PERTURBATION ANALYSIS FOR THE COMPLEX MATRIX EQUATION $Q \pm A^{\mathrm{H}} X^p A - X = 0$

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Abstract. We study the sensitivity of the solution of a general type matrix equation $Q \pm A^{\mathrm{H}} X^p A - X = 0$. Local and nonlocal perturbation bounds are derived. The results are obtained using the technique of Lyapunov majorants and fixed point principles. A numerical example is given.

Full text

References

[1] W. Anderson, T. Morley, E. Trapp, *Positive solutions to $X = A - BX^{-1}B^*$*, Linear Alg. Appl. **134**(1990), 53–62. MR1060009(91c:47031). Zbl 0702.15009.

[2] V. Angelova, *Perturbation analysis for the matrix equation $X = A_1 + \sigma A_2^{\mathrm{H}} X^{-2} A_2$, $\sigma = \pm 1$*, Annual of the Univ. Arch., Civil Eng. and Geodesy, **41**(2000–2001), fasc. II, 33–41, publ. 2005.

[3] R. Bhatia, D. Singh, K. Sinha, *Differentiation of operator functions and perturbation bounds*, Commun. Math. Phys., **191**(1998), 603–611, MR1608543(99c:47017). Zbl 0918.47017.

[4] D. Bini, N. Higham, B. Meini, *Algorithms for the matrix pth root*, Numer. Algorithms **39** (2005), 349–378. MR2134331(2005m:65094).

[5] J. Boneva, M. Konstantinov, P. Petkov, *Frechét derivatives of rational power matrix functions*, Proc. 35 Spring Conf. UBM, Borovetz, 2006, 169–174.

[6] J. Boneva, M. Konstantinov, V. Todorov, P. Petkov, *Affine approximation of rational power matrix functions* Proc. 36 Int. Conf. "Appl. of Math. in Economics and Technique, St. Konstantin & Elena resort, Varna, 2007, 149–153.

[7] J. Engwerda, A. Ran, A. Rijkeboer, *Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $X + A^* X^{-1} A = Q$*, Linear Alg. Appl. **186**(1993), 255–275. MR1217209(94j:15012). Zbl 0778.15008.

[8] C. Guo, *Convergence rate of an iterative method for a nonlinear matrix equation*, SIAM J. Matrix Anal. Appl. **23**(2001), 295–302. MR1856611(2002g:15033). Zbl 0997.65069.

[9] V. Hasanov, I. Ivanov, *Solutions and perturbation theory of a special matrix equation I: Properties of solutions*, Proc. 32 Spring Conf. UBM, Sunny Beach, 2003, 244–248.

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http://www.utgjiu.ro/math/sma

[10] V. Hasanov, I. Ivanov, *Solutions and perturbation theory of a special matrix equation II: Perurbation theory*, Proc. 32 Spring Conf. UBM, Sunny Beach, 2003, 58–264.

[11] V. Hasanov, S. El–Sayed, *On the positive definite solutions of nonlinear matrix equation* $X + A^{\mathrm{H}}X^{-\delta}A = Q$, Linear Alg. Appl.**412**(2-3)(2006), 154–160. MR2182958(2006g:15027). Zbl 1083.15018.

[12] I. Ivanov, N. Georgieva, *On a special positive definite solution of a class of nonlinear matrix equations*, Proc. 32 Spring Conf. UBM, Sunny Beach, 2003, 253–257.

[13] I. Ivanov, S. El–Sayed, *Properties of positive definite solutions of the equation* $X + A^{*}X^{-2}A = I$, Linear Alg. Appl. **297**(1998), 303–316. MR1637909(99c:15019). Zbl 0935.65041.

[14] M. Konstantinov, J. Boneva, V. Todorov, P. Petkov, *Theory of differentiation of matrix power functions*, Proc. 32 Int. Conf., Sozopol, 2006, 90–107.

[15] M. Konstantinov, D. Gu, V. Mehrmann, P. Petkov, *Perturbation Theory for Matrix Equations*, North Holland, Amsterdam, 2003, ISBN 0-444-51315-9. MR1991778(2004g:15019). Zbl 1025.15017.

[16] Z. Peng, S. El–Sayed, *On positive definite solution of a nonlinear matrix equation*, Numer. Linear Alg. Appl. **14**(2)(2007), 99–113. MR2292298.

[17] Z. Peng, S. El–Sayed, X. Zhang, *Iterative methods for the extremal positive definite solution of the matrix equation* $X + A^{\mathrm{H}}X^{-\alpha}A = Q$, J.Comput. Appl. Math. **200**(2)(2007), 520–527. MR2289231.

[18] A. Ran, M. Reurings, *On the nonlinear matrix equation* $X + A^{\mathrm{H}}f(X)A = Q$: *solutions and perturbation theory*, Linear Alg. Appl. **346**(2002), 15–26. MR1897819(2003a:15014). Zbl 1086.15013.

[19] S. El-Sayed, A. Ran, *On an iteration method for solving a class of nonlinear matrix equations*, SIAM J. Matrix Anal. Appl. **23**(2001), 632–645. Zbl 1002.65061.

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