The Role of Real World Scripts in the Teaching and Learning of Addition

Alison Price, Oxford (Great Britain)

Abstract: Analysis of classroom observations of young children learning addition has identified a variety of representations used by both teacher and children to communicate mathematics. One of these representations is categorised as "real world scripts". This paper will consider the characteristics of such scripts and the role they play in enabling children's learning. The reasons for this will then be discussed with reference to Bruner's concept of narrative as a means of learning.

Kurzreferat: *Die Rolle von Alltagsskripten beim Lehren und Lernen der Addition.* Eine Analyse von Beobachtungen kleiner Kinder beim Erlernen der Addition im Unterricht hat eine Vielfalt von Darstellungen identifiziert, die sowohl vom Lehrer wie auch von den Schülern bei der mathematischen Kommunikation benutzt werden. Eine dieser Darstellungsweisen wird als "Alltagsskript" kategorisiert. In diesem Beitrag werden Merkmale solcher Skripten betrachtet sowie deren Rolle beim Lernen der Kinder. Gründe dafür werden in Bezug auf Bruners Konzept der Erzählung als ein Lernmittel diskutiert.

ZDM-Classification: C51, C71

1. Introduction

This paper arises out of a wider research project which sets out to describe and analyse the learning of mathematics in early primary classroom environments. The focus is on the interactions between teacher, child, activity and situation, and on the evidence of learning that ensues. The children observed are aged 4 to 5 years in the Reception class (two years before grade 1) of a primary school. The data is qualitative, collected as a result of observation by the researcher in the classroom. Analysis of the data follows procedures suggested by Strauss and Corbin (1990) who speak of "the discovery of theory from data systematically obtained". Each teaching episode is analysed in depth in order to generate theory rather than to test existing theory.

This theory generation is embedded in a broader theoretical framework of communication through interaction. I have established three theoretical positions relating to the study. The first is a theory of learning which must address both individual and socio-cultural aspects of learning. The second is a model of communication as seen in terms of socio-cultural tools used to communicate understanding. The third is a view of mathematics teaching and learning as mathematization of social context rather than application of formal knowledge. These are not separate but interrelated theories which both underpin and are drawn out from my observations in the classroom, as described below.

One aspect of the analysis has involved consideration of the way in which addition is represented to the children. The key categories of representation identified are manipulative materials, spoken language, pictures, symbols and "real world scripts"- representations where the mathematics is contained within a context or story about the world outside the classroom (following Lesh et al. 1987). Analysis shows that the children show more misunderstandings in teaching episodes which do not contain real world scripts than in those which do. I will explain how this representation emerged and characterise it with reference to the data. Finally I shall relate these characterisations to Bruner's theory of narrative (1996) to explain why "real world scripts" may be effective in introducing children to mathematical concepts.

1.1 Individual and sociocultural aspects of learning

Current theories of teaching and learning in mathematics education consider both the individual as a constructive learner, and the social interaction between the learner, teacher and others as learning takes place. So, von Glasersfeld (1995) describes a radical constructivist view as having two basic principles, that "knowledge is not passively received but built up by the cognizing subject" and that "the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality" (p. 18).

But, von Glasersfeld does not recognise a special and separate place for the role of social interaction in this construction process, asking "how do these 'others', the other people with whom the child populates his or her experiential world, differ from the innumerable physical objects the child constructs?" (p. 12).

In contrast, Vygotsky (1978) emphasises the role of the teacher or more knowledgeable peer within social interaction, in terms of the zone of proximal development. He discusses the role of culturally developed tools for thinking in terms of language and sign systems. "Children solve practical tasks with the help of their speech and communication" (1962). The adult's more experienced understanding of these cultural tools enables them to scaffold the child's learning. Vygotsky's view of the learning is not that of Piaget's isolated learner exploring the environment but that all learning occurs first on an intermental plane, between minds, and then on an intramental plane, within the mind. So, Yackel et al. (Yackel, Cobb, Wood et al. 1990) begin their study from "the view that mathematics is a creative human activity and that social interaction in the classroom plays a crucial role as children learn mathematics" (p. 20). Bruner affirms both the role of the individual and the role of culture when he remarks:

> "I have come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make his knowledge his own, but that he must make it his own in a community of those who share his sense of belonging to a culture." (Bruner 1986, p. 127)

In this study the focus is on the individual learner making sense of addition tasks, but the context is that of the classroom where, through social interaction, the individual culture of the young child newly starting school and the developing shared culture of the classroom come together in teaching and learning.

1.2 Socio-cultural tools used to represent and communicate mathematics

Analysis of the social interaction within the teaching and learning situation requires consideration of the elements of social interaction. Vygotsky emphasises language and sign systems (written language and other symbol systems) as cultural tools, enabling a shared understanding of the physical objects or social situation under discussion.

Lesh, Behr and Post (1987) have identified five ways in which children represent mathematics when carrying out problem solving activities (see Fig. 1 below). This model can be used to identify what representations of mathematics are being used by both the teacher and the pupils. Communication can be seen through the interaction between these representations.

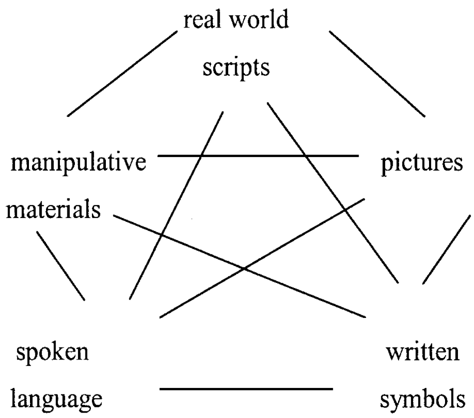


Figure 1: Elements of communication adapted from Lesh et al. 1987

Four of these elements in Figure 1 are common within the context of mathematics education. Spoken language, written symbols, pictures and manipulative materials have been used extensively in primary mathematics classrooms in the United Kingdom. However, the use of story contexts as representations of mathematics, while present in many Nursery and early primary classrooms (see for example Davies and Pettitt 1994), had been little discussed. Lesh et al. use the expression "real script" for one "in which knowledge is organised around 'real world' events that serve as general contexts" (p. 33) in the context of problem solving. To emphasise the "real world" element as well as the story context I shall call these "real world scripts".

These five representations can be seen as socio-cultural tools within the mathematics classroom. However communication requires not only the "sending of a message", but also interpretation of this message on behalf of the hearer (Steinbring 2000). The assumption that the child gives the same meaning to a communicative act as the adult resulted in many of Piaget's conclusions about children's ability to learn which have since been challenged. Donaldson et al. (1978) showed how the child was making *human sense* of the situation through their prior understanding of the mathematical situation, their understanding of the language being used and their understanding of the social situation. Children seemed to rely on those areas of understanding in which they felt most confident and for young children their understanding of the social world seems the most highly developed, while their mathematical understanding is less confident.

So, a theory of social interaction which also allows for the active construction of knowledge by individual learners must also take into account their prior learning, understandings and confidence within the domains of mathematical, linguistic and social understanding. Children's *mis*conceptions can be explained as arising out of alternative readings of the complex social situation which we call the mathematics classroom.

2. Representing mathematics: the identification of "Real World Scripts"

In all the lessons observed the focus of the teaching and learning was on early addition. The children already had some concept of addition; they were able to play a game involving the throwing of two dice and find the total score, usually by counting all.

One series of lessons involved developing the children's concept of addition to include the idea that a number could be partitioned in different ways producing subsets which could then be re-aggregated. The re-aggregation can be described using the language of addition, reinforcing the children's use of mathematically specific language. So, 4 can be partitioned as 0 and 4, 1 and 3, 2 and 2, 3 and 1 or 4 and 0. In each of these lessons a different total number was selected and a different context was used. This enabled analysis of lessons in terms of the children's understanding of the same concept within different contexts.

Three lessons will be presented here, though these are representative of others observed. Each of the lessons presented started with a whole class introduction, following which the teacher, Beth, worked with a group of five children: Emma (5), Angela (5), Charles (4), Ian (5), and Jacob (5).

2.1 Farm animals in fields

The lesson started with the children around the table with Beth. On the table are sets of three farm animals and a card for each child with two irregular fields drawn on.

Beth Now we've all got a little picture of two fields. Can you put your animals in the fields. Let's have a look at our animals. Ian, how many in that field (3) and in that field? (none) How many altogether? (3) So you started with three and you've still got three in all your fields.
Beth Jacob, tell me about your fields.
Jacob Got 2 pigs in that field and 1 pig in that field.
Beth How many altogether? (3) You've still got three. What about you Emma?
The children each explain sets of animals in similar language.
Beth Now, can we all make our sets look like Ian's? Can you tell me a little number sentence about what you've got here?
Angela We've got three and none (Charles interrupts with zero) and altogether got three.
Beth Now, all the cows walk into the other field. Tell me a different number sentence.
Ian We've got zero and three and altogether got three.
Beth Can you make one of your animals walk into another field? What have you got now?
Jacob Got one in that field and two in that one ...

The initial analysis of this lesson showed elements of spoken language, manipulative materials (animals), and later, as the children recorded their work, pictures and number symbols. Further analysis showed that there is a significant element of story, however rudimentary, a "real world script" as the animals move around their fields. The children were all able to use the animals and fields to partition and use appropriate language to explain their work in terms of addition.

The lesson went as planned with no apparent evidence of miscommunication. It is included here as a contrast to the next lesson we will look at.

2.2 Partitioning eight cubes

The children started with a worksheet (see Figure 2) containing rectangles and were asked to select eight multilink cubes which they placed in the top section.

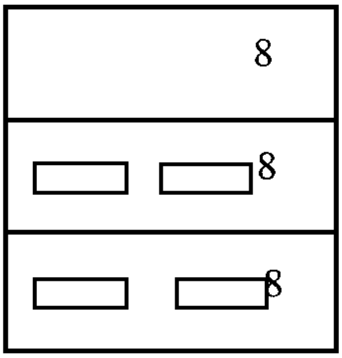


Figure 2: Cubes worksheet

Beth Can you put some of the eight in this set and some of them in this set? (indicating the two rectangles on the middle row.)

There is a problem here since no more than three cubes will fit into each rectangle drawn on the sheet. Angela fixes hers together in a square, Charles tries to squash his in, while the others let them spill out over the lines. All the children place 4 in each subset except Emma who has 3 and 5.

The children took turns to describe their cubes as a number sentence.

Beth So, everybody except Emma made "four and four altogether make eight" and Emma made "five and three altogether make eight". Now ... I want you to change your number story now, but to a different number story in the bottom sets.

All the children create a different number sentence with their cubes except Angela who still has 4 and 4, since she had merely transposed the cubes to the opposite boxes.

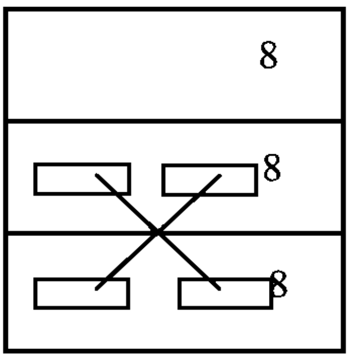


Figure 3: Transposition

Beth It's still 4 and 4. Can you break it up and make it different?

Angela removes one cube from the left hand set and hides it in her hand.

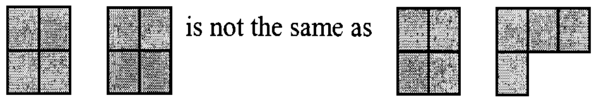
Angela Three and four together make eight (very hesitantly, looked at the extra cube and put it back in the three but this time to make an L shape rather than a square) Four and four together make eight.

Beth Can you change the cubes? (Angela looks puzzled.) Have a look at some of the others to see if it will give you a clue.

Angela repeats her procedure of removing one and counting the rest "1, 2, 3, 4, 5, 6, 7."

In terms of elements of communication this part of the cubes lesson consists of manipulative materials used to model the mathematics and spoken language used to describe it. The teacher has started from the mathematics and used the multilink cubes as a model. However, superficially there seems little difference between this and the animals lesson in terms of mathematics. The numbers used are greater but Angela can confidently count out objects to thirty. So what causes Angela such problems?

First the cubes will link together and, once linked, the mathematical model is changed in a subtle way. For Angela the shape of the resulting structure seems to take over. The cubes are no longer seen as individual items but as an entity. When asked to change them she can no longer focus on the cubes being the same numerically, since they are so obviously different with respect to shape.



The choice of multilink cubes, which seem socially neutral compared with the animals and should therefore be less distracting, causes more distractions. Are the cubes more distracting than the animals because they have characteristics of colour and are able to fix together? But there are also characteristics in the animals that could have distracted the children – they are small, real animals are not made out of plastic, plastic animals cannot walk, and the fields have no grass in them. The children questioned none of these, realising that because it was a story some characteristics were representative rather than real. In the case of the cubes Angela does not know what the cubes are meant to represent and is unable to express her frustration, while Beth is unable to clarify it for her. Communication is therefore limited.

The teacher sees the mathematics as situated in the manipulation of cubes from which generalisations about number pairs that make eight can be generated. However Angela wants the task to make sense in her social, experiential world and is confused when it does not seem to.

2.3 The bus lesson

The bus session starts with the whole class of twenty-seven children sitting together with their classteacher. The teacher, Beth, introduces the session with a rhyme. Each time the rhyme is said, different numbers of children are said to get on the bus creating pairs of numbers for addition.

The teacher takes a situation, children travelling on a bus, which can be used mathematically to exemplify the concept of partitioning ten into two subsets (upstairs and downstairs) and which can then be explained in the language of addition to re-aggregate the subsets. In this example the teacher uses pictures, of children and the bus, to model the situation and uses spoken language and later symbols to describe the situation in mathematical terms. However for the children, the mathematics is also situated in a real world script, that of children travelling on a bus, an experience which they have shared as a class. The teacher is therefore able to relate the mathematics to their shared social experience.

Beth Here comes the bus it soon will stop, Hurry up children in you pop,
5 inside and 5 on top, How many altogether?
The children raise their hands and one is chosen to answer. Each time the total is ten.
Beth Well done ... Now, can we all see the board?
Beth draws a bus outline on the board. She has also prepared ten circles of card with children's faces drawn on them, and with blu-tack on the back so that they will stick onto the board.
Beth Let's see if we have got ten children (counts the faces with the class). Now, all the children want to go on the bus. Do you think they are going to the Garden Centre, or perhaps they are going swimming.
Beth Now, let's put some on the bus (puts five faces onto the top deck counting) 1, 2, 3, 4, 5. So, how many inside (places rest of faces onto lower deck counting) 1, 2, 3, 4, 5.
Charles What about the bus driver?
(this is ignored)
Beth Who can tell me a number story about this?
Emma Five add five altogether make ten. ...
Charles (again) What about the driver?
Beth We're not worried about the driver. I think he has gone for a cup of coffee. Now, while he was having his coffee some of the children started to run about, so ... she went upstairs ... and so did he ... and so did she (moving a face each time.)
Beth Who can tell me something about the numbers now?
Ian It still makes ten
Beth Very good. Ian, can you tell me about this? How many on top?
Ian Eight.
Beth And how many downstairs?
Ian Two
Beth So, can you tell me anything else?
Ian Eight add two altogether makes ten.
...

The fact that the mathematics is contained within the context of a bus journey does not mean that it is ambiguous. Charles is concerned with the driver. However he is able to use his knowledge of bus journeys to articulate his concern and Beth is able to use the story element to deal with his concerns. This is in contrast to Angela's problem in the previous excerpt, where she was unable to articulate her problem so Beth was unable to recognise and deal with it.

2.4 Real world scripts identified

As a result of analysis of these and similar teaching episodes I concluded that where the lesson contained elements of story, grounded in the real world experience of the child, then the children were more able to make sense of the mathematics and more able to talk about problems which did arise. This story element I categorised as a "real world script". The scripts identified a shared social and cultural world in which the mathematics could be communicated through shared meaning. Because the children had been in school for only a short time they had not yet built up, amongst themselves and with the teacher, a shared mathematical world in which to work. They relied on a shared understanding of the wider world. What, then, are the characteristics of a real world script which enable this understanding?

3. Characteristics of real world scripts

In the lessons containing a real world script I identified five common factors: animation, integration, motivation, shared culture and interpretation.

Firstly the objects (people, animals, beetles etc.) were all *animate* objects. In many mathematical situations, and particularly in addition, the mathematics describes change or movement. Martin Hughes (1986) studied children's use of symbols to record addition and subtraction. None of the children used formal addition and subtraction signs (+, −) but several of the children drew a hand effecting a change in the number of cubes present, while one child used pictures of soldiers marching on (+) or off (−) the page. Animate objects are capable of change through movement in a way that inanimate objects are not.

So, Beth is able to instruct the children:

Beth Now, all the cows walk into the other field. Tell me a different number sentence.

Or

Beth Now, while he (the driver) was having his coffee some of the children started to run about, so ... she went upstairs ... and so did he ... and so did she (moving a face each time.)

Whereas in the cubes lesson the instructions have no animate meaning; the emphasis is on the child to effect the change.

Beth Now ... I want you to change your number story now, but to a different number story in the bottom sets ...
Beth It's still 4 and 4. Can you break it up and make it different?

Animation, the first characteristic of a real world script, generates a condition of change which can be described mathematically.

Secondly, because there is animation in the action, the mathematics can be seen as an *integral* part of the real world script and not merely added on for cosmetic purposes, to make the mathematics more interesting. It helps to define the concept. The children moving around on the bus demonstrate ways in which they can be partitioned in different ways. So too do animals moving from field to field, The movement brings about the changed mathematical state, a different combination of numbers which make the total. Representation of mathematics through real world scripts requires not just a story, but a story where the mathematics is an integral part of the story line.

Thirdly, animate objects are considered to be *motivated* to change. There can be a good reason for an animal to want to move from one field to another. Children enjoy running up and down stairs. Later in the bus lesson Beth exclaims

Beth They are a bit naughty these children running up and down the stairs,

demonstrating further the motivational element of the story. Inanimate objects are not mobile, nor are they motivated. Why should multilink cubes be partitioned in different ways except to represent the mathematics? Cubes are not sentient, they have no motivation.

The fourth characteristic of real world scripts is that they allow the teacher and children to have a shared understanding of the context. The contexts chosen arose out of the *shared culture* of the classroom and society. Examples may not reflect actual experience; it is possible that Jacob may not have seen a live pig, farms around the school tending to keep cows or sheep, but pigs are part of the shared culture, in stories and on the television. In the bus story Beth refers directly to their shared experiences:

Beth Now, all the children want to go on the bus. Do you think they are going to the Garden Centre, or perhaps they are going swimming?

The children had recent experience of travelling in buses. The previous week the class had been on a visit to a large garden centre as part of their science work on plant growth. They also went swimming once a week during this term. For each of these they were taken in a bus. So the *shared culture* behind the story helps the children to understand the mathematics contained within it.

Finally, real world scripts allow the teacher and children to negotiate text, e.g. to discuss the absence of a driver, since they allow for *interpretation.* Charles interprets the situation as lacking a driver, Beth is able to reinterpret it to explain the naughty children. This is in contrast to the cubes lesson where the teacher was unable to understand why Angela was having trouble understanding her instruction "Can you break it up and make it different", because there was no context in which to interpret the instruction. Angela makes a literal interpretation, breaks the cubes and makes them look different. It is the nature of real world contexts to allow for interpretation, but without a real world script Beth sees the cubes as representing the mathematics and cannot understand why Angela cannot see it too. This is reminiscent of Paul Cobb's work on the use of manipulatives in teaching mathematics (1987). The teacher sees the mathematics as situated in the manipulative materials, for example sees Dienes apparatus as representing place value, but the children see only the apparatus and have no way of interpreting it because of their lack of place value knowledge. Cobb concludes that the use of manipulatives in teaching place value is therefore problematic.

Each of these factors: animation, integration of the mathematics, motivation, shared cultural knowledge and interpretation, which seem to effect successful learning in mathematics, can be seen as characteristics of real world scripts. But they do not explain why such characteristics should be valuable in teaching addition. For that I want to relate them to Bruner's theory of narrative.

4. Real world scripts as examples of narrative

Bruner (1996) argues that human culture is framed by narratives. Our personal as well as our collective histories are told as narrative, stories which have a common structure even though the detail and the interpretations may change. He argues that

> "[i]t seems evident, then, that narrative construction and narrative understanding is crucial to constructing our lives and a 'place' for ourselves in the possible world we will encounter." (p. 40)

Through narrative, young children learn about their personal and cultural histories, their understanding of the social word and the world of language is developed not only through personal experience but also through the telling and hearing of narrative stories. Bruner asserts that

> "[u]nderstanding is the outcome of organizing and contextualizing essentially contestable, incompletely verifiable propositions in a disciplined way. One of our principle means of doing so is through narrative: by telling a story of what something is about". (1996, p. 90)

Can we apply this to learning mathematics in school?

If we see mathematics as about fixed truths then this may not fit with "contestable, incompletely verifiable propositions". We may then see narrative as applicable to the learning of the social sciences but not to mathematics. The nature of mathematics has been discussed at length (see for example Ernest 1991) and it is beyond the scope of this paper to rehearse these arguments. However these young children have not yet learnt that society may see some disciplines as "factual" and others as "relative". Children, constructing knowledge for themselves, will test all new knowledge in the light of their existing understanding. So, the learning of mathematics through narrative becomes a viable proposition. Is this, then, a description of what is happening in the use of real world scripts?

Bruner identifies nine "universals of narrative realities" (1996, p. 133 ff).

- The structure of committed time;
- generic particularity;
- actions have reasons;
- hermeneutic composition;
- implied canonicity;
- the centrality of trouble;
- ambiguity of reference;
- inherent negotiability; and
- the historical extensibility of narrative.

The real world scripts described above have a simplicity when compared to the complex narratives of literature or history, animals moving around fields do not make an exciting story, but do the characteristics of real world scripts have resonance with Bruner's universals? If we accept with Bruner that narrative is one of the principle ways in which we make sense of the world, then relating the characteristics of real world scripts to those of narrative may help to explain the effectiveness of real world scripts in teaching early addition.

4.1 Animation *and the structure of committed time*

Bruner talks of the "structure of committed time" as a characteristic of narrative explaining this in terms of "time that is bounded not simply by clocks but by the humanly relevant actions that occur within its limits" (p. 133). This can be seen in terms of the animation I identified as a characteristic of a real world script. It is not just a description of a static situation but contains action within time. The numbers of children up and down stairs, numbers of animals in the fields alter with time, while, without animation, the cubes remain the same over time.

4.2 Integration of the mathematics *and generic particularity*

Narratives, according to Bruner, contain particular examples which contain generic truths. So *literary* narratives can be categorised within genres, while *historic* narratives can help us to form principles which guide our future action. Mathematical contexts can also be seen as containing the general, the generic concept, within the particular. The stories of animals in fields or children on the bus contain the mathematics of partitioning and re-aggregation within the particularity of the story. So, real world scripts which exemplify a mathematical concept can be seen as having generic particularity within mathematics.

4.3 Motivation *and actions have reasons*

"What people do in narratives is never by chance, nor is it strictly determined by cause and effect; it is motivated by beliefs, desires, theories, values or other 'intentional states'. Narrative actions imply intentional states" (p. 136). So, Bruner identifies motivation as a narrative element; motivation is both a characteristic of real world scripts and of narrative.

4.4 Interpretation *and hermeneutic composition, ambiguity of reference, inherent negotiability and the historical extensibility of narrative*

Each of these four characteristics of narrative, that it requires interpretation, is ambiguous, that its meaning is inherently negotiable and that the narrative is only part of an extended story, all reflect the view that one of the characteristics of real world scripts is that they allow for interpretation, and as such meaning can be negotiated. So, Beth and Charles were happy to negotiate their reading of the absence of a driver, Beth could use this to extend the story. But she was unable to understand why Angela did not interpret the situation with the cubes in the same way as Beth herself did, since there was no story, nothing to negotiate. The class has yet to establish the "normative patterns of interaction and discourse" of the classroom (Wood 2000) which will allow a shared understanding of more abstract mathematics apart from the "real world".

4.5 Shared cultural knowledge *and implied canonicity, the centrality of trouble*

Bruner identifies that narratives often breach implied rules about how life and society work and it is this that makes them interesting. The implied canonicity assumes a shared cultural knowledge, rules about how life and society work. There is a shared knowledge about behaviour on buses. The "naughtiness" of the children breaches implied rules about how to behave, causing trouble. The children also know that the total number of children on the bus does not alter as they run up and down stairs. This allows Ian to conserve number in a way that he was not able to in the context of cubes:

Beth Now, while he was having his coffee some of the children started to run about, so ... she went upstairs ... and so did he ... and so did she (moving a face each time.)
Who can tell me something about the numbers now?

Ian It still makes ten

This contrasts strongly with Angela trying to change the number sentence represented by the cubes:

Beth It's still 4 and 4. Can you break it up and make it different?

Angela removes one cube from the left hand set and hides it in her hand.

Angela Three and four together make eight (very hesitantly, looked at the extra cube and put it back in the three but this time to make an L shape rather than a square) Four and four together make eight.

Angela has no expectations of how the cubes should behave and seems surprised when the number sentence does not turn out as she expects. She has "broken it up" and "made it different" as asked, but she knows that the number sentence is supposed to still make eight.

5. Implications and conclusions

Characteristics of real world scripts therefore relate to Bruner's nine universals of narrative. Real world scripts, as defined in the teaching of early addition, can be identified as narrative. So, Bruner's assertion that one of the principle ways of understanding is through narrative can help us to see how learning mathematics through real world scripts supports children's understanding. This is not to say that mathematics need always be taught through the use of real world scripts. As children gain more experience of learning mathematics they will develop a shared *mathematical culture* within the classroom to which to relate their new learning. But at the beginning of school mathematics learning the children have no such experience. Perhaps it would be appropriate to introduce each new concept through the use of real world scripts. The introduction can be situated in the social experience of the children, while subsequent work can build on existing mathematical understanding and the developing mathematical culture of the classroom.

Mathematics is only one way of looking at the world into which children need to be inducted and stories, narratives, real world scripts can effect this induction.

6. References

Bruner, J. (1996): The Culture of Education. – Cambridge, MA: Harvard University Press

Bruner, J. S. (1986): Actual Minds Possible Worlds. – Cambridge, MA.: Harvard University Press

Cobb, P. (1987): Information-processing, Psychology and Mathematics Education – a Constructivist Perspective.– In: The Journal of Mathematical Behaviour 6(1), p. 4–40

Davies, A.; Pettitt, D. (1994): Developing Understanding in Primary Mathematics. – Lewes: Falmer Press

Donaldson, M. (1978): Children's Minds. – London: Fontana

Ernest, P. (1991): Philosophy of Mathematics Education. – Lewes: Falmer Press

Hughes, M. (1986): Children and Number. – Oxford: Basil Blackwell

Lesh, R.; Post, T.; Behr, M. (1987): Representations and Translations among Representations in Mathematics Learning and Problem Solving. – In: C. Janvier, Problems of Representation in the Teaching and Learning of Mathematics. Hillsdale, NJ:

Erlbaum, p. 33–40
Steinbring, H. (2000): Interaction Analysis of Mathematical Communication in Primary Teaching: The Epistemological Perspective. – In: ZDM Zentralblatt für Didaktik der Mathematik 32(2000)No. 5, p. 135–147
Strauss, A.; Corbin, J. (1990): Basics of Qualitative Research: Grounded Theory Procedures and Techniques. – Newbury Park, CA: Sage
von Glasersfeld, E. (1995): Radical Constructivism. – London, Falmer Press
Vygotsky, L. S. (1978): Mind In Society. – Cambridge, MA: MIT Press
Wood, T. (2000): Differences in Teaching and Opportunities for Learning in Primary Mathematics Classes. – In: ZDM Zentralblatt für Didaktik der Mathematik 32(2000)No. 5, p. 148–153
Yackel, E.; Cobb, P.; Wood, T.; Wheatley, G.; Merkel, G. (1990): The importance of social interaction in children's construction of mathematical knowledge. – In: T. E. Cooney (Ed.), 1990 Yearbook of The National Council of Teachers of Mathematics. Reston, VA: NCTM, p. 12–21

Author
Price, Alison, Oxford Brookes University, Oxford OX29AT.
E-mail: aprice@brookes.ac.uk