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A COMMON FIXED POINT THEOREM FOR NEW TYPE COMPATIBLE MAPS ON PARTIAL METRIC SPACES

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Abstract. In this paper, we introduce the concept of compatible maps of type (I) and of type (II) in partial metric space and prove a common fixed point theorem for four such maps on complete partial metric space.

1. Introduction

In [12], Matthews introduced a new class of generalized metric spaces, which was named as partial metric spaces, in order to develop and to introduce a new fixed point theory. In partial metric spaces, the self-distance of a point may not be zero. Also, each partial metric on a nonempty set generates a T_0 topology. After the definition of partial metric spaces, Matthews proved the partial metric version of Banach fixed point theorem. Then, as can seen in [1–3,11,13,19–22], the fixed point theory studies in such spaces have been rapidly developed. In the recent paper, Ćirić et al. [4] proved a common fixed point theorem for weakly compatible mappings satisfying generalized nonlinear contractive condition in complete partial metric spaces. In this paper, we introduce the concept of compatible maps of type (I) and of type (II) on partial metric spaces and we give some examples to illustrate that weakly compatible and these two types of compatible maps are independent. Then using these new concepts, we prove a common fixed point theorem on partial metric spaces.

First we recall some definitions and some properties of partial metric spaces.

A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ (nonnegative real numbers) such that for all $x, y, z \in X$:

(i) $x = y \iff p(x,x) = p(x,y) = p(y,y)$ (T_0-separation axiom), (ii) $p(x,x) \leq p(x,y)$ (small self-distance axiom), (iii) $p(x,y) = p(y,x)$ (symmetry) and (iv) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$ (modified triangular inequality).

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A partial metric spaces (for short PMS) is a pair (X, p) such that X is a nonempty set and p is a partial metric on X. It is clear that, if $p(x, y) = 0$, then, from (i) and (ii), $x = y$. But if $x = y$, $p(x, y)$ may not be 0. A basic example of a PMS is the pair $(\mathbb{R}^+ = [0, \infty), p)$, where $p(x, y) = \max\{x, y\}$ for all $x, y \in \mathbb{R}^+$. For another example, let I denote the set of all intervals $[a, b]$ for any real numbers $a \leq b$. Let $p : I \times I \to \mathbb{R}^+$ be the function such that $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (I, p) is a PMS. Other examples may be found in [5–7,12].

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x, \varepsilon) : x \in X, \varepsilon > 0\}$, where

$$B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$$

for all $x \in X$ and $\varepsilon > 0$.

Definition 1. (i) A sequence $\{x_n\}$ in a PMS (X, p) is called a Cauchy sequence if there exists $\lim_{n,m\to\infty} p(x_n, x_m)$ (and is finite).

(ii) A PMS (X, p) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

Remark 1. (i) A sequence $\{x_n\}$ in a PMS (X, p) converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.

(ii) A mapping $F : X \to X$ is said to be continuous at $x_0 \in X$, if for every $\varepsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x_0, \delta)) \subseteq B_p(Fx_0, \varepsilon)$.

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y)$$

is a metric on X. For example, if we consider $p(x, y) = \max\{x, y\}$ on $\mathbb{R}^+$, then it is clear that $p^s(x, y) = |x - y|$. Furthermore, $\lim_{n\to\infty} p^s(x_n, x) = 0$ if and only if

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m).$$

The following lemmas play an important role in obtaining fixed point results on a PMS.

Lemma 1. [12,13] *Let (X, p) be a PMS.*

(a) *$\{x_n\}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, p^s).*

(b) *(X, p) is complete if and only if (X, p^s) is complete.*

Lemma 2. [11] *If $\{x_n\}$ converges to x in (X, p), then $\lim_{n\to\infty} p(x_n, y) \leq p(x, y)$ for all $y \in X$.*

2. Various definitions of compatibility

In this section, first, we recall some definitions and properties of various compatibilities in metric space.

DEFINITION 2. Let (X,d) be a metric space and $A,S\colon X\to X$ be two mappings. Then the pair (A,S) is said to be:

(a) commuting if $ASx = SAx$ for all $x\in X$ and weakly commuting [8] if $d(ASx,SAx)\le d(Ax,Sx)$ for all $x\in X$,

(b) compatible [8] if $\lim_{n\to\infty} d(ASx_n,SAx_n)=0$,

(c) compatible of type (A) [10] if

$$\lim_{n\to\infty} d(ASx_n,SSx_n)=0 \text{ and } \lim_{n\to\infty} d(SAx_n,AAx_n)=0,$$

(d) compatible of type (B) [16] if

$$\lim_{n\to\infty} d(STx_n,TTx_n)\le \frac{1}{2}[\lim_{n\to\infty} d(STx_n,St)+\lim_{n\to\infty} d(St,SSx_n)]$$

and

$$\lim_{n\to\infty} d(TSx_n,SSx_n)\le \frac{1}{2}[\lim_{n\to\infty} d(TSx_n,Tt)+\lim_{n\to\infty} d(Tt,TTx_n)],$$

(e) compatible of type (P) [17] if $\lim_{n\to\infty} d(AAx_n,SSx_n)=0$,

(f) compatible of type (I) [15] if $d(t,St)\le \overline{\lim}_{n\to\infty} d(t,ASx_n)$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n=\lim_{n\to\infty} Sx_n=t$ for some $t\in X$,

(g) compatible of type (II) [15] if (S,A) is compatible of type (I),

(h) weakly compatible [9] if $ASx=SAx$, whenever $Ax=Sx$ for some $x\in X$.

REMARK 2. It is well known that commuting mappings are weakly commuting and weakly commuting mappings are compatible, but neither implication is reversible. Also, if the pair (A,S) is compatible, compatible of type (A), compatible of type (B) or compatible of type (P), then it is weakly compatible. There are some examples in [18] showing that these implications are not reversible. But weakly compatible maps, compatible of type (I) and of type (II) are independent from each other. There are some examples in [14] showing this fact.

Using the concept of weakly compatible maps, Ćirić et al. [4] proved some fixed point result in partial metric spaces. In this paper we introduce the concepts of compatible maps of type (I) and of type (II) in partial metric spaces and show that weakly compatible maps, compatible of type (I) and of type (II) are independent from each other as in metric spaces. Later, we prove a common fixed point theorem using these new concepts.

DEFINITION 3. Let (X,p) be a partial metric space and $A,S\colon X\to X$ be two mappings. Then the pair (A,S) is said to be compatible of type (I) if

$$p(t,St)\le \overline{\lim_{n\to\infty}}\, p(t,ASx_n)$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} p(Ax_n, t) = \lim_{n\to\infty} p(Sx_n, t) = p(t,t)$$

for some $t \in X$. The pair (A, S) is said to be compatible of type (II) if and only if (S, A) is compatible of type (I).

Considering examples given in [14], which show that the concepts of weakly compatible maps, compatible of type (I) and of type (II) are independent in a metric space, we provide the following examples.

EXAMPLE 1. Let $X = [0, \infty)$ be equipped with $p(x, y) = \max\{x, y\}$. Define $A, S\colon X \to X$ by

$$Ax = 2x + 1 \text{ and } Sx = x^2 + 1.$$

Then at $x = 0$, $Ax = Sx$. Also $ASx = 3$ and $SAx = 2$, which shows that A and S are not weakly compatible. Now suppose that $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} p(Ax_n, t) = \lim_{n\to\infty} p(Sx_n, t) = p(t, t)$ for some $t \in X$. By definition of A and S, $t \in [1, 5]$. For $t = 1$ we have $p(t, St) = 2 \le 3 = \overline{\lim}_{n\to\infty} p(t, ASx_n)$, which shows that the pair (S, T) is a pair of compatible mappings of type (I).

EXAMPLE 2. Let $X = [0, \infty)$ be equipped with $p(x, y) = \max\{x, y\}$. Define $A, S\colon X \to X$ by

$$Ax = \begin{cases} \cos x & \text{if } x \ne 1 \\ 0 & \text{if } x = 1 \end{cases} \text{ and } Sx = \begin{cases} e^x & \text{if } x \ne 1 \\ 0 & \text{if } x = 1. \end{cases}$$

Then it is clear that $Ax = Sx$ if and only if $x = 0$ and $x = 1$. Also at these points $ASx = SAx$. It means that A and S are weakly compatible. Now suppose that $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} p(Ax_n, t) = \lim_{n\to\infty} p(Sx_n, t) = p(t, t)$ for some $t \in X$. By definition of A and S, $t \ge 1$. For this value we have $p(t, St) = e^t$ and $\overline{\lim}_{n\to\infty} p(t, ASx_n) = t < e^t$. Therefore the pair (A, S) is not a compatible pair of mappings of type (I).

PROPOSITION 1. *Let $A, S\colon X \to X$ be such that the pair (A, S) is compatible of type (I) (resp. of type (II)) and $Az = Sz$ for some $z \in X$. Then $p(Az, SSz) \le p(Az, ASz)$ (resp. $p(Sz, AAz) \le p(Sz, SAz)$).*

Proof. Let $\{x_n\}$ be a sequence in X defined by $x_n = z$ for $n = 0, 1, 2, \dots$ and $Az = Sz$ for some $z \in X$. Then we have

$$\lim_{n\to\infty} p(Ax_n, Az) = \lim_{n\to\infty} p(Sx_n, Az) = p(Az, Az)$$

Suppose that (A, S) is compatible of type (I). Then

$$p(Az, SSz) \le p(Az, SAz) \le \overline{\lim_{n\to\infty}} p(Az, ASx_n) = p(Az, ASz).$$

Similarly, if the pair (A, S) is compatible of type (II), it can be easily shown that $p(Sz, AAz) \le p(Sz, SAz)$. ∎

3. The main result

In the sequel, for a partial metric space (X,p) and for maps A,B,S and $T\colon X\to X$, we define

$$M(x,y)=\max\{p(Sx,Ty),p(Ax,Sx),p(By,Ty),\tfrac{1}{2}[p(Sx,By)+p(Ax,Ty)]\}$$

and

$$M^*(x,y)=\max\{p(Sx,Ty),\tfrac{1}{2}p(Ax,Sx),\tfrac{1}{2}p(By,Ty),\tfrac{1}{2}[p(Sx,By)+p(Ax,Ty)]\}$$

for all $x,y\in X$. It is clear that $M^*(x,y)\le M(x,y)$ for all $x,y\in X$.

Theorem 1. *Let A,B,S and T be self maps of a complete partial metric space (X,p) such that $AX\subseteq TX$, $BX\subseteq SX$ and*

$$p(Ax,By)\le\varphi(M^*(x,y)) \tag{3.1}$$

for all $x,y\in X$, where $\varphi\colon\mathbb{R}^+\to\mathbb{R}^+$ is upper semicontinuous, non-decreasing and $\varphi(t)<t$ for $t>0$. Suppose one of the following is satisfied:

(a_1) *A is continuous and the pairs (A,S) and (B,T) are compatible of type (II),*

(a_2) *B is continuous and the pairs (A,S) and (B,T) are compatible of type (II),*

(a_3) *S is continuous and the pairs (A,S) and (B,T) are compatible of type (I),*

(a_4) *T is continuous and the pairs (A,S) and (B,T) are compatible of type (I).*

Then A,B,S and T have a unique fixed point in X.

Proof. Let $x_0\in X$ be an arbitrary point. Using $AX\subseteq TX$, $BX\subseteq SX$ we can define two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n}=Tx_{2n+1}=Ax_{2n} \text{ and } y_{2n+1}=Sx_{2n+2}=Bx_{2n+1},\ n=0,1,2,\dots$$

Now since φ is non-decreasing and $M^*(x,y)\le M(x,y)$ for all $x,y\in X$, we have $\varphi(M^*(x,y))\le\varphi(M(x,y))$ for all $x,y\in X$. Therefore as in the proof of Theorem 2.1 of [4], we obtain that $\{y_n\}$ is a Cauchy sequence in (X,p). Therefore there exists $z\in X$ such that

$$p(z,z)=\lim_{n\to\infty}p(y_n,z)=\lim_{n,m\to\infty}p(y_n,y_m).$$

Again as in the proof of Theorem 2.1 of [4] we obtain

$$p(z,z)=\lim_{n\to\infty}p(y_n,z)=\lim_{n,m\to\infty}p(y_n,y_m)=0$$

and

$$\begin{aligned}\lim_{n\to\infty}p(Ax_{2n},z)&=\lim_{n\to\infty}p(Bx_{2n+1},z)=\lim_{n\to\infty}p(Sx_{2n},z)\\&=\lim_{n\to\infty}p(Tx_{2n+1},z)=0.\end{aligned}$$

Now suppose that the condition (a_4) holds. Then, since the pair (B,T) is compatible of type (I) and T is continuous, we have

$$p(z,Tz) \leq \overline{\lim_{n\to\infty}} p(z,BTx_{2n+1}) \text{ and } \lim_{n\to\infty} p(TTx_{2n+1},Tz) = p(Tz,Tz).$$

Setting $x = x_{2n}$ and $y = y_{2n}$ in (3.1), we obtain

$$\begin{aligned} p(Ax_{2n},By_{2n}) = p(Ax_{2n},BTx_{2n+1}) &\leq \varphi(M(x_{2n},Tx_{2n+1})) \\ &\leq \varphi(\max\{p(Sx_{2n},TTx_{2n+1}),\tfrac{1}{2}p(Ax_{2n}, \\ &\quad Sx_{2n}),\tfrac{1}{2}p(BTx_{2n+1},TTx_{2n+1}), \\ &\quad \tfrac{1}{2}[p(Sx_{2n},BTx_{2n+1})+p(Ax_{2n},TTx_{2n+1})]\}) \\ &\leq \varphi(\max\{p(Sx_{2n},TTx_{2n+1}),\tfrac{1}{2}p(Ax_{2n},Sx_{2n}), \\ &\quad \tfrac{1}{2}[p(BTx_{2n+1},Ax_{2n})+p(Ax_{2n},TTx_{2n+1})], \\ &\quad \tfrac{1}{2}[p(Sx_{2n},BTx_{2n+1})+p(Ax_{2n},TTx_{2n+1})]\}) \end{aligned}$$

Suppose $\overline{\lim}_{n\to\infty} p(z,BTx_{2n+1}) \neq 0$. Then by taking the limit superior on both sides of above inequality we have

$$\begin{aligned} \overline{\lim_{n\to\infty}} p(z,BTx_{2n+1}) &\leq \varphi\max(\{p(z,Tz),\tfrac{1}{2}p(z,z), \\ &\quad \tfrac{1}{2}[\overline{\lim_{n\to\infty}} p(BTx_{2n+1},z)+p(z,Tz)], \\ &\quad \tfrac{1}{2}[\overline{\lim_{n\to\infty}} p(z,BTx_{2n+1})+p(z,Tz)]\}) \\ &\leq \varphi(\max\{p(z,Tz),\tfrac{1}{2}[\overline{\lim_{n\to\infty}} p(BTx_{2n+1},z)+p(z,Tz)]\}) \\ &\leq \varphi(\max\{p(z,Tz),\overline{\lim_{n\to\infty}} p(BTx_{2n+1},z)\}) \\ &= \varphi(\overline{\lim_{n\to\infty}} p(BTx_{2n+1},z)) \\ &< \overline{\lim_{n\to\infty}} p(z,BTx_{2n+1}), \end{aligned}$$

a contradiction. Thus we have $\overline{\lim}_{n\to\infty} p(z,BTx_{2n+1}) = 0$. Since the pair (B,T) is compatible of type (I) it follows that $p(z,Tz) = 0$, that is, $z = Tz$.

Again replacing x by x_{2n} and y by z in (3.1) and allowing $n\to\infty$, we have

$$\begin{aligned} p(Ax_{2n},Bz) &\leq \varphi(M(x_{2n},z)) \\ &\leq \varphi(\max\{p(Sx_{2n},Tz),\tfrac{1}{2}p(Ax_{2n},Sx_{2n}),\tfrac{1}{2}p(Bz,Tz), \\ &\quad \tfrac{1}{2}[p(Sx_{2n},Bz)+p(Ax_{2n},Tz)]\}). \end{aligned}$$

So we obtain

$$\begin{aligned} p(z,Bz) &\leq \varphi(\max\{p(z,Tz),\tfrac{1}{2}p(z,z),\tfrac{1}{2}p(Bz,Tz), \\ &\quad \tfrac{1}{2}[p(z,Bz)+p(z,Tz)]\}) \\ &\leq \varphi\max(\{p(z,Tz),\tfrac{1}{2}p(z,z),\tfrac{1}{2}\left[p(Bz,z)+p(z,Tz)\right], \end{aligned}$$

$$\frac{1}{2}[p(z,Bz)+p(z,Tz)]\})$$
$$\leq \varphi(\tfrac{1}{2}p(z,Bz)) < \tfrac{1}{2}p(z,Bz)$$

and hence $z = Bz$. Since $BX \subseteq SX$ there exists a point $u \in X$ such that $Bz = Su = z$. By (3.1), we have, if $p(Au,z) > 0$,

$$\begin{aligned} p(Au,z) = p(Au,Bz) &\leq \varphi(M(u,z)) \\ &\leq \varphi(\max\{p(Su,Tz), \tfrac{1}{2}p(Au,Su), \tfrac{1}{2}p(Bz,Tz), \\ &\quad \tfrac{1}{2}[p(Su,Bz)+p(Au,Tz)]\}) \\ &\leq \varphi(\max\{p(Su,Tz), \tfrac{1}{2}p(Au,Su), \\ &\quad \tfrac{1}{2}[p(Bz,z)+p(z,Tz)], \tfrac{1}{2}[p(Su,Bz)+p(Au,Tz)]\}) \\ &\leq \tfrac{1}{2}p(Au,z), \end{aligned}$$

a contradiction. Thus $Au = z$. Since the pair (A,S) is compatible of type (I) and $Au = Su = z$, by Proposition 1 $p(Au,SSu) \leq p(Au,ASu)$ and so we have $p(z,Sz) \leq p(z,Az)$. Again by (3.1) we have, if $Az \neq z$,

$$\begin{aligned} p(z,Az) = p(Az,Bz &\leq \varphi(M(z,z)) \\ &\leq \varphi(\max\{p(Sz,Tz), \tfrac{1}{2}p(Az,Sz), \tfrac{1}{2}p(Bz,Tz), \\ &\quad \tfrac{1}{2}[p(Sz,Bz)+p(Az,Tz)]\}) \\ &\leq \varphi(p(Sz,z)) \leq \varphi\,(p(z,Az)) < p(z,Az), \end{aligned}$$

which implies that $Az = z$. Therefore $Az = Bz = Sz = Tz = z$ and z is a common fixed point of A,S,B,T. Now we prove the uniqueness of a common fixed point. Let us suppose that z and w are two common fixed points of A,S,B and T, with $p(z,w) > 0$. Using (3.1), we get

$$\begin{aligned} p(z,w) = p(Az,Bw) &\leq \varphi(M(z,w)) \\ &\leq \varphi(\max\{p(Sz,Tw), \tfrac{1}{2}p(Az,Sz), \tfrac{1}{2}p(Bw,Tw), \\ &\quad \tfrac{1}{2}[p(Sz,Bw)+p(Az,Tw)]\}) \\ &\leq \varphi(\max\{p(z,w), \tfrac{1}{2}p(z,z), \tfrac{1}{2}p(w,w), \tfrac{1}{2}[p(z,w)+p(z,w)]\}) \\ &\leq \varphi(\max\{p(z,w), \tfrac{1}{2}p(z,z), \tfrac{1}{2}p(w,w)\}) \\ &\leq \varphi(p(z,w)) < p(z,w), \end{aligned}$$

which is a contradiction. Then we deduce that $z = w$. ■

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