

NON-SIMULTANEOUS BLOW-UP FOR A SEMILINEAR PARABOLIC SYSTEM WITH NONLINEAR MEMORY

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Abstract. In this note, we study the possibility of non-simultaneous blow-up for positive solutions to the following system,

$$u_t - \Delta u = u^{q_1} \int_0^t v^{p_1}(x, s) ds, \quad v_t - \Delta v = v^{q_2} \int_0^t u^{p_2}(x, s) ds.$$

Under appropriate hypotheses, we prove that u blows up while v fails to blow up if and only if $q_1 > 1$ and $p_2 < 2(q_1 - 1)$.

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