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# GROUPOID ENRICHED CATEGORIES AND NATURAL SYSTEMS

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ABSTRACT. We generalize the Baues-Jibladze descent theorem to a large class of groupoid enriched categories.

## 1. Introduction

A natural system $D$ on a category $\boldsymbol{C}$ is a covariant functor on the category of factorizations $\mathbf{F}\boldsymbol{C}$ of $\boldsymbol{C}$ (also called twisted arrow category of $\boldsymbol{C}$, see [Mac Lane 1971]). Cohomology of $\boldsymbol{C}$ with coefficients in a natural system was first defined in [Baues & Wirsching 1985] and plays important rôle in many areas of algebra and topology [Baues 1989, Baues 2003, Jibladze & Pirashvili 1991, Jibladze & Pirashvili 2005, Pirashvili 1990]. One of the main points for the applications is the fact that the third cohomology group $H^3(\boldsymbol{C}; D)$ classifies the so called linear track extensions of $\boldsymbol{C}$ by $D$ [Pirashvili 1988, Pirashvili 1990]. Recently in [Baues & Jibladze 2002] it was proved that linear track extensions are essentially the same as groupoid enriched categories such that automorphism groups of all 1-arrows are abelian (=abelian track categories, see below). The proof of this important result relies on the fact that in any abelian track category $\mathscr{T}$, automorphism groups $\mathrm{Aut}_{\mathscr{T}}(f)$ of 1-arrows can be "descended" to the homotopy category $\mathscr{T}_{\simeq}$, i. e. they only depend on the isomorphism class of $f$ in a nice way – see Theorem 2.4 in [Baues & Jibladze 2002].

The aim of this work is to generalize this descent result for a large class of non-abelian natural systems equipped with certain type of descent data.

## 2. Preliminaries

A groupoid is called *abelian* if the automorphism group of each object is an abelian group. We will use the following notation for 2-categories. Composition of 1-arrows will be denoted by juxtaposition; for 2-arrows we will use additive notation, so composition is + and identity 2-arrows are denoted by 0. The hom-category for objects $A$, $B$ of a 2-category will be denoted by $[\![A, B]\!]$.

There are several categories associated with a 2-category $\mathscr{T}$. The category $\mathscr{T}_0$ has the same objects as $\mathscr{T}$, while morphisms in $\mathscr{T}_0$ are 1-arrows of $\mathscr{T}$. The category $\mathscr{T}_1$ has the same objects as $\mathscr{T}_0$. The morphisms $A \to B$ in $\mathscr{T}_1$ are 2-arrows $\alpha : f \Rightarrow f_1$ where

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$f, f_1 : A \to B$ are 1-arrows in $\mathscr{T}$. The composition in $\mathscr{T}_1$ is given by $(\beta : x \Rightarrow x_1)(\alpha : f \Rightarrow f_1) := (\beta\alpha : xf \Rightarrow x_1 f_1)$, where

$$\beta\alpha = \beta f_1 + x\alpha = x_1\alpha + \beta f.$$

One furthermore has the source and target functors

$$\mathscr{T}_1 \underset{t}{\overset{s}{\rightrightarrows}} \mathscr{T}_0\ ,$$

where $s(\alpha : f \Rightarrow f_1) = f$ and $t(\alpha : f \Rightarrow f_1) = f_1$, the "identity" functor $i : \mathscr{T}_0 \to \mathscr{T}_1$ assigning to an 1-arrow $f$ the triple $0_f : f \Rightarrow f$. Moreover, consider the pullback diagram

$$(*) \qquad \begin{array}{ccc} \mathscr{T}_1 \times_{\mathscr{T}_0} \mathscr{T}_1 & \xrightarrow{p_2} & \mathscr{T}_1\ ; \\ {\scriptstyle p_1}\downarrow & & \downarrow{\scriptstyle t} \\ \mathscr{T}_1 & \xrightarrow{s} & \mathscr{T}_0 \end{array}$$

there is also the "composition" functor $m : \mathscr{T}_1 \times_{\mathscr{T}_0} \mathscr{T}_1 \to \mathscr{T}_1$ sending $(\alpha : f \Rightarrow f_1, \alpha' : f_2 \Rightarrow f)$ to $\alpha + \alpha' : f_2 \Rightarrow f_1$. Note that these functors satisfy the identities $sp_1 = tp_2$, $sm = sp_2$, $tm = tp_1$ and $si = ti = \mathrm{id}_{\mathscr{T}_0}$. Sometimes we will also simply write $\mathscr{T}_1 \rightrightarrows \mathscr{T}_0$ to indicate a 2-category $\mathscr{T}$.

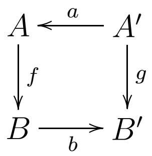
A *track category* $\mathscr{T}$ is a *category enriched in groupoids*, i. e. is the same as a 2-category all of whose 2-arrows are invertible. If the groupoids $[\![A, B]\!]$ are abelian for all $A, B \in \mathrm{Ob}\mathscr{T}$, then $\mathscr{T}$ is called an *abelian* track category. For track categories we might occasionally talk about *maps* instead of 1-arrows and *homotopies* or *tracks* instead of 2-arrows. If there is a homotopy $\alpha : f \Rightarrow g$ between maps $f, g \in \mathrm{Ob}([\![A, B]\!])$, we will say that $f$ and $g$ are homotopic and write $f \simeq g$. Since the homotopy relation is a natural equivalence relation on morphisms of $\mathscr{T}_0$, it determines the *homotopy category* $\mathscr{T}_{\simeq} = \mathscr{T}_0/\simeq$. Objects of $\mathscr{T}_{\simeq}$ are once again objects in $\mathrm{Ob}(\mathscr{T})$, while morphisms of $\mathscr{T}_{\simeq}$ are homotopy classes of morphisms in $\mathscr{T}_0$. For objects $A$ and $B$ we let $[A, B]$ denote the set of morphisms from $A$ to $B$ in the category $\mathscr{T}_{\simeq}$. Thus

$$[A, B] = [\![A, B]\!] / \simeq\ .$$

Usually we let $q : \mathscr{T}_0 \to \mathscr{T}_{\simeq}$ denote the quotient functor. Sometimes for a 1-arrow $f$ in $\mathscr{T}$ we will denote $q(f)$ by $[f]$. A map $f : A \to B$ is a *homotopy equivalence* if there exists a map $g : B \to A$ and tracks $fg \simeq 1$ and $gf \simeq 1$. This is the case if and only if $q(f)$ is an isomorphism in the homotopy category $\mathscr{T}_{\simeq}$. In this case $A$ and $B$ are called *homotopy equivalent* objects.

A *track functor* $F : \mathscr{T} \to \mathscr{T}'$ between track categories is by definition a groupoid enriched functor. Let $\boldsymbol{C}$ be a category. Then the category $\mathbf{F}\boldsymbol{C}$ of factorizations in $\boldsymbol{C}$ is defined as follows. Objects of $\mathbf{F}\boldsymbol{C}$ are morphisms $f : A \to B$ in $\boldsymbol{C}$ and morphisms

$(a,b) : f \to g$ in $\mathbf{F}\boldsymbol{C}$ are commutative diagrams



in the category $\boldsymbol{C}$. A *natural system* on $\boldsymbol{C}$ with values in a category $\mathscr{C}$ is a covariant functor $D : \mathbf{F}\boldsymbol{C} \to \mathscr{C}$. We write $D(f) = D_f$. If $a : C \to D$, $f : A \to C$ and $g : D \to B$ are morphisms in $\boldsymbol{C}$, then the morphism $D_f \to D_{af}$ induced by the morphism $(1_A, a) : f \to af$ in $\mathbf{F}\boldsymbol{C}$ will be denoted by $a_*$, while the morphism $D_g \to D_{ga}$ induced by $(a, 1_B) : g \to ga$ will be denoted by $a^*$.

A morphism of natural systems is just a natural transformation. For a functor $q : \boldsymbol{C}' \to \boldsymbol{C}$, any natural system $D$ on $\boldsymbol{C}$ gives a natural system $D \circ (\mathbf{F}q)$ on $\boldsymbol{C}'$ which we will denote $q^*(D)$.

For us a crucial observation is that any 2-category gives rise to a natural system. Indeed let $\mathscr{B}$ be a 2-category. There is a natural system $\mathrm{End}_{\mathscr{B}}$ of monoids on $\mathscr{B}_0$ (i. e. a functor $\mathbf{F}\mathscr{B}_0 \to \mathbf{Monoids}$) which assigns to an 1-arrow $f : A \to B$ the monoid of all 2-arrows $f \Rightarrow f$ in $\mathscr{B}$. Indeed for $g : B \to B'$, $h : A' \to A$ morphisms in $\mathscr{B}_0$ we already defined the induced homomorphisms:

$$(\varepsilon \mapsto g_*\varepsilon = g\varepsilon) : \mathrm{Hom}_{\mathscr{B}}(f, f) \to \mathrm{Hom}_{\mathscr{B}}(gf, gf),$$
$$(\varepsilon \mapsto h^*\varepsilon = \varepsilon h) : \mathrm{Hom}_{\mathscr{B}}(f, f) \to \mathrm{Hom}_{\mathscr{B}}(fh, fh).$$

For a track category $\mathscr{T}$, clearly $\mathrm{End}_{\mathscr{T}} = \mathrm{Aut}_{\mathscr{T}}$ takes values in the category of groups.

## 3. $\mathscr{T}$-natural systems

To state our main result we need to introduce some more notions.

3.1. Definition. *Consider a track category $\mathscr{T}$. A $\mathscr{T}$-*natural system *with values in a category $\mathscr{C}$ is a natural system $D : \mathbf{F}\mathscr{T}_0 \to \mathscr{C}$ on $\mathscr{T}_0$ together with a family of morphisms*

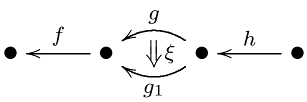
$$\nabla_\xi : D_f \to D_g$$

*in the category $\mathscr{C}$, one for each track $\xi : f \Rightarrow g$ in $\mathscr{T}$, such that the following conditions are satisfied:*

*i) $\nabla_{0_f} = \mathrm{id}_{D_f}$ for all 1-arrows $f$ in $\mathscr{T}$.*

*ii) For $\xi : f \Rightarrow g$, $\eta : g \Rightarrow h$ one has $\nabla_{\eta+\xi} = \nabla_\eta \circ \nabla_\xi$.*

*iii) For a diagram*

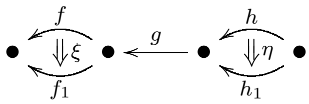


*the following diagram*

$$\begin{array}{ccccc} D_{fg} & \xleftarrow{f_*} & D_g & \xrightarrow{h^*} & D_{gh} \\ {\scriptstyle \nabla_{f\xi}}\downarrow & & {\scriptstyle \nabla_{\xi}}\downarrow & & {\scriptstyle \nabla_{\xi h}}\downarrow \\ D_{fg_1} & \xleftarrow{f_*} & D_{g_1} & \xrightarrow{h^*} & D_{g_1 h} \end{array}$$

*commutes.*

iv) *For a diagram*




*the diagram*

$$\begin{array}{ccccc} D_{fg} & & & & D_{gh} \\ & \nwarrow^{f_*} & & \nearrow^{h^*} & \\ {\scriptstyle \nabla_{\xi g}}\downarrow & & D_g & & \downarrow{\scriptstyle \nabla_{g\eta}} \\ & \swarrow_{f_{1*}} & & \searrow_{h_1{}^*} & \\ D_{f_1 g} & & & & D_{gh_1} \end{array}$$

*commutes.*

*A morphism* $\Phi : (D, \nabla) \to (D', \nabla')$ *of* $\mathscr{T}$*-natural systems is a natural transformation* $\Phi$ *between the functors* $D, D' : \mathbf{F}\mathscr{T}_0 \to \mathscr{C}$*, such that the diagram*

$$\begin{array}{ccc} D \circ \mathbf{F}s & \xrightarrow{\Phi \mathbf{F}s} & D' \circ \mathbf{F}s \\ {\scriptstyle \nabla}\downarrow & & \downarrow{\scriptstyle \nabla'} \\ D \circ \mathbf{F}t & \xrightarrow{\Phi \mathbf{F}t} & D' \circ \mathbf{F}t \end{array}$$

*commutes. We denote by* $\mathscr{T}$-Nat *the category of* $\mathscr{T}$*-natural systems.*

Let $G : \mathscr{T}' \to \mathscr{T}$ be a track functor. For any $\mathscr{T}$-natural system $(D, \nabla)$ one defines a $\mathscr{T}'$-natural system $G^*(D, \nabla) = (D \circ \mathbf{F}G, \nabla G)$, where for $\xi' : f' \Rightarrow g'$ in $\mathscr{T}'$, $(\nabla G)_{\xi'} : D_{Gf'} \to D_{Gg'}$ is defined to be $\nabla_{G\xi'}$. In this way one obtains a functor

$$G^* : \mathscr{T}\text{-Nat} \to \mathscr{T}'\text{-Nat}.$$

3.2. Example. *For a track category* $\mathscr{T}$*, the group-valued natural system* $\mathrm{Aut}_{\mathscr{T}}$ *is equipped with a canonical structure of a* $\mathscr{T}$*-natural system given by*

$$\nabla_\xi(a) = \xi + a - \xi.$$

Let $D$ be a natural system on $\mathscr{T}_{\simeq}$. Then $q^*D$ is a natural system on $\mathscr{T}_0$ given by $(q^*D)_f = D_{q(f)}$. Here $q : \mathscr{T}_0 \to \mathscr{T}_{\simeq}$ is the canonical projection. Define the structure of a $\mathscr{T}$-natural system on $q^*D$ by $\nabla = \mathrm{id} : D \circ \mathbf{F}q \circ \mathbf{F}s = D \circ \mathbf{F}q \circ \mathbf{F}t$. In this way one obtains the functor $q^* : \mathrm{Nat}(\mathscr{T}_{\simeq}) \to \mathscr{T}$-Nat. Our Theorem 4.1 claims that the functor $q^*$ is a full embedding. Actually we also identify the essential image of the functor $q^*$. We need the following definition.

3.3. Definition. *A $\mathcal{T}$-natural system $(D, \nabla)$ is called* inert *if $\nabla_\varepsilon = \mathrm{id}_f$ for all $\varepsilon : f \Rightarrow f$.*

Inert $\mathcal{T}$-natural systems form a full subcategory of the category of $\mathcal{T}$-natural systems, which is denoted by $\mathcal{T}$-Inert. It is clear that the image of the functor $q^*$ lies in $\mathcal{T}$-Inert. It is also clear that $\mathrm{Aut}_{\mathcal{T}}$ equipped with the canonical $\mathcal{T}$-natural system structure defined in Example 3.2 is inert if and only if $\mathcal{T}$ is an abelian track category.

Let us observe that for any track functor $G : \mathcal{T}' \to \mathcal{T}$ restriction of the functor $G^* : \mathcal{T}\text{-Nat} \to \mathcal{T}'\text{-Nat}$ yields the functor $G^* : \mathcal{T}\text{-Inert} \to \mathcal{T}'\text{-Inert}$.

## 4. The main result

4.1. Theorem. *Let $\mathcal{T}$ be a track category. Then $q^* : \mathrm{Nat}(\mathcal{T}_\simeq) \to \mathcal{T}$-Inert is an equivalence of categories. Furthermore, for any track functor $G : \mathcal{T}' \to \mathcal{T}$ the diagram*

$$\begin{array}{ccc} \mathrm{Nat}(\mathcal{T}_\simeq) & \xrightarrow{q^*} & \mathcal{T}\text{-Inert} \\ \downarrow{\scriptstyle G^*_\simeq} & & \downarrow{\scriptstyle G^*} \\ \mathrm{Nat}(\mathcal{T}'_\simeq) & \xrightarrow{q'^*} & \mathcal{T}'\text{-Inert} \end{array}$$

*commutes.*

Proof. Let $E$ and $E'$ be natural systems on $\mathcal{T}_\simeq$ and let $\Phi : q^*E \to q^*E'$ be a morphism of $\mathcal{T}$-natural systems. We claim that if $f$ and $g$ are homotopic maps in $\mathcal{T}_0$ (and therefore $qf = qg$), then the homomorphisms $\Phi_f : E_{qf} \to E'_{qf}$ and $\Phi_g : E_{qg} \to E'_{qg}$ are the same. Indeed, we can choose a track $\xi : f \Rightarrow g$. Then we have the following commutative diagram:

$$\begin{array}{ccc} (q^*E)_f & \xrightarrow{\nabla_\xi} & (q^*E)_g \\ {\scriptstyle \Phi_f}\downarrow & & \downarrow{\scriptstyle \Phi_g} \\ (q^*E')_f & \xrightarrow[\nabla'_\xi]{} & (q^*E')_g \end{array}$$

By definition of the $\mathcal{T}$-natural system structure on $q^*E$ and $q^*E'$ the morphisms $\nabla_\xi$ and $\nabla'_\xi$ are the identity morphisms, hence the claim. This shows that the functor $q^*$ is full and faithful.

It remains to show that for any inert $\mathcal{T}$-natural system $(D, \nabla)$ there exists a natural system $E$ on $\mathcal{T}_\simeq$ and an isomorphism $\Delta : D \to q^*E$ of $\mathcal{T}$-natural systems. First of all one observes that if $\xi, \eta : f \Rightarrow g$ are tracks, then $\nabla_\xi = \nabla_\eta : D_f \to D_g$. Indeed, thanks to the property ii) of Definition 3.1 we have

$$\nabla_\xi = \nabla_{\xi - \eta + \eta} = \nabla_{\xi - \eta} \nabla_\eta = \nabla_\eta,$$

because $\xi - \eta : g \Rightarrow g$ and $D$ is inert. Therefore for $qf = qg$ there is a well defined homomorphism $\nabla_{f,g} : D_f \to D_g$ induced by any track $f \Rightarrow g$. Then the relation ii) of Definition 3.1 shows that $\nabla_{g,h}\nabla_{f,g} = \nabla_{f,h}$ for any composable 1-arrows $f, g, h$. By harmless abuse of notation we will just write $\nabla$ instead of $\nabla_{f,g}$ in what follows.

Since the functor $q : \mathscr{T}_0 \to \mathscr{T}_\simeq$ is identity on objects and full, we can choose for any arrow $a$ in $\mathscr{T}_\simeq$ a map $u(a)$ in $\mathscr{T}_0$ such that $qu(a) = a$. Moreover for any map $f$ in $\mathscr{T}_0$ we can choose a track $\delta(f) : f \Rightarrow u(qf)$. Now we put

$$E_a := D_{u(a)} \text{ and } \Delta_f := \nabla = \nabla_{f,u(qf)} = \nabla_{\delta(f)} : D_f \to D_{u(qf)} = E_{qf}.$$

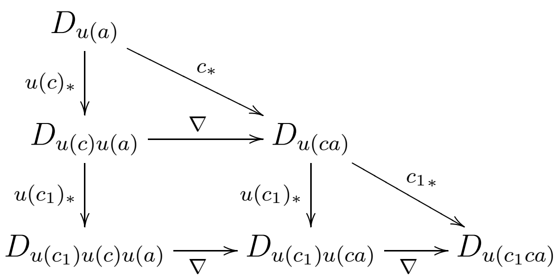
For a diagram $\xleftarrow{c}\xleftarrow{a}\xleftarrow{b}$ in the category $\mathscr{T}_\simeq$ we define the homomorphism $c_* : E_a \to E_{ca}$ to be the following composite:

$$E_a = D_{u(a)} \xrightarrow{u(c)_*} D_{u(c)u(a)} \xrightarrow{\nabla} D_{u(ca)} = E_{ca}.$$

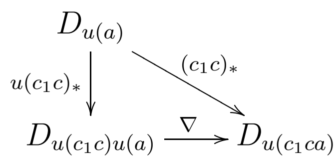
Similarly we define the homomorphisms $b^* : E_a \to E_{ab}$ to be the following composites:

$$E_a = D_{u(a)} \xrightarrow{u(b)^*} D_{u(a)u(b)} \xrightarrow{\nabla} D_{u(ab)} = E_{ab}.$$

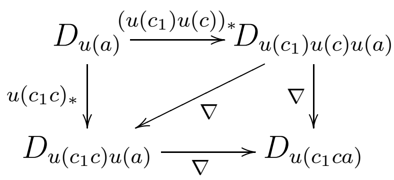
It follows from the property iii) of Definition 3.1 that for any diagram $\xleftarrow{c_1}\xleftarrow{c}\xleftarrow{a}$ in the category $\mathscr{T}_\simeq$ we have the following commutative diagram:




Thus $c_1(c_*) = \nabla(u(c_1)(u(c)_*))$. On the other hand by definition we have the commutative diagram:




It follows from the property iv) of Definition 3.1 that one has also the following commutative diagram




Therefore

$$(c_1c)_* = \nabla(u(c_1c)_*) = \nabla(\nabla(u(c_1)(u(c)_*))) = c_1(c_*).$$

Similarly $(b_1 b)^* = ({b_1}^*)b$ and $E$ is a well-defined natural system on $\mathscr{T}_{\simeq}$. It remains to show that $\Delta : D \to q^*E$ is a natural transformation of functors defined on $\mathbf{F}\mathscr{T}_0$. To this end, one observes that for any composable morphisms $g, f$ in the category $\mathscr{T}_0$ we have the following commutative diagram

$$\begin{array}{ccccccc}
D_f & \xrightarrow{\nabla} & D_{u(qf)} & & & & \\
\downarrow{\scriptstyle g_*} & & \downarrow{\scriptstyle g_*} & \searrow^{u(qg)_*} & & & \\
D_{gf} & \xrightarrow{\nabla} & D_{gu(qf)} & \xrightarrow{\nabla} & D_{u(qg)u(qf)} & \xrightarrow{\nabla} & D_{uq(gf)}
\end{array}$$

$$D_{gf} \xrightarrow{\nabla} D_{uq(gf)}$$

This means that the following diagram also commutes:

$$\begin{array}{ccc}
D_f & \xrightarrow{\Delta_f} & E_{qf} \\
\downarrow{\scriptstyle g_*} & & \downarrow{\scriptstyle (qg)_*} \\
D_{gf} & \xrightarrow[\Delta_{gf}]{} & E_{q(gf)}
\end{array}$$

Similarly the diagram

$$\begin{array}{ccc}
D_g & \xrightarrow{\Delta_g} & E_{qg} \\
\downarrow{\scriptstyle f^*} & & \downarrow{\scriptstyle (qf)^*} \\
D_{gf} & \xrightarrow[\Delta_{gf}]{} & E_{q(gf)}
\end{array}$$

also commutes and therefore $\Delta$ is indeed a natural transformation. ∎

Now let $\mathscr{T}$ be an abelian track category, so that $\mathrm{Aut}_{\mathscr{T}}$ is a natural system on $\mathscr{T}_0$ with values in the category of abelian groups. According to Example 3.2 it is equipped with the canonical structure of a $\mathscr{T}$-natural system, which is moreover inert, because $\mathscr{T}$ is abelian. Thus one can use Theorem 4.1 to conclude that there is a natural system $D$ defined on $\mathscr{T}_{\simeq}$ and an isomorphism of $\mathscr{T}$-natural systems $\tau : \mathrm{Aut}_{\mathscr{T}} \to q^*D$ defined on $\mathscr{T}_0$. This was the main result of [Baues & Jibladze 2002].

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