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ON A TYPE OF SEMISYMMETRIC METRIC CONNECTION ON A RIEMANNIAN MANIFOLD

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Abstract. We study a type of semisymmetric metric connection on a Riemannian manifold whose torsion tensor is almost pseudo symmetric and the associated 1-form of almost pseudo symmetric manifold is equal to the associated 1-form of the semisymmetric metric connection.

1. Introduction

In 1924, Friedmann and Schouten [**13**] introduced the idea of semisymmetric connection on a differentiable manifold. Let M be an n-dimensional Riemannian manifold of class C^∞ endowed with a Riemannian metric g.

A linear connection $\bar{\nabla}$ defined on (M^n, g) is said to be semisymmetric [**13**] if its torsion tensor T is of the form $T(X,Y) = \pi(Y)X - \pi(X)Y$, where π is a 1-form and ρ is a vector field given by $\pi(X) = g(X,\rho)$, for all vector fields $X \in \chi(M)$, $\chi(M)$ being the set of all differentiable vector fields on M.

In 1932, Hayden [**14**] introduced the idea of semisymmetric metric connections on a Riemannian manifold (M,g). A semisymmetric connection $\bar{\nabla}$ is called a semisymmetric metric connection [**14**] if it further satisfies $\bar{\nabla} g = 0$. The study of the semisymmetric metric connection was further developed by Yano [**28**], Amur and Pujar [**1**], Prvanović [**19, 20, 21**], Prvanović and Pušić [**22**], Chaki and Konar [**5**], De [**8, 7**], De and Biswas [**11**], De and De [**9, 10**], Sharfuddin and Hussain [**25**], Binh [**2**], Özen, Aynur and Altay [**16**], Ozgur and Sular [**17, 18**] and many others.

We consider a type of the semisymmetric metric connection introduced by Yano [**28**]. A relation between the semisymmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ on (M,g) has been obtained by Yano [**28**] and it is given by $\bar{\nabla}_X Y = \nabla_X Y + \pi(Y)X - g(X,Y)\rho$. We also have [**28**]

$$(\bar{\nabla}_X \pi)(Y) = (\nabla_X \pi)Y - \pi(X)\pi(Y) + \pi(\rho)g(X,Y). \tag{1.1}$$

Further, a relation between the curvature tensor $\bar{R}$ of the semisymmetric metric connection $\bar{\nabla}$ and the curvature tensor R of the Levi-Civita connection ∇ [**28**] is

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given by

(1.2) $\bar{R}(X,Y)Z = R(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X + g(X,Z)QY - g(Y,Z)QX,$

where α is a tensor field of type (0,2) and Q is a tensor field of type (1,1) given by

$$\alpha(Y,Z) = g(QY,Z) = (\nabla_Y \pi)(Z) - \pi(Y)\pi(Z) + \tfrac{1}{2}\pi(\rho)g(Y,Z). \tag{1.3}$$

Combining (1.2) and (1.3), we obtain

$$\begin{aligned}\widetilde{\bar{R}}(X,Y,Z,W) = \widetilde{R}(X,Y,Z,W) - \alpha(Y,Z)g(X,W) + \alpha(X,Z)g(Y,W)\\ - g(Y,Z)\alpha(X,W) + g(X,Z)\alpha(Y,W),\end{aligned} \tag{1.4}$$

where $\widetilde{\bar{R}}(X,Y,Z,W) = g(\bar{R}(X,Y)Z,W)$ and $\widetilde{R}(X,Y,Z,W) = g(R(X,Y)Z,W)$.

In 1967, Sen and Chaki [**24**] studied curvature restrictions of a certain kind of conformally flat Riemannian space of class one and they obtained the following expression of the covariant derivative of the curvature tensor

$$R^h_{ijk,l} = 2\lambda_l R^h_{ijk} + \lambda_i R^h_{ljk} + \lambda_j R^h_{ilk} + \lambda_k R^h_{ijl} + \lambda^h R_{lijk}, \tag{1.5}$$

where R^h_{ijk} are the components of the curvature tensor R, $R_{ijkl} = g_{hl}R^h_{ijk}$, λ_i is a nonzero covariant vector and "," denotes covariant differentiation with respect to the metric tensor g_{ij}.

Later in 1987, Chaki [**4**] called a manifold whose curvature tensor satisfies (1.5), a pseudo symmetric manifold. In index free notation, this can be stated as follows: A nonflat Riemannian manifold (M,g), $n \geqslant 2$ is said to be a pseudo symmetric manifold [**4**] if its curvature tensor R satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z)W = 2A(X)R(Y,Z)W + A(Y)R(X,Z)W\\ + A(Z)R(Y,X)W + A(W)R(Y,Z)X + g(R(Y,Z)W,X)U,\end{aligned}$$

where A is a nonzero 1-form, U is a vector field defined by $A(X) = g(X,U)$, for all X, and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. The 1-form A is called the associated 1-form of the manifold. If $A = 0$, then the manifold reduces to a symmetric manifold in the sense of Cartan [**3**]. An n-dimensional pseudo symmetric manifold is denoted by $(PS)_n$. In this context, we can mention that the notion of weakly symmetric manifold was introduced by Tamassy and Binh [**26**]. Such a manifold was denoted by $(WS)_n$.

In a recent paper De and Gazi [**12**] introduced a type of nonflat Riemannian manifold (M^n,g), $n \geqslant 2$ whose curvature tensor R of type (1,3) satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y,Z)W = [A(X) + B(X)]R(Y,Z)W + A(Y)R(X,Z)W\\ + A(Z)R(Y,X)W + A(W)R(Y,Z)X + g(R(Y,Z)W,X)U,\end{aligned}$$

where A, U and ∇ have the meaning already mentioned and B is a nonzero 1-form, V is a vector field defined by $B(X) = g(X,V)$, for all X.

Such a manifold was called an almost pseudo symmetric manifold and was denoted by $(APS)_n$.

If $B = A$, then from the definitions it follows that $(APS)_n$ reduces to a $(PS)_n$. In the same paper the authors constructed two nontrivial examples of $(APS)_n$. It may be mentioned that almost pseudo symmetric manifolds are not a particular case of weakly symmetric manifolds.

A Riemannian manifold of quasiconstant curvature was given by Chen and Yano [**6**] as a conformally flat manifold with the curvature tensor $\widetilde{R}$ of type $(0,4)$ which satisfies the condition

$$\begin{aligned}\widetilde{R}(X,Y,Z,W) = p[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] \\ + q[g(X,W)T(Y)T(Z) - g(X,Z)T(Y)T(W) \\ + g(Y,Z)T(X)T(W) - g(Y,W)T(X)T(Z)],\end{aligned} \tag{1.6}$$

where p and q are scalar functions, T is a nonzero 1-form and λ is a unit vector field defined by $g(X,\lambda) = T(X)$.

It can be easily seen that if the curvature tensor $\widetilde{R}$ has the form (1.6), then the manifold is conformally flat. On the other hand, Vranceanu [**27**] defined the notion of almost constant curvature by the same expression (1.6). Later Mocanu [**15**] pointed out that the manifold introduced by Chen and Yano [**6**] and the manifold introduced by Vranceanu [**27**] are the same. Hence a Riemannian manifold is said to be a quasiconstant curvature manifold if the curvature tensor $\widetilde{R}$ satisfies the relation (1.6). If putting $q = 0$ in (1.6), then the manifold reduces to a manifold of constant curvature.

Here we consider a Riemannian manifold endowed with a type of semisymmetric metric connection whose torsion tensor is almost pseudo symmetric.

After introduction in Section 2, we first obtain the expressions of the curvature tensor and the Ricci tensor of the semisymmetric metric connection. In this section we prove that if a Riemannian manifold admits a semisymmetric metric connection whose curvature tensor vanishes and the torsion tensor is almost pseudo symmetric with respect to the semisymmetric metric connection, the associated 1-form A of the manifold is equal to the associated 1-form π of the semisymmetric metric connection, then the manifold is of quasiconstant curvature with respect to the Levi-Civita connection and also determine the sectional curvature of the plane by two vectors. Finally, we investigate a Riemannian manifold admitting a semisymmetric metric connection whose torsion tensor is almost pseudo symmetric and the Ricci tensor is symmetric with respect to the semisymmetric metric connection.

2. Expression for the curvature tensor of the semisymmetric metric connection

Definition 2.1. Let (M^n, g), $(n > 3)$ be a Riemannian manifold admitting a semisymmetric metric connection whose torsion tensor is almost pseudo symmetric, that is,

$$\begin{aligned}(\bar{\nabla}_X T)(Y,Z) = [A(X) + B(X)]T(Y,Z) + A(Y)T(X,Z) \\ + A(Z)T(Y,X) + g(T(Y,Z),X)U,\end{aligned} \tag{2.1}$$

where A and B are defined earlier.

Contrating Y in (2.1), it follows that

$$(n-1)(\bar{\nabla}_X\pi)Z = (n+1)A(X)\pi(Z) + (n-1)B(X)\pi(Z) + (n-2)A(Z)\pi(X) - \pi(U)g(X,Z). \tag{2.2}$$

Combining (1.1) and (1.3), we get

$$(\bar{\nabla}_X\pi)Z = \alpha(X,Z) + \tfrac{1}{2}\pi(\rho)g(X,Z). \tag{2.3}$$

Using (2.3) in (2.2) yields,

$$(n-1)\alpha(X,Z) = (n+1)A(X)\pi(Z) + (n-1)B(X)\pi(Z) + (n-2)A(Z)\pi(X) - \pi(J)g(X,Z), \tag{2.4}$$

where $J = \frac{1}{2}(n-1)\rho + U$. From (1.4), we have

$$\begin{aligned}(n-1)\tilde{\tilde{R}}(X,Y,Z,W) = {} & (n-1)\tilde{R}(X,Y,Z,W) - (n-1)\alpha(Y,Z)g(X,W) \\ & + (n-1)\alpha(X,Z)g(Y,W) - (n-1)g(Y,Z)\alpha(X,W) \\ & + (n-1)g(X,Z)\alpha(Y,W).\end{aligned} \tag{2.5}$$

Using (2.4) in (2.5), we obtain

$$\begin{aligned}(n-1)\tilde{\tilde{R}}(X,Y,Z,W) = {} & (n-1)\tilde{R}(X,Y,Z,W) - (n+1)[A(Y)\pi(Z)g(X,W) \\ & - A(X)\pi(Z)g(Y,W) + A(X)\pi(W)g(Y,Z) \\ & - A(Y)\pi(W)g(X,Z)] - (n-1)[B(Y)\pi(Z)g(X,W) \\ & - B(X)\pi(Z)g(Y,W) + B(X)\pi(W)g(Y,Z) \\ & - B(Y)\pi(W)g(X,Z)] - (n-2)[A(Z)\pi(Y)g(X,W) \\ & - A(Z)\pi(X)g(Y,W) + A(W)\pi(X)g(Y,Z) \\ & - A(W)\pi(Y)g(X,Z)] + 2\pi(J)[g(X,W)g(Y,Z) \\ & - g(X,Z)g(Y,W)].\end{aligned} \tag{2.6}$$

Now we take that the associated 1-form A of the manifold is equal to the associated 1-form π of the semisymmetric metric connection. Then from (2.6), it follows that

$$\begin{aligned}(n-1)\tilde{\tilde{R}}(X,Y,Z,W) = {} & (n-1)\tilde{R}(X,Y,Z,W) - (2n-1)[\pi(Y)\pi(Z)g(X,W) \\ & - \pi(X)\pi(Z)g(Y,W) + \pi(X)\pi(W)g(Y,Z) \\ & - \pi(Y)\pi(W)g(X,Z)] - (n-1)[B(Y)\pi(Z)g(X,W) \\ & - B(X)\pi(Z)g(Y,W) + B(X)\pi(W)g(Y,Z) \\ & - B(Y)\pi(W)g(X,Z)] + (n+1)\pi(\rho)[g(X,W)g(Y,Z) \\ & - g(X,Z)g(Y,W)].\end{aligned} \tag{2.7}$$

Putting $X = W = e_i$ in (2.7), where $\{e_i\}$, $1 \leqslant i \leqslant n$ is an orthonormal basis of the tangent space at any point of the manifold M^n and then summing over i, we obtain

$$(2.8)\qquad \begin{aligned}(n-1)\bar{S}(Y,Z) = {} & (n-1)S(Y,Z) - (2n-1)(n-2)\pi(Y)\pi(Z) \\ & - (n-1)(n-2)B(Y)\pi(Z) - (n-1)\pi(V)g(Y,Z) \\ & + n(n-2)\pi(\rho)g(Y,Z),\end{aligned}$$

where $\bar{S}$ and S are the Ricci tensors with respect to the semisymmetric metric connection and the Levi-Civita connection respectively.

Thus from the above discussions we can state the following:

PROPOSITION 2.1. *If a Riemannian manifold admits a semisymmetric metric connection whose torsion tensor is almost pseudo symmetric with respect to the semisymmetric metric connection and the associated* 1*-form* A *is equal to the associated* 1*-form* π *of the semisymmetric metric connection* $\bar{\nabla}$*, then*

(i) *the curvature tensor* $\bar{R}$ *of* $\bar{\nabla}$ *is given by* (2.7)*,*
(ii) *the Ricci tensor* $\bar{S}$ *of* $\bar{\nabla}$ *is given by* (2.8)*,*
(iii) *the Ricci tensor* $\bar{S}$ *with respect to the semisymmetric metric connection is symmetric if and only if* $B(X)\pi(Y) = B(Y)\pi(X)$*.*

Next, we prove the following:

THEOREM 2.1. *If a Riemannian manifold admits a semisymmetric metric connection whose curvature tensor vanishes and the torsion tensor is almost pseudo symmetric with respect to the semisymmetric metric connection and the associated* 1*-form* A *of the manifold is equal to the associated* 1*-form* π *of the semisymmetric metric connection, then the manifold is of quasiconstant curvature with respect to the Levi-Civita connection.*

PROOF. Suppose $\tilde{\bar{R}} = 0$, then the Ricci tensor with respect to the semisymmetric metric connection is also zero, that is, $\bar{S} = 0$. Using this in (2.8), we get

$$(2.9)\qquad B(X)\pi(Y) = B(Y)\pi(X).$$

Putting $Y = \rho$ in (2.9), it follows that $B(X) = \frac{B(\rho)}{\pi(\rho)}\pi(X)$, that is,

$$(2.10)\qquad B(X) = f\pi(X),$$

where $f = \frac{B(\rho)}{\pi(\rho)}$.

Then using (2.10) in (2.7), we obtain

$$(2.11)\qquad \begin{aligned}\widetilde{R}(X,Y,Z,W) = {} & a'[\pi(Y)\pi(Z)g(X,W) - \pi(X)\pi(Z)g(Y,W) \\ & + \pi(X)\pi(W)g(Y,Z) - \pi(Y)\pi(W)g(X,Z)] \\ & + b'[g(X,W)g(Y,Z) - g(X,Z)g(Y,W)],\end{aligned}$$

where

$$a' = \frac{2n-1}{n+1} + f \text{ and } b' = -\frac{n+1}{n-1}\pi(\rho),$$

which implies that the manifold is of quasiconstant curvature. □

If $R = 0$ and $A = B = 0$, then from (2.6) it follows that the manifold is a group manifold and the manifold is of constant curvature.

The above result has been proved by Yano in [**28**].

Theorem 2.2. *If a Riemannian manifold admits a semisymmetric metric connection whose curvature tensor vanishes and the torsion tensor is almost pseudo symmetric with respect to the semisymmetric metric connection, the associated* 1-*form A is equal to the associated* 1-*form π of the semisymmetric metric connection, then the sectional curvature of the plane determined by two vectors $X, Y \in \rho^{\perp}$ is $-\frac{n+1}{n-1}\pi(\rho)$.*

Proof. Let $\rho^{\perp}$ denoted the $(n-1)$ dimensional distribution orthogonal to ρ in a Riemannian manifold admitting a semisymmetric metric connection whose curvature tensor vanishes and the torsion tensor is almost pseudo symmetric. Then for any $X \in \rho^{\perp}$, $g(X,\rho) = 0$ or, $\pi(X) = 0$. Now we shall determine the sectional curvature $'R$ at the plane determine by the vectors $X, Y \in \rho^{\perp}$. Putting $Z = Y$, $W = X$ in (2.11), we get

$$\widetilde{R}(X,Y,Y,X) = b'[g(X,X)g(Y,Y) - g(X,Y)g(X,Y)].$$

Then

$$'R(X,Y) = \frac{\widetilde{R}(X,Y,Y,X)}{g(X,X)g(Y,Y) - g(X,Y)^2} = -\frac{n+1}{n-1}\pi(\rho). \qquad \square$$

3. Existence of a torseforming vector field of Riemannian manifolds admitting a semisymmetric metric connection $\bar{\nabla}$

Definition 3.1. A vector field ρ is said to be a torseforming vector field [**23**] if

$$(\nabla_X \pi)Y = \alpha g(X,Y) + \beta\pi(X)\pi(Y),$$

where α and β are two scalars and π is a nonzero 1-form defined by $g(X,\rho) = \pi(X)$.

Theorem 3.1. *If a Riemannian manifold admits a semisymmetric metric connection whose torsion tensor is almost pseudo symmetric, the Ricci tensor of the semisymmetric metric connection is symmetric and the associated* 1-*form A of manifold is equal to the associated* 1-*form π of the semisymmetric metric connection, then*

(i) *the* 1-*form π is closed,*
(ii) *the vector field ρ is irrotational,*
(iii) *the integral curves of the vector field ρ are geodesics provided ρ is a unit vector field and*
(iv) *the associated vector field of the semisymmetric metric connection is a torseforming vector field.*

Proof. Suppose the associated 1-form A is equal to the associated 1-form π of the semisymmetric metric connection and the Ricci tensor of the semisymmetric metric connection is symmetric.

Now putting $A(X) = \pi(X)$ in (2.2) and using Proposition 2.1 and (2.10), we have

$$(n-1)(\bar{\nabla}_X \pi)Z = [(2n-1) + f(n-1)]\pi(X)\pi(Z) - \pi(\rho)g(X,Z). \tag{3.1}$$

From (3.1), it follows that

$$[(\bar{\nabla}_X \pi)Z - (\bar{\nabla}_Z \pi)X] = 0, \tag{3.2}$$

that is, $d\pi(X,Z) = 0$, which implies that the 1-form π is closed.

From (3.2), we obtain $(\bar{\nabla}_X \pi)Z = (\bar{\nabla}_Z \pi)X$. Since $\pi(X) = g(X,\rho)$, we get from this

$$g(X, \bar{\nabla}_Z \rho) = g(Z, \bar{\nabla}_X \rho), \tag{3.3}$$

which implies that the vector field ρ is irrotational.

Next if we take ρ as a unit vector field, then from (3.3), we have

$$g(X, \bar{\nabla}_\rho \rho) = 0,$$

that is, $\bar{\nabla}_\rho \rho = 0$, which implies that the integral curves of ρ are geodesics.

Now using (1.1) in (3.1), we get

$$(\nabla_X \pi)Z = ag(X,Z) + b\pi(X)\pi(Z),$$

where

$$a = -\Big[\frac{n}{n-1}\Big]\pi(\rho) \text{ and } b = \frac{3n-2}{n-1} + f,$$

or, $\nabla_X \rho = aX + b\pi(X)\rho$, which implies that vector field ρ is a torseforming vector field. □

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