polycyclic group in which the derived series has cyclic factors of prime order then $IA(G)$ is polycyclic.*

Proof. Let $1 = G_m \trianglelefteq G_{m-1} \trianglelefteq \ldots \trianglelefteq G_1 \trianglelefteq G_0 = G$ be the derived series of G. We have G_{k-1}/G_k a cyclic group of prime order. For any $k \in \{1, 2 \ldots, m\}$, we denote by γ_k the set of all IA-automorphisms of G which stabilize the derived series $G_k \trianglelefteq G_{k-1} \trianglelefteq \ldots \trianglelefteq G_1 \trianglelefteq G_0 = G$ (we put $\gamma_1 = IA(G)$). Clearly by proposition 2.1, $\gamma_1, \ldots, \gamma_m$ forms a decreasing sequence of normal subgroups of $IA(G)$. Using Lemma 1.2, we can assert that γ_m is nilpotent (of class at most $m-1$) and as it is finitely generated, and hence it is polycyclic. It remains to prove that $IA(G)/\gamma_m$ is polycyclic. For each integer k (with $1 \leq k \leq m-1$), consider the homomorphism $\psi_k : \gamma_k \longrightarrow Aut(G_k/G_{k+1})$ where for any $f \in \gamma_k$, $\psi_k(f)$ is defined as the automorphism induced by f on G_k/G_{k+1}. We observe that $Aut(G_k/G_{k+1})$ is finite cyclic and that $ker\ \psi_k = \gamma_{k+1}$. So by the fundamental theorem of homomorphism, γ_k/γ_{k+1} is finite cyclic. It follows that $1 = \gamma_m/\gamma_m \trianglelefteq \gamma_{m-1}/\gamma_m \trianglelefteq \ldots \trianglelefteq \gamma_1/\gamma_m = IA(G)/\gamma_m$ forms a normal series in which each factor is finite and cyclic. Thus $IA(G)/\gamma_m$ is polycyclic, and hence $IA(G)$ is polycyclic. ■

If we consider the dihedral group D_{2n}, it follows that the collection of all automorphisms which are both central and IA forms an abelian group. Indeed this can be generalized to the following result.

THEOREM 2.4. *For a finite group G, the set of all automorphisms which are both central and IA forms an abelian normal subgroup of $Aut(G)$, and every prime divisor of the order of this group divides the order of G.*

Proof. Let A denote group of all automorphisms which are central as well as IA. It is obvious that A is a normal subgroup of $Aut(G)$. Consider $[G, A] \equiv \{x^{-1}\sigma(x) \mid x \in G \text{ and } \sigma \in A\}$. But as $x^{-1}\sigma(x) \in G'$, $[G, A] \subseteq G'$. Further, as elements of A are central, they preserve elements of G'. So A acts trivially on $[G, A]$. Thus $[G, A, A] \equiv [[G, A], A] = 1$, and hence by [7], A is solvable group of derived length at most one. This implies that A is an abelian group.

To prove the second part, let us assume that p is a prime factor of $|A|$ where p does not divide $|G|$. Let P be a p-sylow subgroup of A. By [7], $(|G|, |P|) = 1 \Rightarrow [G, P, P] = [G, P]$. Clearly, $[G, P, P] \subseteq [G, A, A] = 1$. So $[G, P] = 1$. It implies that P acts trivially on G, i.e., $|P| = 1$. This is not possible, and hence p must divide order of G. ■

COROLLARY 2.4.1. *$IA(G)$ is abelian if and only if G is a nilpotent group of class at most 2.*

Proof. If G is nilpotent with nilpotency class ≤ 2 then $G' \leq Z(G)$. So every IA-automorphism is central and hence by above theorem, $IA(G)$ is an abelian group. Conversely, let $IA(G)$ be an abelian group. As I_G is normal subgroup of $IA(G)$, I_G is also an abelian group. But $I_G \cong G/Z(G)$. Thus $G/Z(G)$ is an abelian group, and this implies that $G' \leq Z(G)$. Hence the result follows. ■

COROLLARY 2.4.2. *Let G be a 2-generated group of class 2. Then $IA(G) \cong G' \times G'$.*

Proof. In [4], Caranti and Scoppola proved that for a 2-generated metabelian group G, $Z(IA(G)) \cong G' \cap Z(G) \ \times \ G' \cap Z(G)$. Since G is nilpotent of class 2, it is metabelian. By the above corollary, $IA(G)$ is abelian. As G is nilpotent of class 2, $G' \leq Z(G)$.

$$\therefore IA(G) \cong G' \ \times \ G'. \quad \blacksquare$$

Adney [1] proved that if G is a purely non-abelian p-group then Aut_c(G) is also a p-group. An analogous result can be proved for $IA(G)$. To this end, we first prove the following proposition.

Proposition 2.5. *Let N be a normal subgroup of G. Let σ be an automorphism which fixes all cosets of N. Suppose $o(\sigma) = p$ where p does not divide $o(N)$. Then $G = NC$ where C is a subgroup of all elements of G which are fixed by σ.*

Proof. Since σ fixes all cosets of N and p does not divide the order of N, it follows that σ permutes the elements of cosets of N and the order of each orbit is 1 or p. But as p does not divide the order of N, each coset of N contains at least one element of C. Thus $G = NC$. ■

Theorem 2.6. *If G is a finite p group then $IA(G)$ is also a p group.*

Proof. Since G is a p-group, it is nilpotent. Hence $G' \leq \phi(G)$ (where $\phi(G)$ is the Frattini subgroup of G). Let σ be an IA-automorphism with order q. Suppose q does not divide the order of G'. As σ is an IA-automorphism, it preserves all cosets of G'. So by Proposition 2.5, $G = G'C$ where C is the subgroup of all elements of G which are fixed by σ. G is generated by elements of $G'C$, i.e. $G = \{\langle x, C\rangle, x \in G'\}$. But $G' \leq \phi(G)$ which is the set of all non-generators. So G is generated by elements of C. However this implies that σ is an identity on G, i.e. $o(\sigma) = 1$. This is a contradiction. So the order of every IA-automorphism divides the order of G', a non-trivial subgroup of p-group G. Hence the result follows. ■

3. Extension of a result by Schur and related results

Schur [8] proved that if $G/Z(G)$ is finite then so is G'. Hegarty [6] proved an analogous result stated below.

Let $L(G) = \{g \in G \mid \alpha(g) = g, \forall\ \alpha \in\ Aut\ G\}$ and let $G^* = \langle g^{-1}\alpha(g) \mid g \in G,\ \alpha \in Aut(G)\rangle$. Hegarty showed that if $G/L(G)$ is finite then so is G^*.

On the lines of the above results of Schur and Hegarty, a similar result is rather trivially provable for IA-automorphisms. Following Hegarty, we introduce the following

Definition 3.1. $S(G) = \{g \in G \mid \alpha(g) = g, \alpha \in IA(G)\}$.

In general, the set $S(G)$ need not be trivial as the following result shows.

Proposition 3.2. *For a finite p-group G, $S(G)$ is non-trivial.*

Proof. From Theorem 2.6, the group $IA(G)$ is a p-group. Take an action of $IA(G)$ on G in a natural way: $IA(G) \times G \longrightarrow G$ such that $(f, x) \longrightarrow f(x)$. This restricts to an action of $IA(G)$ on $G \setminus \{e\}$. By the orbit-stabilizer theorem, for any y in $G \setminus \{e\}$, $|IA(G)| =$ [order of orbit(y)].[order of stabilizer(y)]. Since $IA(G)$ is a p-group, the orbit of y has order 1 or power of p. If all orbits in $G \setminus \{e\}$ have order a positive power of p, then order of $G \setminus \{e\}$ will be divisible by p. This is a contradiction. Hence there is some y in $G \setminus \{e\}$, whose orbit under $IA(G)$ is a singleton. Such a y must be in $S(G)$. ■

We further introduce the following

Definition 3.3. $G^{**} = \langle g^{-1}\alpha(g) \mid g \in G, \alpha \in IA(G)\rangle$.

Obviously, we have $L(G) \trianglelefteq S(G) \trianglelefteq Z(G)$. Similarly $G' \leq G^{**} \leq G^{*}$. However from the definition of IA-automorphisms, $g^{-1}\alpha(g) \in G^{'}$, $\forall\ g \in\ G,\ \alpha \in IA(G)$. Therefore $G^{**} \leq G^{'}$. Thus, $G^{'} = G^{**} \leq G^{*}$.

Theorem 3.4. *If $G/S(G)$ is finite then so is G^{**}.*

Proof. We can define a homomorphism from $G/S(G)$ onto $G/Z(G)$ as $gS(G) \longrightarrow gZ(G)$, $\forall g \in G$, with kernel $Z(G)/S(G)$. So, if $G/S(G)$ is finite then $G/Z(G)$ will also be finite. By Schur's result, G' will be finite. That is G^{**} is finite. ■

However, an overall generalisation of these results is not true. That is, if $I_G \trianglelefteq N \trianglelefteq Aut(G)$ then the corresponding result may not hold for N. The following example was suggested to us by Prof. Martin Issacs in personal communication.

Example 3.5. Let G be an infinite elementary abelian p-group with minimum generating set $= \langle x_0, x_1, x_2, \dots\rangle$. Let $S_i = \{\alpha \in Aut(G) : \alpha(x_i) = x_i,$ and $\alpha(x_0) = x_0x_i\}$ for all $i > 0$. Let A be the group generated by all S_i, and $C_G[A]$ be the subgroup of G of elements fixed by A. Then $C_G[A] = \langle x_1, x_2, \dots\rangle$ has index p in G, but $[G, A] = \langle x_1, x_2, \dots\rangle$ is infinite. Since G is abelian, I_G is trivial, and hence A contains I_G. Thus, while index of $C_G[A]$ is finite(p), the order of $[G, A]$ is infinite.

A weak converse of Hegarty's result is proved below.

Theorem 3.6. *Let G be an arbitrary group. If G^{*} is finite and Aut G is finitely generated then $[G : L(G)]$ is finite.*

Proof. Let $Aut(G) = \langle\alpha_1, \alpha_2, \dots, \alpha_n\rangle$. We define $f : G/L(G) \longrightarrow G^{*} \times \dots \times G^{*}$($n$ times) by $\bar{g} \longrightarrow (g^{-1}\alpha_1(g), \dots, g^{-1}\alpha_n(g))$, where $\bar{g} = gL$, for some $g \in G$. For $\bar{x}, \bar{y} \in G/L(G)$, $\bar{x} = \bar{y} \Rightarrow xL = yL \Longrightarrow x = yl$ for some $l \in L$. Therefore $f(\bar{x}) = ((yl)^{-1}\alpha_1(yl), \dots, (yl)^{-1}\alpha_n(yl)) = (y^{-1}\alpha_1(y), \dots y^{-1}\alpha_n(y)) = f(\bar{y})$. So, f is well defined.

It is enough to prove that f is injective. For this, let $f(\bar{y}) = f(\bar{x})$ for some $\bar{x}, \bar{y} \in G/L(G)$. Thus, we have $y^{-1}\alpha_i(y) = x^{-1}\alpha_i(x)$ for all $1 \leq i \leq n$, i.e.

$\alpha_i(y) = yx^{-1}\alpha_i(x)$. But then, $\alpha_i(yx^{-1}) = \alpha_i(y)\alpha_i(x^{-1}) = yx^{-1}\alpha_i(x)\alpha_i(x^{-1}) = yx^{-1},\ 1 \le i \le n$.

$$\therefore yx^{-1} \in L(G), i.e.\ \bar{y} = \bar{x}.$$

This completes the proof. ■

As in Hegarty [6], we introduce

Definition 3.7. $Var(G) = \{\alpha \in\ Aut\ G \mid g^{-1}\alpha\ (g) \in L(G), \forall\ g \in G\}$.

On the similar lines, we introduce

Definition 3.8. $Ivar(G) = \{\alpha \in\ IA(G) \mid g^{-1}\alpha\ (g) \in S(G), \forall\ g \in G\}$.

Attar[2] proved that $Aut_c^z(G) \approx\ Hom(G/Z(G), Z(G))$, where $Aut_c^z(G)$ is the group of those central automorphisms of G, which preserve centre $Z(G)$ element-wise. A similar result replacing $Z(G)$ by $L(G)$ is true, as proved below.

Theorem 3.9. *For a finite group* G, $Var(G) \approx\ Hom(G/L(G), L(G))$.

Proof. For any $\mu \in Var(G)$, define the map $\psi_\mu : G/L(G) \longrightarrow L(G)$ as $\psi_\mu(gL(G)) = g^{-1}\mu(g)$. We first show that ψ_μ is well defined. Let $gL(G) = hL(G)$. Then $h = gl$ for some $l \in L(G)$, and $\psi_\mu(hL(G)) = h^{-1}\mu(h) = (gl)^{-1}\mu(gl) = l^{-1}g^{-1}\mu(g)l = g^{-1}\mu(g) = \psi_\mu(gL(G))$. Further, it is clear from the definition that ψ_μ is a homomorphism.

We now define a map $\psi : Var(G) \longrightarrow Hom(G/L(G), L(G))$ as $\psi(\mu) = \psi_\mu$. We show that ψ is an isomorphism. For $f, g \in Var(G)$, consider $\psi(fg)$. For $h \in G$, $\psi(fg)(hL(G)) = \psi_{fg}(hL(G)) = h^{-1}fg(h) = h^{-1}f(hh^{-1}g(h)) = h^{-1}f(h)h^{-1}g(h){=}\psi_f(hL(G)\ \psi_g(hL(G)$. $\therefore \psi(fg) = \psi(f)\psi(g)$. Consider $\psi(\mu_1) = \psi(\mu_2)$. So $\psi_{\mu_1} = \psi_{\mu_2}$, and this implies $\psi_{\mu_1}(gL(G)) = \psi_{\mu_2}(gL(G)) \Rightarrow g^{-1}\mu_1(g) = g^{-1}\mu_2(g) \Rightarrow \mu_1 = \mu_2$, as g is an arbitrary element of G. Therefore ψ is a monomorphism.

We next show that ψ is onto. For any $\tau \in\ Hom((G/L(G), L(G))$, define the map $\mu : G \to G$ as $\mu(g) = g\tau(gL(G))$. We show that $\mu \in Var(G)$.

For $g_1, g_2 \in G$, $\mu(g_1g_2) = g_1g_2\tau(g_1g_2L(G)) = g_1\tau(g_1L(G))g_2\tau(g_2L(G)) = \mu(g_1)\mu(g_2)$. $\therefore \mu$ is a homomorphism.

Further, let $\mu(g) = 1$. This implies $g\tau(gL(G)) = 1 \Rightarrow \tau(gL(G)) = g^{-1} \Rightarrow g^{-1} \in L(G)$. $\therefore gL(G) = L(G) \Rightarrow \tau(gL(G)) = 1 \Rightarrow g = 1$ as $g\tau(gL(G)) = 1$. Hence μ is one-one. As G is finite, μ is also onto.

As $g^{-1}\mu(g) = g^{-1}g\tau(gL(G)) = \tau(gL(G)) \in L(G)$, for all $g \in G$, we have $\mu \in Var(G)$. It is obvious that $\psi(\mu) = \tau$. Thus, given $\tau \in\ Hom((G/L(G), L(G))$, we can find $\mu \in Var(G)$ such that $\psi(\mu) = \tau$. Hence the result follows. ■

Corollary 3.9.1. *For a p-group G, Var(G) is also a p-group.*

Proof. As G is a p-group, $Hom(G/L(G), L(G))$ is also a p-group. Hence by above theorem, $Var(G)$ is a p-group. ■

Lastly we prove a sufficient condition for $IA(G)$ to be finite.

THEOREM 3.10. *If G is a group with $G/S(G)$ finite then so is $IA(G)$.*

Proof. Let $|G/S(G)| = n$. Then $IA(G)/Ivar(G)$ is isomorphic to a subgroup of S_n and by theorem 3.4, G^{**} is finite. We thus need only to show that G^{**} finite $\Rightarrow Ivar(G)$ is finite.

Let $\alpha \in Ivar(G)$. Define $\alpha^* : G/S(G) \longrightarrow S(G)$ as $\alpha^*(gS(G)) = g^{-1}\alpha(g)\, \forall g \in G$. Since $(yz)^{-1}\alpha(yz) = z^{-1}y^{-1}\alpha(y)z = y^{-1}\alpha(y)$ (as $z \in S(G)$), α^* is well defined. Clearly $\alpha^* \in Hom(G/S(G), S(G))$ and $\alpha^*(G/S(G))$ is a finite subgroup of $S(G)$.

Let $\sigma : Ivar(G) \longrightarrow Hom(G/S(G),\ S(G))$ be defined by $\sigma(\alpha) = \alpha^*$. Clearly, σ is a monomorphism from $Ivar(G)$ into $Hom(G/S(G), S(G))$. If $Ivar(G)$ were infinite, then $S(G)$ would contain infinitely many elements of the form $g^{-1}\alpha\ (g)$, and so G^{**} would, by definition be infinite. Hence G^{**} finite $\Longrightarrow Ivar(G)$ finite. ∎

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