Ernest, Paul:

Social Constructivism as a Philosophy of Mathematics

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The attempt to formulate a philosophy of mathematics based on social constructivism is both a monumental and risky undertaking. The search for an epistemological underpinning of mathematical certainties has been a live battlefield for quite some time and warriors from differing camps have entered the fray and been wounded. One of the achievements of this book is that Ernest, although he makes no bones about the banner he is carrying, manages to give a very fair picture of past and present disputants, and the educated reader will gain a very useful overview. No doubt, experts of the field will quibble about his interpretations. The effort to render some of the opposing views understandable brings with it the risk of terse objections from every quarter. Ernest obviously foresaw this and therefore supplied a list of six explicit points for which, in his view, "an adequate philosophy of mathematics should account" (p. 56):

1. Mathematical knowledge: its character, genesis and justification, with special attention to the role of proof.
2. Mathematical theories, both constructive and structural: their character and development, and the issues in their appraisal and evaluation.
3. The objects of mathematics: their character, origins, and relationship with the language of mathematics.
4. The applications of mathematics: its effectiveness in science, technology, and other realms and, more generally, the relationship of mathematics with other areas of knowledge and values.
5. Mathematical practice: its character, and the mathematical activities of mathematicians, in the present and past.
6. The learning of mathematics: its character, and its role in the onward transmission of mathematical knowledge and in the creativity of individual mathematicians.

Most mathematicians would, I believe, agree with the first four of these points. The fifth they may consider quaint or downright superfluous. But for the social-constructivist approach it is not at all irrelevant. From that perspective, mathematics is what mathematicians *do*. It is not a domain of crystalline objects that reside in an ulterior absolute reality, but is continuously constituted by the actions and interactions of members of the discipline.

The sixth point, sadly neglected by the traditional philosophers of mathematics, obviously gains relevance when mathematics is considered a social phenomenon.

Readers who at all cost want to hold on to their metaphysical realism (Platonic or other) will hate this book from beginning to end, because it brings up a great many arguments that threaten their position. On the first page of the Introduction, Ernest characterizes mathematics as a "virtual reality" (p. xi) and throughout the subsequent text one is not allowed to forget that he considers it a human construction.

Chapter 1 is a spirited deconstruction of "absolutism in mathematics" and a plea for "fallibilism". Ernest shows that even relatively transparent areas of mathematical knowledge, such as Euclid's geometry, involve the acceptance of "basic truths" that were considered to need no justification, but can, as the invention of non-Euclidean geometries demonstrated, be successfully questioned and denied without contradiction. He does an excellent job of criticizing the traditional quest for absolute foundations of mathematics in the epistemologies of philosophers and contrasts this attempt "with the opposing view, ... that mathematical truth is fallible and corrigible and should never be regarded as being above revision and correction" (p. 9–10). Of the three schools of philosophy of mathematics, logicism and formalism are easy victims of this line of criticism; but Ernest also discards intuitionism, because he contends that it claimed "intuition" as an absolute foundation.

In Chapter 2 he lays out the grounds for a new beginning. "Only since the Second World War, and especially in the past three decades, has a more genuinely philosophical (as opposed to mathematical) philosophy of mathematics emerged ... This includes more emphasis on ontological questions, as well as including a 'maverick' tradition concerned with mathematical practice and its methodology" (p. 41).

Being a constructivist, albeit of a somewhat different kind, this reviewer finds himself in full agreement with this undertaking. Traditionalists may say that practice and methods of procedure can have no role in establishing foundations, but mathematics is an occupation that differs from all others. While it is doubtful that the practice of bricklaying reveals a great deal about architecture, the situation in mathematics is another. Practicing mathematicians are constantly concerned with developing new mathematical structures and, therefore, with decisions as to what is and what is not legitimate in their field. This is to say, whether they like it or not, they have to concern themselves with metatheory and thus with foundations that are contingent rather than absolute.

Chapters 3 and 4 are focused on the approaches to the philosophy of mathematics taken respectively by Wittgenstein and Lakatos. The work of both these authors may be subject to varying interpretations, but Ernest provides a detailed and very reasonable introduction.

"The Social Construction of Objective Knowledge" is the title of Chapter 5, and it is here that Ernest lays out the positive core of his thesis. He begins with the observation that "Any explicit human formulations of doubt, belief, or knowledge, ... presuppose the social institution of language" (p. 131). Philosophers, he says, can therefore, not afford to neglect the role that the social aspect of linguistic conventions and meaning plays in their field. He anticipates "the standard philosophical rejoinder" that this may be so with regard to the genesis of individual knowhow, but does not demonstrate that social factors make a "necessary contribution to the constitution or justification of knowledge" (p. 131–132). He then examines a number of arguments to substantiate that "social phenomena such as language, conversation, and group acceptance cannot be accounted for in purely individual or objectivist terms" (p. 135–136).

New ideas in mathematics, he observes, do not become mathematical *knowledge* until they are accepted by "representatives of the academic community of mathematicians" (p. 149). New ideas have to be examined and approved, and this involves a more or less formal dialectical process of conversation and thus language and the interpretation of meaning. Even more importantly, it involves the notions of *proof* and *rigor*. Ernest shows, I believe convincingly, that these notions have changed considerably in the course of history, which turns the picture of mathematics as a timeless, objectively "true" edifice into something of an illusion.

Under the term "conversation" Ernest subsumes the various specific forms of social interaction that may impinge on a mathematician's search for and justification of novelty. In Chapter 6, he highlights and examines the role of rhetorical devices in this process and shows that it is practically impossible to separate the elements of rhetoric from a purely mathematical content (p. 175). In short, as do other constructivist schools, the social constructivism replaces objectivity with negotiated intersubjectivity. Although this is unlikely to change the minds of convinced objectivists, it brings to the fore a good many arguments they will find difficult to push aside.

Chapter 7 begins with a detailed exposition of Vygotsky's theory, according to which the higher levels of thought develop in children as a result of the language they have internalized from interactions with adults. "[N]orms, rules, and conventions of linguistic behavior that every speaker meets, in some form, when entering into a linguistic community are part of a preexisting form of life ..." (p. 213). For Ernest, this is the basis of the "social construction of knowledge" (the title of this chapter), but where mathematics is concerned, it takes a little more: "the public representation of collective, socially accepted mathematical knowledge within a teaching-learning conversation ... is necessary but not sufficient for such knowledge to become the personally appropriated mathematical knowledge of an individual learner. Sustained two-way participation in such conversations is also necessary to generate, test, correct, and validate mathematical performances" (p. 221). The echo of Wittgenstein's "language games" is quite deliberate. Ernest summarizes the individual's acquisition as "the interrelated social construction of subjective and objective (read 'intersubjective') knowledge of mathematics in a creative and reproductive cycle" that he graphically represents in a circular diagram. Conversation and interpersonal negotiation are the mechanisms which, by means of criticism and both public and personal reformulation, constitute public mathematical knowledge (p. 241–243).

Having provided, early in his book, six criteria according to which a philosophy of mathematics could be evaluated, Ernest now uses Chapter 8, the last of the book, to apply them to the theory he has proposed. This gives him the opportunity to counter the major criticisms of his proposal that he expects will be voiced by philosophers and mathematicians. He does this, I think, very fairly and without overbearing confidence. He is careful to remind the reader that "as modern philosophers of science assert, our theories are logically underdetermined by our observations" (p. 251). "Social constructivism" he says, is offered both as a philosophical position, largely worked out, and as a research program needing further elaboration" (p. 268).

As I mentioned, I am not a social constructivist. However, I consider this book a very valuable piece of work. Written without undue proselytizing, it provides a wide-ranging basis for thought and discussion and is apt to enrich and expand every reader's view of the field. My criticisms spring from my own epistemological bias concerning the genesis of knowledge.

Ernest presents a solid case for the view that mathematics can be seen as a human construction rather than a god-given or in some other way pre-ordained complex of absolute truths. As human construction it is influenced by the historical context and the agreement of thinkers who cannot in any way compensate for the uncertainties inherent in language and the interpretation of meaning.

From my perspective, the active constructing of knowledge is under all circumstances an individual's enterprise – hemmed in, constrained, and guided, if you will, by interactions with others, but having access to no other raw material than the "stuff" of the individual's own experience. Consequently I would say that, although in the domain of mathematics much knowledge is socially *induced*, its actual formation requires the cognitive effort of an individual. At the end of his Introduction, we learn that the author has shifted away from a Piagetian/constructivist view, but he suggests no other conceptual tools that would enable individuals to profit from social "transmission". I see no reason why the mechanisms of assimilation, accommodation, and reflective abstraction should be considered incompatible with social constructivism.

Ernest occasionally refers to Euclid's *Elements* and their displacement as absolutes, but geometry as a source of mathematical knowledge is absent from his text. Yet, geometry provides clear examples that new knowledge can be generated by an individual *without social interaction* from simple graphic designs – for instance, the *visual* proof of the Pythagorean theorem.

Referring to Quine, Ernest writes: "Naturalized epistemology explores the grounds for individuals' knowledge which are found in their experiences and environment and in scientific theories and explanations of knowing" (p. 51). Society is made up of such individuals. "At any moment there seems to be no more than a vast collection of individuals, but their interrelationships, expectations, traditions, and histories of negotiations together make up the mortar that joins these individuals into a whole that is more than the sum of its human parts." Thus the socially objective knowledge "is based on shared language use, rules, and understandings, embedded in shared forms of life. It is essentially supported by the subjective knowledge of individuals, but because of their interrelations, it is correlated in a complex and ever changing way" (p. 146). The notion of a "whole that is more than the sum of its human parts" rests on the assumption that linguistic and other interactions lead to *social* knowledge that is *shared* by the members of the community. Given the inherent uncertainties in communication and the interpretation of meaning that Ernest has made amply clear in his critique of absolute truth, rigor, and proof, the word "shared" applied now to understandings and knowledge seems to me misleading. The result of continuous social negotiation and concomitant individual accommodations leads to a relative *compatibility* of individuals' ways of thinking and conceptions. But compatibility does not entail sameness – it merely entails the absence of noticed friction or contradiction. Observers, including philosophers, who set themselves apart and try to describe the understandings they have abstracted from their interactions with society and the phenomena that go by the name of mathematics, are still confined to the domain of their personal experience. Whether or not their description will be deemed compatible with the experience of other observers can be found out only by further interaction and conversation. I consider it therefore very important that Ernest explicitly presents the social constructivist perspective as a research program and contemplates its further elaboration.

A different kind of criticism regards the production of the book. Although the text is remarkably free of printer's errors, the pagination was apparently changed after the index had been compiled. This led to the irritating fact that, to find the indexed items, one has to increase all page numbers after 34 by 2.

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