How Social Interactions Within a Class Depend on The Teacher's Assessment of the Students' Various Mathematical Capabilities
A Case Study[1]

Alain Mercier, Marseille (France)
Gérard Sensevy, Rennes (France)
Maria-Luisa Schubauer-Leoni, Genève (Switzerland)

Abstract: In this contribution, we will address from a *clinical* point of view the issue of the interrelations between the knowledge acquiring processes and the social interactions within a class of mathematics: a) how can the knowledge that is to be acquired determine the kind of social relationship established during a didactic interaction, and b) reciprocally, how can the social relationship already established within the class influence each and every student's acquisition of knowledge?

Kurzreferat: *Soziale Interaktionen im Unterricht: ihre Abhängigkeit davon, wie der Lehrer die verschiedenen Fähigkeiten der Schüler bewertet. Eine Fallstudie.* In diesem Beitrag werden aus klinischer Sicht die Beziehungen zwischen Prozessen der Wissensaneignung und sozialen Interaktionen im Mathematikunterricht behandelt: a) Wie kann das zu erwerbende Wissen die Art der sozialen Beziehung in einer didaktischen Interaktion bestimmen? b) Wie kann umgekehrt die bereits bestehende soziale Beziehung in der Klasse den Wissenserwerb eines jeden Schülers beeinflussen?

ZDM-Classification: C50, C60, C70

1. Framework

The second part of the question (b) (see Abstract) has been amply debated upon, since Rosenthal & Jacobson (1968) showed the influence of the teacher's expectancies upon a student's cursus. Many other studies have gone further, emphasising that such expectancies may refer either to presupposed capabilities or may concern more precisely social stereotypes (Mehan 1978) within a labelling theory. The question goes beyond both teachers' expectancies and the labelling theory since some studies (Sirota 1986, Broccolichi 1995) clearly show how both the teacher's questioning and the student's ensuing answers or his/her subsequent speaking initiatives depend hugely upon the student's position within the school ratings, even more than upon their parents' social backgrounds. However, whether in those latest studies or in the previous ones, the very object of the interactions i.e. how expertise and knowledge might depend on the teacher's expectancies has not been precisely addressed. We never found any study showing their effective weight, hic et nunc within the class, upon the student's acquisition of a piece of knowledge.

That's why we are addressing the question from its primary perspective (a): how can a student's expertise give him/her such or such a social position within the class? Specifically, does the teacher's assessment of a student's mathematical capabilities influence the expectancy process within a social interaction which stakes are and remain "studying the piece of knowledge taught"?

The French speaking didacticians and social psychologists are currently using a fundamental notion: the *didactic contract* (Brousseau 1984, 1986, 1997, Chevallard 1988, 1992, Schubauer-Leoni 1986, 1996). According to us, this notion can contribute to raise in a convincing way the issue we want to address. Brousseau defines the didactic contract as a system of reciprocal expectancies between teacher and students, concerning knowledge, that establishes the context for both student's and teacher's acts related to the subject matter to be taught and learnt, and that can offer a framework in which to explain these teaching and learning acts. Using a sociological theorisation, some didacticians (Schubauer-Leoni 1988) have then claimed that this covenant shaped a group habitus (Bourdieu 1990) within the class. Reay (1995) shows how the notion of habitus helps to make visible the taken-for-granted inequalities embedded in social processes; we emphasize their conversion into inequalities in the process of teaching, setting studies, and learning.

The notion of didactic contract seems quite momentous to us, so that we agree with Krummheuer and Wood in this ZDM-issue: both of them think at the mathematical class as a place where reciprocal expectancies may produce social and cognitive interactions. We emphasize the point, according to Krummheuer, that argumentative work may exist if a narrative work was overdone (Krummheuer 2000) and, according to Wood, that inquiry and argument patterns may exist when an action was done and when exposition patterns were overdone (Wood 2000): we recognise here Brousseau's *adidactical* dimensions of the didactic relation (Mercier 1995). But, although emphasizing the didactic standards and categories which are suitable within a given class of mathematics – considered then as an institution (Douglas 1986), the notion of didactic contract stricto sensu fails to help us understand how each and every student inhabits (privately) those standards (Sarrazy 1996, Schubauer-Leoni 1997a). So, in order for us to bring out this, we first show in an empirical way how each and every student occupies specific niches within the didactic contract. The following lines may be considered in this respect as a first approach, and we will base our contribution on recent research from a clinical point of view in didactics (Mercier 1997, Schubauer-Leoni 1997a, Sensevy 1997, Salin 1997). Then, as the notion of a didactical contract refers to the class as a whole, and as different students are offered different didactic standards within the same class, according to their position in the school hierarchy, we'll try to understand this phenomenon.

2. Social interactions and acquisition of knowledge within the class of mathematics: the case of Jerome and Louis

The context is a lesson on numeration in a primary school class (4th grade, in France CM1) during which the students work upon how to write large figures, through a number's dictation. The study was based on the observation of the class, and we developed several series of anal-

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ysis levels of the videotaped interactions. A transcription of these interactions was devised in the following way. We noted the public utterances of each actor implicated during the lesson together with their public graphic gestures when they came to the blackboard to explain their answers and strategy.

We wanted to achieve a representation of the *didactic space,* by way of modelling the sequence of events through our theory. The mathematical analysis of the problem gave us the didactical problem, from the teacher's and from the students' point of view (Mercier 1997). The first level of conversational analysis led us to the teaching problem for the teacher. Then the analysis of the teaching team's preparation led us to the teacher's project (Schubauer-Leoni 1997b); and the sequencing of the lesson, for which we analysed some students' acts and the teacher's teaching gestures. This analysis enabled us to understand some of the problems the students encountered and the way the teacher conducted her project when meeting a didactically delicate teaching situation (Sensevy 1997). Through this observatory clinical method, we had the opportunity to offer a description of this mathematics class, seen as a social space set by *didactics stakes* (i.e. collective production and personal acquisition of mathematical knowledge). These didactic stakes determine *social stakes* (i.e. increased acknowledgement of one's capability to acquire or produce mathematical expertise): a space where one and each student follow his/her proper route. So, as to give a bird's-eye view of these analyses, we'll present a few moments of two selected students' public work at the blackboard. Each of them spent more than a quarter of the teaching period there; Jerome came without any question, and he learned mathematics while staying at the blackboard, when Louis came with a good question and did not find any answer: we want to explain the difference.

During the lessons about large figures, all the students were able to read such a large number as 2340105, when the teacher writes 2 340 105, separating classes of millions and thousands, so that they read the orders "two" for the class of millions, "three hundred and forty" for the class of thousands and "one hundred and five" for the class of units: they have a name for each place value. At this moment, they were ordered to write "seventeen million two thousand and fifty eight". The teacher is aware that some students will write "17 200 058": referring to the teaching team preparation, this is the expected mistake; the teacher chooses one of the students that has in fact made this, Jerome, to go and write his solution onto the blackboard. Jerome is one of the best in mathematics, one can therefore deduce that the teacher takes it for granted that he is capable of explaining the strategy he uses: in her lesson's preparation, the teacher wrote that she should choose "a fairly good student". So Jerome would seem to play the part of a teaching auxiliary.

2.1 Jerome

Here is a summary of the whole set of interrelationships between teacher, class, and Jerome, when this latter is in front of the blackboard. First, Jerome fails to see where his mistake arises from (notwithstanding his teacher's asking the whole class for help) and he tries to prove that he is right. Let's look at Christian's help for example, in front of what Jerome has written on the blackboard:

seventeen | million | two | thousand | fifty eight
17 200

Minute 23
Christian: this (showing the 2) are the units ... and this are the tens and these (showing the two 0 from the right hand to the left hand side) are [the hun
Jerome: [but no ... this (showing the 2) is the hundreds ...
Christian: this (showing the 2 then the words two thousand) are the thousands ...
(several students are speaking at the same time)
The teacher: hush ... hush ...
Jérome: (while writing at the blackboard) there we write an *h* (for hundreds of thousands) ... there we write a *t* (for tens of thousands) ... and there the units (he writes a *u* for units of thousands) ...
(on the blackboard we can see ...)

seventeen | million | two | thousand | fifty eight
h t u
17 200

A student: he is wrong ...
(the teacher rubs out the *h, t,* and *u* Jerome has just written, so that Jerome's idea will be forgotten)
The teacher: then ... (taking Christian by the shoulders) they still disagree ... (one of her hands is on Jerome's shoulder, while she pushes Christian back to his seat with her other hand) they can't find a common ground (she has rubbed out "*h*", "*t*", and "*u*",) ... She then says "who can come and give them a clear solution which both can agree on ...?"
Some students: me ... me ...
The teacher: Fatia ... okay ... come ... let her come ...
Minute 24
Jérome: oh yeah ... (whispering)

One can see, at the end of that series of turns that Jerome, whose mistake everybody tries to explain, shows a new understanding (oh yeah), a dawning appearing almost by itself.

As for Fatia, the student called to help Jerome, she develops her own explanation: she leads. However, noticing her first hesitation (in fact, Fatia was looking for a piece of chalk), the teacher gives back the leadership to Jerome (who had kept the piece of chalk). This decision attests the fact that the teacher had noticed Jerome whispering (oh yeah), so that the whole scenario (lasting ten minutes) of the successive presence of five students in front of the blackboard seems to have had but one function: give Jerome some respite, so as to enable him to look for a solution to his problem.

Which he does ... Despite the teacher's rubbing out the marks for hundreds, tens and units (the three orders in the class of thousands): Jerome keeps thinking with them. The mathematical analysis of Jerome's gestures shows that Jerome, when making the mistake and writing *17 200 058* on his rough copy, used two rules which dialectics he had been unable to use in a higher efficiency structure encompassing and furthering both of them. Here are these implicit rules:

R1: put the figures into the proper class;
R2: put three figures for a class;

but for Jerome, both these rules remain unconnected, so to speak self centered, devoid of any dialectic unit. Jerome's mistake may have been produced in such a way: he wrote figures into the proper class, 17 for seventeen (million), 2 for two (thousand), and 58 for fifty eight (units), writing then three figures for a class (completing with three 0s). In the episode we have just briefly introduced, he discovers a new rule R3, encompassing and furthering the previous ones so that he can articulate them.

R3: give each order a figure

Jerome obtains this new rule by reconstructing "the large figures' numeration system", starting from the small figures' one, as the following dialogue shows, when he is writing ... *fifty eight:*

Minute 27

Jerome: as three are left there ... there ... three are left ... (shows on the blackboard) so
The teacher: wait ... wait ... I think [that
Jerome: [zero hundred (writes "0") zero hundred ... there is zero hundred ...
The teacher: and [then ...
Jerome: [or else we would have said one ... one hundred and f ... fifty eight ... and as there are none we write this (points at the last "0") then we write the fifty (writes "5") and eight (writes "8" so that we can read "17 002 058") ...

Jerome managed to solve the problem, thanks to his present numbering knowledge. For example, he writes *058* because *0* must occupy the order that would be occupied by *1* if he had had to write one hundred and fifty eight, so that the rule R3 (give each order a figure) is achieved. So, he can give a figure for each place value, but he doesn't count the place values for "seventeen millions" if he doesn't write down *17 000 000.* This mathematical analysis of Jerome's utterances and writing gestures shows that he has really performed a creative work, which has enabled him to acquire the knowledge at stake in an original way. So he seems to have fullfilled the teacher's expectations, which would confirm his staying long minutes in front of the blackboard, protected by the teacher as in a kind of airball where he could "snatch" here and there what suited him from other students' help, while keeping thinking aloud. This conjunction of favorable elements enables him to build a new personal knowledge which he displays. However the teacher does not repeat, for all the other students. Studying her preparation shows, by the way, that she never had the least intention to rely on the small figures' writing rules to describe and prove how large figures' writing rules work: every rule in that field remains implicit.

2.2 Louis

Jerome's error was expected from some of the students, but Jerome's mathematical learning was not expected from any one of them: the teacher is the only one who expects the learning. But Louis' idea disturbs the teacher: "Why write the 0, he says ? Either we put all of them or we put none": he suggests the writing *17 2 58,* such intervals showing here the successive class names million and thousand. When asking Louis to come to the blackboard, the teacher seems to expect that the class will help Louis. Thus a very long episode (from minute 27 to minute 45) starts where the teacher, helped by students, endeavours to bring the heretic Louis to reason, but to no avail. Nevertheless, Louis does not write anything on the blackboard. We can then notice this remarkable fact: Jerome will write Louis' proposition in such a way that the teacher together with the students has but one strategy to attempt ensuring Louis' agreement at hand, which consists in asking him to read the number anew.

On the blackboard, since minute 30, we can see:

seventeen	million	two	thousand	fifty eight
17		**002**		**058**
				17 2 58

Minute 45

The teacher: (to one student) no you ... shut up ... enough is enough ... (to Louis) look Louis ... honestly ... that number over there ... were you told that this seventeen was somewhere when it was written that way ... read this number for me ... if I give you the name of this number read good god ... hush ... what is this number here? come on, read ... shut up class ...

And Louis reads correctly! As the study of numbering is based on no mathematical reference which might enable the students to check that their conjecture is correct, the teacher has no solution available but to ask the student again and again to give his answer, until the expected answer appears. The teacher's failure with Louis allows us a better understanding of the teacher's success with Jerome. In this didactical environment, Jerome's position in the class can then become absolutely dominant. This is obvious through the following exchanges (see the minutes 30-32) when Nathalie, who came to back Louis in his aberration, is grilled by Jerome, who makes a brilliant comeback.

Jerome: (from his seat) no ... but because if we remove the "0" ... the two zeros over there ... well it isn't any longer the same number ...
The teacher: okay ... come here and show this to us ...
Nathalie: yes ... it still does give the same number ...
The teacher: hush ...
Jerome: still the same number ... I ask you ... read it to me and then you see ...
The teacher: okay ... go ahead ... do it ... explain it to them ...
Jerome: wait ... here is the chalk ... (to Nathalie) so we are going to do it like you do ... aren't we ...
The teacher: (to Jerome) well ... wait ... because they aren't going to see ... there I am afraid they won't see ... yes write it for us there ... write for us there ... and move a bit ... wait for us ... (to the class) you'll see all the same at last ... well we are going to see what he wants to show us ... (to Jerome) don't write in too large a hand ... I'm afraid that there won't be enough space there (Jerome writes on the right side of the blackboard

17 2 58

(background noises)

The teacher: hush ... hush ...
Jerome: (to Nathalie) well come on ... read it to me now ...
Minute 31
Nathalie: it is still the same number ...
(protests can be heard)
The teacher: shush ... shush ... well ... here I think that Jerome has ... as for the others well ... do you think like Nathalie that this (shows 17 2 58) is the same number as that (shows 17 002 058) ...
(one can hear yes ! yes ! no ! ...)
The teacher: tell us Louis ... hush ...
Louis: because here there are intervals ... so this always means millions ...

One can see how Jerome plays the part of deputy teacher, who shuffles pieces of chalk and who behaves with Nathalie as a teacher would with a student. The teacher completely agrees with this behavior, she may hope that Jerome, as a didactic collaborator, will manage to clear up the situation. He has the status of deputy teacher, which is proved by this teacher's question (minute 38) to him, as he is standing up near the blackboard while two other students are sent back: "Come on and ask Louis something which would enable him to become aware of his mistake".

3. Classroom game, mathematical game, didactical game

"Modelling a teaching situation consists of producing a game specific to the target knowledge, among different subsystems: the educational system, the student system, the milieu, etc. There is no question of precisely describing these subsystems except in terms of the relationships they have within the game." (Brousseau 1997, p. 47)

These few minutes of the classroom's work *shape a mathematical* and *a social game.* A rapid study of them show, according to us, how closely knit they are with the *didactical game.* In other words, the ranking and categorisation process of an individual does not seem to be generated by any outside circumstance, but primarily from its particular position as a student, in the class' didactic covenant. Thus Jerome is a strong leader. He builds at the blackboard a new personal knowledge in mathematics in an almost private way, though he is helped by all the other students. He also interferes within the class in the capacity of a deputy teacher who is given the task to convert the dissidents and to explain Louis' mistake to him while nobody, even the teacher, managed to do so.

What would have happened if Jerome had made the same mistake as Louis? A very different interactional network, doubtlessly.

One must therefore understand how within this context didactical interactions are social ones as well. The students are assessed by the teacher, keeping in mind the background of a common didactic experience which gives them, or fails to do so, some sort of didactic legitimacy, which seems to produce most of social power in the classroom. In this episode, instead of being their own personal mathematical job, students' work is nothing but the strict obedience to behavioural rules, since no adidactic situation for a precise mathematical problem has ever been designed by the teacher (Brousseau 1986, 1997). In this class' didactic contract, students' solving numbering exercises presupposes harnessing the rules but that is not enough. Social legitimacy is given to those assessed students whom the teacher can trust, and so who can have (yet again) the upperhand over the actual mathematical work.

The analysis of the here and now of the class then shows that some students such as Jerome don't have to respect the same working methods as the whole class, and that the didactic insight they were able to develop in the past is a capital which they can invest in the present situation. The interaction analyses thus show that a few students are not corseted by the rules and the didactic contract, and may allow themselves to think by themselves, as Jerome does. Most students, like Louis, cannot refuse to "play by the rules" because their position is impossible to maintain since it questions the very legitimacy of what is learnt: indeed, why accept one convention rather than another one if the one chosen can't be explained in a more convincing way ? But such a claim does not afford any legitimacy.

Here, Jerome's mathematical production (min. 26–27) is not taken for granted, as the teacher does not repeat for the class. Nevertheless Jerome waves from his seat (min. 30), so that he is expected to explain the rules (min. 31), which legitimate him. He then uses the very knowledge that he produced twenty minutes previously. His mathematical legitimacy (min. 31) is a didactical effect of his social legitimacy (min. 20–26), which gives him back more didactical power (min. 30–32). This is not the case for Christian (min. 23) or Fatia (min. 24): have they less social or mathematical legitimacy than Jerome? Louis' question asks for the rationality of the implicit rules, taking these rules for mathematical knowledge (min. 27). The consequence is that his didactical power is not reinforced during the competition with Jerome: and his social and mathematical legitimacy fall, just as Nathalie's do (min. 30).

In the didactic interaction that we are observing, only those students who are conversant enough with the school work to accept its social and didactical rules may sometimes bypass them for their own mathematical use. Referring to the observed episodes, Jerome is one of them, Louis or Nathalie are not. Such a student learns mathematics: his legitimacy and domination is thus increasingly justified, as "the teacher's assessment of his mathematical capabilities influence the expectancy process". Other students may learn, the assessment of their capabilities will not afford any legitimacy but this one: they remain students and are allowed to follow on studying.

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Authors

Mercier, Alain, Prof., HDR, INRP, Pôle technologique de Château Gombert, UNIMECA, 60 rue Joliot Curie, F-13453 Marseille Cedex 13, France
E-mail: mercier@univ-aix.fr
Sensevy, Gérard, Prof., HDR, IUFM de Bretagne, Site de Rennes, 153, rue Saint-Malo, F-35043 Rennes Cedex, France.
E-mail: gerard.sensevy@bretagne-iufm.fr
Schubauer-Leoni, Maria-Luisa, Prof., Dr. Université de Genève, FPSE, 9. Route de Grize, CH-1227 Carouge, Suisse. E-mail: schubauer.leoni@bluewin.ch