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## Mathematics Teaching and Values Education – An Intersection in Need of Research

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**Abstract:** This paper raises issues concerning the teaching of values in the context of mathematics education. It argues that a focus on education for democracy inevitably involves educating about values. It reviews the major relevant research and theoretical perspectives and argues for more research attention to be paid to this area. Although there has been relevant research done in the affective domain, both in general and in relation to mathematics, and in social and cultural issues, there is little direct research focus on either values or valuing. Teachers are rarely aware of teaching values either explicitly or implicitly, yet values teaching clearly does take place, mostly implicitly. If there are desires to change the directions of mathematics teaching to be more attuned to life in modern democratic societies then this aspect of mathematics education needs to be better understood in order that it can be better taught.

**Kurzreferat:** *Mathematikunterricht und Werteerziehung – eine Schnittstelle, die der Forschung bedarf.* In diesem Beitrag werden Fragen zur Werteerziehung im Rahmen des Mathematikunterrichts aufgeworfen. Es wird argumentiert, daß Erziehung zur Demokratie immer auch Werteerziehung miteinschließt. Es wird ein Überblick über relevante Forschungsarbeiten und theoretische Perspektiven gegeben und für verstärkte Forschungsaktivitäten in diesem Bereich plädiert. Obwohl es relevante Forschung im affektiven Bereich gibt, sowohl allgemein wie auch auf Mathematik bezogen, im sozialen und im kulturellen Kontext, so gibt es wenig direkte Forschung, die sich auf Werte oder Wertsetzung bezieht. Lehrer sind sich selten darüber bewußt, daß sie implizit oder explizit Werte vermitteln, aber Wertevermittlung findet statt, und zwar meistens implizit. Wenn der Mathematikunterricht dahingehend verändert und verbessert werden soll, daß er dem Leben in modernen demokratischen Gesellschaften gerechter wird, dann muß zunächst dieser Aspekt der mathematischen Erziehung besser verstanden werden.

**ZDM-Classification:** C20, C70, D30

### 1. Introduction and context

Modern society is demanding much greater mathematical knowledge of its citizens than ever before and the essential challenge for mathematics educators concerned with issues of democracy is how to provide an adequate mathematics education for the greatest number of citizens. Computer developments are simultaneously facing us with some of our greatest dilemmas, and offering us some of the most exciting educational possibilities. They are not only changing the way we think about mathematics teaching, they are also changing the nature of mathematical activity itself. Societies are also becoming more multi-cultural and because the nature of mathematics is being re-examined, mathematics educators are becoming increasingly concerned about the goals which should be formulated for mathematics education. There are important developments in the last few years which could have widespread benefits for mathematics learners around the world, for example in the areas of ethnomathematics (see Gerdes, 1995) and critical mathematics education (see Skovsmose, 1994, 1996) where both the nature and the role of mathematics in education is being looked at anew.

What is particular provocative about this general situation and these examples is that there is a strong interest in examining, and in changing, the values being taught through mathematics education. But only rarely does one find explicit values teaching going on in mathematics classrooms. Why would this be? The reason lies in the widespread belief that mathematics is a value-free subject, a myth which has been exploded in the last two decades.

Dealing with issues of democracy in mathematics education clearly requires engaging with values, and this is problematic at the present time because not only do we not know what currently happens with values teaching in mathematics classrooms, or why, but we have even less idea of how potentially controllable such values teaching is by teachers. In addition, whereas it appears to be relatively easy in the teaching of humanities and arts subjects to record, discuss and develop values teaching and learning, this is not the case at present in mathematics teaching. Most mathematics teachers would not even consider that they are teaching any values when they teach mathemat-

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ics. Changing that perception may prove to be one of the biggest hurdles to be overcome.

There are therefore several important questions which are worth considering here: What is the current situation regarding values teaching in mathematics classrooms? What values do mathematics teachers think they are teaching? What values are being learnt by students? Can teachers gain sufficient control over their values teaching to teach other values besides those which they currently teach? Sadly there is very little research into any of these questions, which is creating a huge lacuna in our understanding of how to affect the current situation. Before discussing those questions in more detail, however, it is necessary to clarify what we mean when we talk about values in mathematics education.

## 2. Values and mathematics education: three sources of conceptualisations

Values in mathematics education are the deep affective qualities which education fosters through the school subject of mathematics. They appear to survive longer in peoples' memories than does conceptual and procedural knowledge, which unless it is regularly used tends to fade. Research indicates that the negative features of these values lead subsequently to a dislike of mathematics as adults and hence to negative parental influence (Cockcroft, 1982).

If we consider the relevant research fields, we can find three principal sources for theoretical ideas which can be used to think about developing values teaching in mathematics. These are the literatures on the affective domain and values education generally, on affective aspects of mathematics education, and on social and cultural aspects of mathematics education.

### 3. Affective domain and values education

The first framework offered to address these issues was Krathwohl et al.'s (1964) analysis of the affective domain of the taxonomy of educational objectives, which introduced the difference between "values" and "valuing". Their analysis suggested five levels of response to a phenomenon in increasing degrees of commitment. Of particular interest here are levels 3 and 4 which are summarised as follows:

- |                            |                                     |
|----------------------------|-------------------------------------|
| 3. Valuing                 | 4. Organization                     |
| 3.1 acceptance of value    | 4.1 conceptualization of a value    |
| 3.2 preference for a value |                                     |
| 3.3 commitment             | 4.2 organization of a value system. |

The following quotation is important for clarifying the focus of our concerns: "Behavior categorized at this level is sufficiently consistent and stable to have taken on the characteristics of a belief or an attitude. The learner displays this behavior with sufficient consistency in appropriate situations that he comes to be perceived as holding a value." (p. 180)

Raths, Harmin and Simon (1987), summarising their often-quoted book, approach the problem in another way, and offer seven criteria for calling something a value. They say (p. 199): "Unless something satisfies *all* seven of the

criteria noted below, we do not call it a value, but rather a 'belief' or 'attitude' or something other than a value." They summarise their criteria in the following terms:

- |  |                            |
|--|----------------------------|
| 1. Choosing freely   | alternative                |
| 2. Choosing from alternatives  | 4. Prizings and cherishing |
| 3. Choosing after thoughtful consideration of the consequences of each | 5. Affirming               |
|  | 6. Acting upon choices     |
|  | 7. Repeating.              |

They add "Those processes collectively define valuing. Results of this valuing process are called values." (p. 201)

Both the taxonomy and the criteria from Raths et al. emphasise the following aspects of valuing which are important for our consideration:

- |                                   |                   |
|-----------------------------------|-------------------|
| (i) the existence of alternatives | (iii) preferences |
| (ii) choices and choosing         | (iv) consistency. |

In relation to values education, the work of Tomlinson and Quinton (1986) is particularly important since it moves the discussion from earlier reliance on the work of Kohlberg (1984) and his followers into the mainstream subject curriculum. They argue strongly that when considering values, due attention should be paid to three elements (p. 3): aims or intended outcomes; means or teaching/learning processes; and effects or actual outcomes. This same triad shaped the work of the IEA comparative research on mathematics teaching (see Garden, 1987) which focused attention on three levels of the curriculum: the intended level, the implemented level, and the attained level. These are clearly important ideas for us to consider here.

### 4. Affective aspects of mathematics education

Regarding the second literature source, McLeod (1992), in one of the most up-to-date and comprehensive summaries of research into affective aspects of mathematics education separates the field into studies of beliefs, attitudes, and emotions. He, like others who have surveyed this field, cites no research on values, although the tone of his discussion makes it clear that, rather like Krathwohl et al. and Raths et al. above, ideas about both beliefs and attitudes towards mathematics do relate to values held by both teachers and students.

In another chapter in the same book Thompson (1992) also discusses the research on teacher beliefs, particularly in relation to teachers' actions in the classroom. She points to a repeated finding that teachers' actions frequently bore no relation to their professed beliefs about mathematics and mathematics teaching. The research by Sosniak et al. (1991) also found striking inconsistencies between different belief statements given by the same teachers. We would contend that this discrepancy is precisely why it is necessary to focus on values rather than beliefs, in order to determine the deeper affective qualities that are likely to underpin teachers' preferred decisions and actions.

Taking into consideration the findings from other research on beliefs and attitudes in mathematics education (e.g. Buxton, 1981, Fasheh, 1982), of more concern is the fact that there appear to have been few studies of the behavioural aspects of affect, such as those behaviours

described earlier related to valuing – namely, the choosing, the preferring, the consistency of behaviour etc. The behavioural component certainly appears to be one significant focus for the development of attitudes and beliefs on the one hand, and values on the other.

### 5. Social and cultural aspects of mathematics education

This third literature source has been helpful in clarifying what is the *content* or the *focus* of the values which should be addressed. As was stated in the opening paragraph of this paper, there are three principal sources of values in the mathematics classroom; society, mathematics, and mathematics education.

Wilson's (1986) review, whilst pointing out the paucity of writing and research on values in mathematics teaching did discuss two values, *a respect for truth*, and *the authority of mathematics*. Later analyses by Bishop (1988 and 1991) sought to build more broadly on the wide literature on mathematical history and culture. Using White's (1959) three component analysis and terminology, he proposed that, in "Western" mathematics development, the predominant ideological values concern the ideas of "rationalism" and "objectism", the sentimental values (which is White's term for individuals' feelings about their relationship to knowledge) are those of "control" and "progress", while the sociological values refer to societal relationships regarding mathematical knowledge, such as "openness" and "mystery". Wilson's (1986) first value is an ideological one, while the second fits comfortably within White's "sentimental" component.

It seems therefore that the three conceptualisations which will be important for values research to consider in the future are the following:

- Ideological: referring to the values of individuals towards the mathematics they are either teaching or learning
- Individual: referring to the values of individuals towards themselves, their self-respect etc. in the context of learning or teaching mathematics
- Social: referring to the values of individuals towards society, in relation to mathematics education.

### 6. The teacher and values education in mathematics

As was said above, only rarely does one find explicit values teaching going on in mathematics classrooms, the reason being the widespread belief that mathematics is a value-free subject. Indeed, many parents and politicians might initially be concerned about explicit values teaching in mathematics. What parents and others should be concerned about is that values teaching and learning does go on in mathematics classrooms, and because most of it appears to be done implicitly, there is only a limited understanding at present of what values are being transmitted, and of how effectively they are being transmitted. Given the often-quoted negative views expressed by adults about their bad mathematics learning experiences, one could speculate that the values transmitted to them were not necessarily the most desirable, but that they were transmitted rather effectively!

At present we have no research which is documenting the extent of values teaching. We have no idea what either the explicit or the implicit forms of values teaching are. Several questions come to mind here: Are values *explicitly* expounded, discussed or raised as teaching "content"? As they do not appear in detailed syllabus descriptions (Howson, 1991) but only, if at all, in the aims statements of curriculum documents, it is unlikely that they will be considered as content to be taught. The assumption will therefore be that they will be addressed (if at all) across and through the mathematical content or process topics.

Do textbooks have explicit values-focused exercises or activities? A look at several textbooks fails to reveal any activities of this nature, and again one would suspect that as values do not appear as content they would not be addressed by classroom texts. Do teachers use values-clarification exercises, etc.?

From a research perspective the International Handbook on Mathematics Education (Bishop et al., 1996) is revealing. It has no specific chapter on values, although several of the chapters clearly refer to value aspects of mathematics education, and stress their importance. For example, Brown (1996) discusses the work of the Humanistic Mathematics Network and quotes one of its aims which states: "An understanding of the value judgements implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated and *why* it is investigated" (p. 1302). Ernest (1996) also implicitly discusses values in his chapter on "Popularization: myths, massmedia and modernism" as do Leder et al. (1996) in the chapter on gender issues.

Skovsmose's (1996) chapter is perhaps the one which most nearly addresses values and valuing explicitly, when he argues that "Critical mathematics education is concerned with the development of citizens who are able to take part in discussions and are able to make their own decisions. We therefore have to take into consideration the fact that students will also want, and should be given the opportunity, to 'evaluate' what happens in the classroom. This turns the focus on students' interest." (p. 1267)

This comment echoes the idea above, that for values education to develop there is a necessity to ensure that the mathematics classroom is a place of choices, and of choosing, for the students. Teachers could, and in my view should, be presenting students with activities which encourage them to make choices; for example, about the selection of problems to be solved; about the solution approaches to be taken; about the criteria for judging the worth of solutions; and about the wider appropriateness of the mathematical models being taught. It should be a natural part of the teacher's repertoire, to present activities which require choices to be made: for example, a task such as "Describe and compare three different proofs of the Pythagorean theorem" would inevitably engage students in discussing the values associated with proving. Even the simple act of presenting different problem-solving solutions to be compared and contrasted by the students stimulates the ideas of choice, criteria, and values. What Skovsmose's focus on students' interests does is to remind us that rather than thinking of mathematics teaching as just

teaching mathematics to students, we are also teaching students through mathematics. They *are* learning values through how they are being taught.

This is also why more attitude-focused research needs to be refocused onto values, and choices. We need studies which not only investigate what students *say* about their attitudes to different aspects of mathematics, but also which look at the choices students make in different situations, which will indicate the influence of certain values.

The acceptability of these ideas will of course depend ultimately on the capacity of teachers to engage with this issue. For example, when choices are offered to the students and made by them, how do teachers respond? Do in fact teachers know what values they are currently implicitly teaching in the ways they respond to students? Are they in that sense in control of their own values teaching? These are of course crucial questions. Many development projects are predicated on the assumption that teachers *are* in control of their values teaching and that they *will* be able to change the values to which they teach. However it is largely an unexplored area. Perhaps only when teachers give students more choices will they themselves be faced with responses which are new to them, and which will therefore require them to become more aware of their own values. Perhaps indeed this is another inhibiting factor in the process: perhaps one reason mathematics teachers do not give their students more opportunities for choice is precisely because it will require them to examine and reveal the values about which they themselves are unsure.

This area is one which is fundamental not just to research, but also to much teacher training and in-service education, and it needs to be thoroughly investigated by both teachers and researchers. The results of any such investigations would do much to enlarge our understandings of why mathematics teachers teach in the ways they do, of how to educate mathematically our future citizens, and of what are desirable, and feasible, goals for mathematics education in democratic societies as we move towards the next century.

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