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A NOTE RELATED TO A PAPER OF NOIRI

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In [4] Noiri gave a counterexample to Lemma 1.1 in [1] which reads: If $f : X \to Y$ is an almost closed and almost continuous mapping, then $f^{-1}(V)$ is regularly open (regularly closed) in X for each regularly open (regularly closed) set V in Y. In this counterexample f is not a surjection. There exists also another counterexample, where f is a surjection. There exists also another counterexample, where f is a surjection (Example 1 in [2]). But, Lemma A is necessarily true if a new condition is added.

LEMMA. *If $f : X \to Y$ is an almost closed almost continuous and α-irreducible mapping of a space X onto a space Y, then $f^{-1}(V)$ is regularly open* (*regularly closed*) *set V in Y* (Corollary 1 in [2]).

If f is an α-irreducible mapping with other properties as in Theorem B of [4] (Theorem 2.1 in [1] and Lemma 5 in [3]), then Theorem B is true (Theorem 1 in [2]).

Noiri in [4] has asked the following question: If X is an almost regular space and $f : X \to Y$ is an almost continuous and almost closed surjection such that $f^{-1}(y)$ is α-nearly compact for each point $y \in Y$, is Y almost regular (Theorem B (1) in [4]. Lemma 5 in [3])?

We construct here an example answering this question.

EXAMPLE. Let $X = \{n, \alpha, \beta, \gamma : n \in N\}$. Let β and each point n be isolated. Let $\{\alpha, \beta\}$ be the fundamental system of neighborhoods of α and let the fundamental system of neighborhoods of γ be $\{V^k(\gamma) : k \in N\}$ where $V^k(\gamma) = \{\gamma, n : n \geq k\}$. The space X is almost regular but not regular (for the closed set $\{\alpha\}$ and $\beta \notin \{\alpha\}$ there are no disjoint open neighborhoods).

Let $Y = \{\alpha, \beta, n : n \in N\}$. Let β and each point n be isolated. Let $\{V^k(\alpha) : k \in N\}$ be the fundamental system of neighborhoods of α, where $V^k(\alpha) = \{\alpha, \beta, n : n \geq k\}$. The space Y is not almost regular (for the regularly closed set $N \cup \{\alpha\}$ and $\beta \notin N \cup \{\alpha\}$ there are no disjoint open neighborhoods). Let $f : X \to Y$ be a mapping of the space X onto the space Y defined by: $f(n) = n$, $n \in N$; $f(\beta) = \beta$;

$f(\alpha) = f(\gamma) = \alpha$. The mapping f is an almost closed and almost continuous (in fact f is closed and continuous) mapping of the almost regular space X onto the space Y such that $f^{-1}(y)$ is α-nearly compact (in fact compact) for each $y \in Y$, but Y is not almost regular. Note that the inverse image of the regularly open set N in Y is not regularly open in X $(\mathrm{Int}\,(\mathrm{Cl}\,(N)) = \mathrm{Int}\,(N \cup \{\gamma\}) = N \cup \{\gamma\})$. By using Lemma A, I have proved some theorems are not correct. Further comments on, when these theorems are true can be found in [2].

REFERENCES

[1] I. Kovačević, *Almost continuity and nearly (almost) paracompactness*, Publ. Inst. Math. (Beograd) (N.S.) **30** (**44**) (1981), 73–79.

[2] I. Kovačević, *A note on almost closed mappings and nearly paracompactness*, to appear in: Univ. u Novom Sadu Zb. Rad. Prirod. Mat. Fak.

[3] I. Kovačević, *Locally nearly paracompact spaces*, Publ. Inst. Math. (Beograd) (N.S.) **29** (**43**) (1981), 117–124.

[4] T. Noiri, *A note on inverse-preservations of regular open sets*, Publ. Inst. Math. (Beograd) (N.S.), this issue.

Errata to the papers

1. *On nearly strongly paracompact spaces*, Publ. Inst. Math. (Beograd) (N.S.) **27** (**41**) (1980), 125–134.

2. *Locally nearly paracompact spaces*, Publ. Inst. Math. (Beograd) (N.S.) **29** (**43**) (1981), 117–124 (by Ilija Kovačević).

After these papers appeared it was noted that besides some typographical errors which can be easily seen and corrected, a few parts of the text have been omitted in the process of printing. In order to complete these papers we give here the integral text of the corresponding statements.

In the first paper:

THEOREM 2.1. *Let X be an almost regular space such that every open star finite family is closure preserving and let A be any α-nearly strongly paracompact subset of X. Then $\bar{A}$ is an α-nearly strongly paracompact subset of X.*

THEOREM 2.6. *The product of an α-nearly strongly paracompact subset and an α-nearly compact subset is α-nearly strongly paracompact.*

PROOF. It is similar to the proof of the corresponding theorem for nearly strongly paracompact and nearly compact spaces (see [1]).

THEOREM 3.6. *Let E be an α-nearly strongly paracompact subset of a locally nearly strongly paracompact almost regular space X such that every open star finite family is closure preserving; and let ...*

Also in 133_6 instead of "closed star finite cover" read "closed cover".

In the second paper:

COROLLARY 1. *If X is an almost regular topological space such that every open star finite family is closure preserving, and A is an α-nearly strongly paracompact subset and U a regular open neighborhood of A, then there exists a regular closed neighborhood V of A such that $A \subset V \subset U$.*

PROOF. It is similar to the proof of Theorem 2.

Also in 120^1 instead of "Every locally nearly" read "Every Hausdorff locally nearly" and in 120^7 instead of "But X is not locally strongly" read "But X is not locally nearly strongly".

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