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EXISTENCE OF VIABLE SOLUTIONS FOR NONCONVEX-VALUED DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract: A local existence result is proved for the viability problem

$$\dot{x}(t) \in F(t,x(t))\,, \qquad x(0)=x_0, \quad x(t)\in K\ ,$$

where $F(\cdot,\cdot)$ is an integrably bounded, (strongly) measurable in t, Lipschitz continuous in x multifunction with closed values and K is a closed subset of a separable Banach space.

1 – Introduction

The viability problem

$$\text{(1)} \qquad \begin{cases} \dot{x}(t) \in F(t,x(t)), \\ x(0)=x_0, \\ x(t)\in K\ , \end{cases}$$

has been the subject of many papers (see [1, 2, 10, 14] and the references therein). In particular, Bressan ([5, 6]) and Bressan–Cortesi ([7]) solved the viability problem (1) for jointly lower semicontinuous F. V.V. Goncharov ([11]) developed an original fixed point argument, in order to handle the Caratheodory case. However, his result requires the convexity of the set K. Using the technique of directionally continuous selection of [5], Colombo ([9]) had obtained recently the local existence result for (1) when F is a Caratheodory lower semicontinuous, integrably bounded, closed valued multifunction and K is a locally compact subset

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of a reflexive Banach space. K. Deimling ([10], Theorem 9.3) and D. Bothe ([4], Theorem 4.1) give existence results for "almost lsc" multifunctions defined on the graph of another multifunction $t \to K(t)$ under an assumption that involves the measure of noncompactness.

In this note we consider the viability problem (1) in the case when $F(\cdot,\cdot)$ is an integrably bounded, closed valued map which is measurable with respect to the first argument and Lipschitz continuous with respect to the second argument. We do not require the Banach space to be reflexive. However, we must place on the multifunction F and on the set K a condition which is more strict than the classical tangency condition. Nevertheless, our condition is closely related to the usual assumptions, as Proposition 4.4 in [3] shows. The technique we shall adapt here is motivated by that used by Larrieu in [12] for studying (1) in the case when F is a singleton valued map in a Banach space.

2 – Existence of solutions to the viability problem (1)

Let E be a separable Banach space and let $K \subset E$ be a nonempty closed set. For measurability purpose, E (resp. $\Omega \subset E$) is endowed with the σ-algebra $B(E)$ (resp. $\mathcal{B}(\Omega)$) of Borel subsets for the strong topology. Let $I = [0,1]$ be endowed with Lebesgue measure and the σ-algebra $\mathcal{L}([0,1])$ of Lebesgue measurable subsets. For nonempty sets A, B of E and $a \in A$ we denote $d(a,B) = \inf\{d(a,b),\, b \in B\}$, $d(A,B) = \sup\{d(a,B),\, a \in A\}$, $H(A,B) = \max\{d(A,B),\, d(B,A)\}$.

Let $F\colon\ I \times E \to E$ be a multifunction with nonempty closed values.

The main result of this note is the following

Theorem 2.1. *Assume that*

i) *For all $x \in E$, $F(\cdot,x)$ is measurable for the above σ-algebras;*

ii) *There is a function $k \in L^1(I,R^+)$ such that for all $t \in I$, all $x_1, x_2 \in E$*

$$H\Big(F(t,x_1), F(t,x_2)\Big) \le k(t)\, \|x_1 - x_2\| \ ;$$

iii) *For any bounded set B of K, there is a function $g_B \in L^1(I,R^+)$ such that for all $t \in I$ and all $x \in B$*

$$\|F(t,x)\| =: \sup_{y \in F(t,x)} \|y\| \le g_B(t) \ ;$$

iv) *For every $x \in K$, the following equality*

$$\liminf_{h\to 0^+} \frac{1}{h}\, d\Big(x + \int_t^{t+h} F(r,x)\, dr,\ K\Big) = 0 \tag{2}$$

holds.

Then for any $t_0 \in [0,1)$ and any $x_0 \in K$ there exists $a \in (t_0, 1]$ such that the viability problem (1) has solutions on $[t_0, a]$, i.e. there is an absolutely continuous function x from $[t_0, a]$ into E satisfying

$$\begin{cases} \dot{x}(t) \in F(t, x(t)) & \text{a.e. } [t_0, a], \\ x(t_0) = x_0, & \\ x(t) \in K & [t_0, a] \ . \end{cases}$$

Firstly, let us prove the existence of approximate solutions.

Proposition 2.2. *Suppose that the multifunction F satisfies conditions i)–iv) of Theorem 2.1 and $t_0 \in [0,1)$, $x_0 \in K$, $M > 1$ are given. Let $g_M \in L^1(I, R^+)$ and $b \in (t_0, 1]$ be such that*

$$\|F(t,x)\| \leq g_M(t) \tag{3}$$

for all $t \in I$, $x \in K \cap B$ and

$$\int_{t_0}^{b} g_M(r)\, dr \leq M - 1 \ , \tag{4}$$

where B is the closed ball centered at x_0 and with radius M.

Then for any $\epsilon \in (0,1)$, $a \in (t_0, b)$ and $y \in L^1([t_0, a], E)$ there are a function $f \in L^1([t_0, a], E)$ and a step function z from $[t_0, a]$ into K such that

1) *$z(t) \in K \cap B$ for all $t \in [t_0, a]$;*

2) *$f(t) \in F(t, z(t))$ for almost all $t \in [t_0, a]$;*

3) *$\|f(t) - y(t)\| \leq d(y(t), F(t, z(t))) + \epsilon$ for almost all $t \in [t_0, a]$;*

4) *$d(z(t),\, x_0 + \int_0^t f(r)\, dr) \leq \epsilon$ for all $t \in [t_0, a]$.*

For the proof of this proposition we need the following results concerning measurable multifunctions in Banach spaces. The reader interested in the theory of measurable multifunctions is referred to [8].

Lemma 2.3 [13]. *Let Ω be a nonempty set in a Banach space E. Assume that $F\colon\ [a,b] \times \Omega \to E$ is a multifunction with nonempty closed values satisfying:*

a) *For every $x \in \Omega$, $F(\cdot, x)$ is measurable on $[a,b]$;*

b) *For every $t \in [a,b]$, $F(t, \cdot)$ is (Hausdorff) continuous on Ω.*

Then for any measurable function $x\colon\ [a,b] \to \Omega$, the multifunction $F(\cdot, x(\cdot))$ is measurable on $[a,b]$.

Lemma 2.4 [13]. *Let $G\colon [a,b] \to E$ be a measurable multifunction and $y\colon [a,b] \to E$ a measurable function. Then for any positive measurable function $r\colon [a,b] \to R^+$, there exists a measurable selection g of G such that for almost all $t \in [a,b]$*

$$\|g(t) - y(t)\| \le D\Big(y(t), G(t)\Big) + r(t) \ .$$

Proof of Proposition 2.2: For the sake of convenience we take $t_0 = 0$. Denote by $\eta(\cdot)$ the modulus of uniform continuity of the function $G(t)$ defined by

$$G(t) = \int_0^t g_M(r)\, dr \ ,$$

so that $|G(t) - G(t')| \le \epsilon$ if $|t - t'| \le \eta(\epsilon)$.

Denote

$$\alpha = \min\Big\{b, \frac{\epsilon}{4}, \eta\Big(\frac{\epsilon}{4}\Big)\Big\} \ . \tag{5}$$

We now define inductively a finite sequence $\{t_i\}_{i=0}^n \subset [0,b]$ with $t_n \in [a,b]$, a function θ from $[0,t_n]$ into $[0,t_n]$ such that for all $i \le n-1$ and $t \in [t_i, t_{i+1})$, $\theta(t) = t_i$ and a finite sequence $\{z_i\}_{i=0}^n$ with $z_0 = x_0$ such that

a) $|t_{i+1} - t_i| \le \alpha$ for $i = 0, 1, ..., n-1$;

b) $z_i \in K \cap B$ for $i = 0, 1, ..., n$;

c) $f(t) \in F(t, z_{\theta(t)})$ for almost all $t \in [0,a]$, where $z_{\theta(t)} = z_i$ if $\theta(t) = t_i$;

d) $\|f(t) - y(t)\| \le d(y(t), F(t, z_{\theta(t)})) + \epsilon$ for almost all $t \in [0,a]$;

e) $\|z_i - x_0 - \int_0^{t_i} f(r)\, dr\| \le \alpha\, t_i$, $i = 0, 1, ..., n$.

For $i = 0$, let $t_0 = 0$ and $z_0 = x_0$ ($f(0)$ need not be defined).

Suppose that the inductive procedure is realized on $[0, t_i]$ with $t_i < b$, i.e. the following conditions hold:

a) $|t_{j+1} - t_j| \le \alpha$ for $j = 0, 1, ..., i-1$;

b) $z_i \in K \cap B$ for $j = 0, 1, ..., i$;

c) $f(t) \in F(t, z_{\theta(t)})$ for almost all $t \in [0, t_i]$;

d) $\|f(t) - y(t)\| \le d(y(t), F(t, z_{\theta(t)})) + \epsilon$ for almost all $t \in [0, t_i]$;

e) $\|z_j - z_0 - \int_0^{t_j} f(r)\, dr\| \le \alpha\, t_j$ for $j = 0, 1, ..., i$.

Let

$$h_i = \max\Big\{h' \le \alpha,\ h' \le b - t_i \mid d\Big(z_i + \int_{t_i}^{t_i + h'} F(r, z_i)\, dr,\ K\Big) \le \frac{\alpha\, h'}{4}\Big\} \ .$$

Set $t_{i+1} = t_i + h_i$. In view of Lemma 2.4, there is a function $f_{i+1} \in L^1([t_i, t_{i+1}], E)$ such that $f_{i+1}(t) \in F(t, z_i)$ for all $t \in [t_i, t_{i+1}]$ and

$$\|f_{i+1}(t) - y(t)\| \leq d\Big(y(t), F(t, z_i)\Big) + \epsilon$$

for almost all $t \in [t_i, t_{i+1}]$.

Extend f to $[t_i, t_{i+1})$ by setting, for all $t \in [t_i, t_{i+1})$

$$f(t) = f_{i+1}(t) \ .$$

Let z_{i+1} be a point of K such that

$$d\Big(z_i + \int_{t_i}^{t_i+h_i} f(r)\,dr,\ z_{i+1}\Big) \leq \frac{\alpha\, h_i}{2} < \alpha\, h_i = \alpha(t_{i+1} - t_i) \ . \tag{6}$$

Then we have

$$\begin{aligned} &\Big\|z_{i+1} - z_0 - \int_0^{t_{i+1}} f(r)\,dr\Big\| \leq \\ &\qquad \leq \Big\|z_{i+1} - z_i - \int_{t_i}^{t_{i+1}} f(r)\,dr\Big\| + \Big\|z_i - z_0 - \int_0^{t_i} f(r)\,dr\Big\| \\ &\qquad \leq \alpha(t_{i+1} - t_i) + \alpha\, t_i = \alpha\, t_{i+1} \ , \end{aligned} \tag{7}$$

which together with (4) gives

$$\begin{aligned} \|z_{i+1} - z_0\| &\leq \alpha\, t_{i+1} + \Big\|\int_0^{t_{i+1}} f(r)\,dr\Big\| \leq \alpha\, t_{i+1} + \int_0^{t_{i+1}} g_M(r)\,dr \\ &\leq \alpha\, t_{i+1} + M - 1 \leq 1 + M - 1 = M \ , \end{aligned}$$

which means that $z_{i+1} \in B$. Thus the conditions a)–e) are satisfied on $[0, t_{i+1}]$.

The inductive procedure can be now continued further. We claim that there is a positive integer n such that $t_n > a$. Suppose the contrary: $t_n \leq a$ for all $n \geq 1$. Then the bounded increasing sequence $\{t_i\}_{i \in N}$ converges to $\overline{t} \leq a < b$. Therefore, the sequence $\{z_i\}_{i \in N}$ converges to a point $\overline{z} \in K \cap B$. Indeed, by construction we have

$$\begin{aligned} \|z_{n+1} - z_n\| &\leq \alpha(t_{n+1} - t_n) + \Big\|\int_{t_n}^{t_{n+1}} f(r)\,dr\Big\| \\ &\leq \alpha(t_{n+1} - t_n) + \int_{t_n}^{t_{n+1}} g_M(r)\,dr \end{aligned}$$

and, for $n > m$

$$\|z_n - z_m\| \leq \alpha(t_n - t_m) + \int_{t_m}^{t_n} g_M(r)\,dr \ .$$

Since $\{t_i\}_{i=0}^{\infty}$ is a Cauchy sequence, the sequence $\{z_i\}_{i=0}^{\infty}$ is also Cauchy and, therefore, z_n converges to a point $\overline{z} \in K$.

Choose a scalar $h \in (0, b - \overline{t})$ and a positive integer n_0 such that for $n \geq n_0$

$$(8) \qquad \begin{cases} d\Big(\overline{z} + \int_{\overline{t}}^{\overline{t}+h} F(r, \overline{z})\, dr, K\Big) \leq \dfrac{\alpha\, h}{25}, \\ |\overline{t} - t_n| < \eta\Big(\dfrac{\alpha\, h}{25}\Big), \\ \|\overline{z} - z_n\| < \dfrac{\alpha\, h}{25}, \\ \int_{\overline{t}}^{\overline{t}+h} k(r)\, \|z_n - \overline{z}\|\, dr \leq \dfrac{\alpha\, h}{25}\ . \end{cases}$$

Let $n > n_0$ be given. For an arbitrary measurable selection ϕ_n of $F(t, z_n)$ on $[0, \overline{t} + h]$ there exists a measurable selection ϕ of $F(t, \overline{z})$ on $[0, \overline{t} + h]$ such that

$$(9) \qquad \begin{aligned} \|\phi_n(t) - \phi(t)\| &\leq d\Big(\phi_n(t), F(t, \overline{z})\Big) + \frac{\alpha}{25} \\ &\leq k(t)\, \|z_n - \overline{z}\| + \frac{\alpha}{25}\ . \end{aligned}$$

Then inequalities (8)–(9) give

$$\begin{aligned} d\Big(z_n + \int_{t_n}^{t_n+h} \phi_n(r)\, dr,\ K\Big) \leq& \\ &\leq \|z_n - \overline{z}\| + d\Big(\overline{z} + \int_{\overline{t}}^{\overline{t}+h} \phi(r)\, dr,\ K\Big) + \Big\|\int_{t_n+h}^{\overline{t}+h} \phi(r)\, dr\Big\| \\ &\quad + \Big\|\int_{t_n}^{\overline{t}} \phi_n(r)\, dr\Big\| + \Big\|\int_{\overline{t}}^{t_n+h} \Big(\phi_n(r) - \phi(r)\Big)\, dr\Big\| \\ &\leq \|z_n - \overline{z}\| + d\Big(\overline{z} + \int_{\overline{t}}^{\overline{t}+h} F(r, \overline{z})\, dr,\ K\Big) + \int_{t_n+h}^{\overline{t}+h} g_M(r)\, dr \\ &\quad + \int_{t_n}^{\overline{t}} g_M(r)\, dr + \int_{\overline{t}}^{t_n+h} \|\phi_n(r) - \phi(r)\|\, dr \\ &\leq \|z_n - \overline{z}\| + d\Big(\overline{z} + \int_{\overline{t}}^{\overline{t}+h} F(r, \overline{z})\, dr,\ K\Big) + \int_{t_n+h}^{\overline{t}+h} g_M(r)\, dr \\ &\quad + \int_{t_n}^{\overline{t}} g_M(r)\, dr + \int_{\overline{t}}^{\overline{t}+h} k(r)\, \|z_n - \overline{z}\|\, dr + \frac{\alpha\, h}{25} \leq \end{aligned}$$

$$\le \frac{\alpha\,h}{25} + \frac{\alpha\,h}{25} + \frac{\alpha\,h}{25} + \frac{\alpha\,h}{25} + \frac{\alpha\,h}{25} + \frac{\alpha\,h}{25}$$
$$< \frac{\alpha\,h}{4} \;.$$

Since ϕ_n is an arbitrary measurable selection of $F(t,z_n)$ on $[0,\bar{t}+h]$ it follows

$$d\Big(z_n + \int_{t_n}^{t_n+h} F(r,z_n)\,dr,\ K\Big) \le \frac{\alpha\,h}{4} \;.$$

On the other hand, by construction, for n large enough we have $t_{n+1} < \bar{t} < t_n + h \le b$, i.e. $h > t_{n+1} - t_n = h_n$. Thus the last inequality contradicts the definition of h_n. Therefore, there is a positive integer n such that $t_n \ge a$.

Now we define the function z on $[0,a]$ by setting

$$z(t) = z_{\theta(t)} \;. \tag{10}$$

It is clear that $z(t) \in K \cap B$ for all $t \in [0,a]$. Assume that $t \in [t_i, t_{i+1}) \cap [0,a]$. Then (6), (7) and (10) imply

$$\begin{aligned}
&\Big\|z(t) - z_0 - \int_0^t f(r)\,dr\Big\| = \\
&\qquad = \Big\|z_i - z_0 + \int_0^t f(r)\,dr\Big\| \\
&\qquad \le \|z_i - z_{i+1}\| + \Big\|z_{i+1} - z_0 - \int_0^{t_{i+1}} f(r)\,dr\Big\| + \Big\|\int_t^{t_{i+1}} f(r)\,dr\Big\| \\
&\qquad \le \alpha(t_{i+1} - t_i) + \Big\|\int_{t_i}^{t_{i+1}} f(r)\,dr\Big\| + \alpha\,t_{i+1} + \Big\|\int_{t_i}^{t_{i+1}} f(r)\,dr\Big\| \\
&\qquad \le 2\alpha + 2\int_{t_i}^{t_{i+1}} g_{(M)}(r)\,dr \;.
\end{aligned}$$

Taking account of (5) we get

$$\Big\|z(t) - z_0 - \int_0^t f(r)\,dr\Big\| \le \epsilon \;,$$

which concludes the proof. ∎

Proof of Theorem 2.1: Without loss of generality we can assume that $t_0 = 0$.

Let $(\epsilon_n)_{n=1}^{\infty}$ be a strictly decreasing sequence of positive scalars such that $\sum_{n=1}^{\infty} \epsilon_n < \infty$. Let M, g_M and b be as in the Proposition 2.2, $a \in (0,b)$ and f_0 a measurable selection of $F(t,x_0)$ on $[0,a]$.

In view of Proposition 2.2, we can define inductively sequences $\{f_n\}_{n=1}^{\infty} \subset L^1([0,a],E)$ and $\{z_n\}_{n=1}^{\infty} \subset S([0,a],E)$ ($S([0,a],E)$ is the space of step functions from $[0,a]$ into E) such that

1) $z_n(t) \in K \cap B$ for all $t \in [0,a]$;

2) $f_n(t) \in F(t, z_n(t))$ for almost all $t \in [0,a]$, $n \geq 1$;

3) $\|f_{n+1}(t) - f_n(t)\| \leq d(f_n(t), F(t, z_{n+1}(t))) + \epsilon_{n+1}$ for almost all $t \in [0,a]$ and $n \geq 1$;

4) $d(z_n(t), z_0 + \int_0^t f_n(r)\,dr) \leq \epsilon_n$ for all $t \in [0,a]$.

Observe that 2) and 3) imply

$$\begin{aligned} \|f_{n+1}(t) - f_n(t)\| &\leq H\Big(F(t, z_n(t)), F(t, z_{n+1}(t))\Big) + \epsilon_{n+1} \\ &\leq k(t)\, \|z_n(t) - z_{n+1}(t)\| + \epsilon_{n+1} \ . \end{aligned} \tag{11}$$

Combining 4) and (11) yields

$$\begin{aligned} \|z_{n+1}(t) - z_n(t)\| \leq &\Big\|z_{n+1}(t) - z_0 - \int_0^t f_{n+1}(r)\,dr\Big\| + \\ &+ \Big\|z_n(t) - z_0 - \int_0^t f_n(r)\,dr\Big\| + \int_0^t \|f_{n+1}(r) - f_n(r)\|\,dr \\ &\leq \epsilon_n + \epsilon_{n+1} + \int_0^t \|f_{n+1}(r) - f_n(r)\|\,dr \\ &\leq 2\,\epsilon_n + \int_0^t k(r)\,\|z_{n+1}(r) - z_n(r)\|\,dr + \epsilon_{n+1} \\ &\leq 3\,\epsilon_n + \int_0^t k(r)\,\|z_{n+1}(r) - z_n(r)\|\,dr \ . \end{aligned}$$

It follows then from the Gronwall's inequality (see e.g. [8, Proposition VI-9]) that

$$\|z_{n+1}(t) - z_n(t)\| \leq 3\,\epsilon_n\, e^L \ ,$$

where $L = \int_0^a k(r)\,dr$.

Therefore we have, for $n < m$:

$$\|z_n(t) - z_m(t)\| \leq 3\, e^L \sum_{i=n}^{m-1} \epsilon_i \ .$$

Thus the sequence $\{z_n(\cdot)\}_{n=1}^{\infty}$ converges uniformly on $[0,a]$ to a function $x(\cdot)$. Since $z_n(t) \in K \cap B$ for every $t \in [0,a]$ and the set K is closed, it follows that $x(t) \in K \cap B$ for all $t \in [0,1]$.

Observe that for every $n \geq 1$ we have

$$\begin{aligned}\|f_{n+1}(t) - f_n(t)\| &\leq H\Big(F(t, z_{n+1}(t)), F(t, z_n(t))\Big) + \epsilon_{n+1} \\ &\leq k(t)\, \|z_{n+1}(t) - z_n(t)\| + \epsilon_{n+1} \\ &\leq \epsilon_n \Big[3\, k(t)\, e^L + 1\Big] \;.\end{aligned}$$

This implies (as above) that $\{f_n(t)\}_{i=1}^n$ is a Cauchy sequence and $f_n(t)$ converges to $f(t)$.

Further, since $\|f_n(t)\| \leq g_M(t)$, by 4) and Lebesgue's theorem we have

$$x(t) = \lim z_n(t) = \lim\Big(x_0 + \int_0^t f_n(r)\, dr\Big) = x_0 + \int_0^t f(r)\, dr \;.$$

Finally, observe that by 2),

$$\begin{aligned}d\Big(f(t), F(t, x(t))\Big) &\leq \|f(t) - f_n(t)\| + H\Big(F(t, z_n(t)), F(t, x(t))\Big) \\ &\leq \|f(t) - f_n(t)\| + k(t)\, \|z_n(t) - x(t)\|\end{aligned}$$

so that $\dot{x}(t) = f(t) \in F(t, x(t))$ a.e. in $[0, a]$. The proof is complete. ∎

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