# Book Reviews

Burn, Bob; Appleby, John; Maher, Philip. (Eds):

## Teaching Undergraduate Mathematics

London: Imperial College Press, 1998. – 266 p.
ISBN 1-86094-115-X

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### 1. Purpose of the book

This highly interesting book is not quite like any other, neither in style nor in subject matter; and in an ideal world it would be read by all who teach mathematics in a tertiary institution. (For brevity, I shall call their students undergraduates and the institution a university). As we shall see, that "all" includes especially those whom others call "theorem-proving" mathematicians, because they appear to regard their teaching role as highly subordinate to that of doing mathematical research. Briefly put, the book is a compilation by the editors, of extracts from the proceedings (up to 1993) of the annual "Nottingham Conferences" sponsored by the London Mathematical Society. These go back to 1975 and still persist, although they have recently moved from Nottingham to other universities. Now all this may seem very British and parochial, but others may learn a lot from it, because it was the structure of the conferences that made them highly unusual, and which is of wider validity. The editors do not explain as much about this important aspect as needs to be known, so part of this review will fill in some details. Here, the reader is warned that I may be biassed, because I am an interested party, as will be shown below.

The conferences were structured to generate discussion by groups of mathematicians about teaching mathematics in universities, in ways better than was usual; but the groups were to be small, so as to focus discussion, and they were expected to raise questions about the meaning of "better" and the associated "ways". I will give more detail below, but first – why were they involved in such things at all? Most participants were theorem-provers, not on the whole accustomed to such "unmathematical" discussion in any depth. They were unlikely to take so much trouble without serious reason, because most were likely to feel that mathematics education was trivial and could safely be left to others. (Yet, who else but mathematicians can be properly serious about it?) In 1975 the catalyst was political (it was a move by the State), and may well return in other disguises, and in contexts other than the British.

Here, the book explains that this catalyst was the potential introduction in Britain (other than in Scotland, which is always a special case) of what were known as "Normal and Further" (or N and F) examinations; these would give pupils a broader education than before, but leave them less well-prepared for the immediate specialisation of existing first-year university courses, which would then have to be modified. (As it turned out, the proposed examinations were never introduced after all.) But that was not the whole reason for the first conference, as will be seen.

### 2. Education and politics

I must begin by explaining a bit of the British social context of the first, 1975, conference. In Scotland, the university course lasts for 4 years, and the students have had one year less of secondary education than elsewhere in Britain. With this exception, pupils in British secondary schools during their final two years in school, concentrate (even nowadays) on only three subjects and end by taking the "A-level" examination. With sufficiently high-grade passes, they can then enter a university, to take a degree course in (traditionally) one of their A-level subjects, although in the last decade many students take a mix of vocationally-oriented options. The outside world frequently finds this education too narrow, and in 1975 there were public calls to impose the N and F system, with 5 subjects, which would be more like the Scottish system.

This change never came to fruition because the 1979 Thatcher government refused to consider any change to the A-level system, which Mrs Thatcher called the educational "Flagship"; no explicit reasons were given, other than the implication that the "Flagship" had produced herself and many of her supporters, so what could be wrong with it? Long before the run-up to her 1979 election, Right-wing politicians had generated much public criticism of the expansion of education in general, and the rhetoric was usually of very poor quality, even though many participants were (not necessarily Right-wing) academic conservatives (see, e.g., the *Bulletin of the Institute of Mathematics and its Applications*, Southend-on-Sea, Essex, UK, of that period, and the earlier "Black Papers": some details are in Griffiths 1999, p. 178). It was sad but illuminating to see how eminent thinkers from different disciplines could set aside the rigour of their own discipline to speak and write

about education without defining terms or checking facts. This was "confident ignorance" indeed, of which we shall see other examples below. Before 1979, the previous government had called a "Great Debate" about the alleged failings of the educational system, and set up the Cockcroft Inquiry into the teaching of mathematics in schools, which was a model of careful work, given the constraint that the inquiry had to exclude mathematics in universities. Cockcroft reported in 1980, and its official influence was soon killed off by the Thatcherites, although it did lead to their funding the interesting LAMP and RAMP projects (Griffiths 1999, p. 180), until these developed in "incorrect" ways.

Another problem with education was its rising cost, because of the economic consequences (in many countries) of the 1973 oil crisis. This caused energy to become very expensive. In Britain, funding of universities fell, and departments needed to recruit more students because of the per capita income they carried. But demographic trends suggested a likely fall in numbers, and universities did not want their recruiting figures made public. So, the Council of the London Mathematical Society (President, Michael Atiyah) decided to have a national conference, to be attended by mathematicians from universities and secondary schools – a combination quite unheard of in those days. If the advertised topic were to be concerned with the likely impact of N's and F's, contact with the teachers might help to increase recruitment of their pupils.

**3. A new structure for a conference**

Heini Halberstam and the reviewer were asked to organise the conference, and we agreed between ourselves to move away from the conventional structure of the earlier educational conferences that we had seen. At these, all sessions were plenary, never leading to practical action, with eminent mathematicians as speakers who made ignorant remarks about education. They often sneered about schoolteachers especially; it seemed not to occur to them that if the university intake was as poorly taught as they claimed, then it might be because they themselves had perhaps taught the teachers badly. And they ignored the effects of various social changes on the attitudes of young people to learning and culture. There was little opportunity to correct such speakers, since the matter is more complicated than saying in a mathematics lecture "there's a mistake on the second line" (which would immediately be accepted). We needed to induce a professional outlook on such matters.

Since there were no proved experts in mathematics education, we decided to confine plenary sessions to the presentation of information only; small groups were to be the place for expressing opinion and challenging it through serious argument. Now, university Mathematicians normally dealt with educational matters by speaking off the cuff; by contrast and most importantly, each group was expected to present a collective view in the form of a written (yes, written) report. To focus ideas, they were given a written "Brief"; that is to say, a problem to solve that would lead them to define terms of discourse, consider known arguments, and advise on a course of action. (I shall give examples later.) Each report was to be passed to another group for criticism in writing, for the first group to amend on return. In this way, it was hoped that the participants might experience something of the growing discipline of mathematics education, even though time would be insufficient for the reports to be really polished.

**4. The first conference and its "style"**

The first conference had about 150 members, and was held in the Shell Centre for Mathematics Education at Nottingham University, where Heini Halberstam was professor of pure mathematics. He was very good at getting funds, and in the early years we could support some schoolteachers; unlike the university members, they often could not get expenses from their employers. Several senior academics, led by Michael Atiyah, came to the first conference, which began with a plenary session to hear instructions. Here, some seemed to fear that rigorous mathematics was to be corrupted and trivialised by the hot air of education. Revealing the "confident ignorance" (mentioned in Sect. 2), one theorem-prover asked whether we couldn't simply tell schools that we wanted all recruits to have 2 of their 3 A-levels in mathematics (as was once traditional) and then go home; but he was quickly informed by a woman tutor from Oxford that many schools could no longer offer such "double mathematics" because they lacked suitable teachers. (This clash immediately illuminated the problems facing us.) But the conference was given whole-hearted support by other seniors, who helped young lecturers to feel that they were pursuing a respectable activity. In later years, another source of respectability came from lecturers of the Open University, whose daily work was not in closed classrooms, but open for all to see on television. They teach mature students, not naïve 19-year olds, so they are constantly forced to ask important questions about teaching mathematics.

Many participants found the first conference to be a surprising stimulant. It was agreed to have another in the same style, next year, and a small committee was elected to run it. This pattern was repeated each year thereafter but the number of schoolteachers soon fell to zero as funds became scarcer (and Heini had moved to the USA). Reduction of funds also caused the university representation to fall, but we began to see an increase from the then polytechnics (which became universities in the mid 80's), so the conferences changed to their present name in 1983. The pattern also changed slightly to allow groups to have a small choice of Brief, and to introduce some practical work such as computing and film-making. Before Heini's move, the Shell Centre had acquired Hugh Burkhardt, who was happy to add his abundant energy to the running of the conferences. Through his advocacy, their style was adopted in modified form at later Congresses of Mathematical Education.

**5. Language, and re-inventing the wheel**

As soon as it was clear that similar conferences would be held annually , we decided to collect the corrected reports into a volume of (duplicated) proceedings, to grow a body of "Literature" which future participants could be expected to read, so as to stop them from re-inventing

the wheel. (Over the years, experience shows that when a group first meets, the members air their views, thinking they are original, obvious and not in need of further analysis, and that the session is a waste of time. Later, a common language emerges, and the group-intelligence starts to do interesting things.)

It is from these proceedings (1975–1993) that the editors have extracted the materials for the book under review. They might have conveyed the flavour more quickly at the start, by displaying an example of a full Brief there, but an extract from one occurs on p. 6 of the book, and many others are later given in full. (It would have been useful to have listed these in the Index.) One Brief occupies the whole of p. 236, and concerns the balance of learning activities that students need, following suggestions made in the Cockcroft Inquiry. Often, the editors merely indicate a Brief by introducing an extract from a working group with a heading such as "Working group on X", where X may be (for example) the objectives of a first-year course, or self-paced learning, or rigour, or enthusiasm, or mastery, or geometry and geometrical thinking, or intuition and experiment, or teaching students to use mathematics, etc., etc. The detailed comments by the group are usually more interesting than those of the Brief, which was merely designed to provoke comment.

**6. Training for academic survival**

Eventually, the nature of some Briefs changed, to reflect the advent of the Thatcher government in 1979, which was aggressively introducing serious problems throughout the whole educational system in Britain. Thus, some Briefs were formulated as exercises in how to marshal arguments to defend one's speciality against being closed down. It is a pity that the book contains no examples of these. Indeed, even before these real threats, it had been found helpful to express some problems in terms of an imaginary, rather downmarket "University of Bognor". (Warning: do not confuse this with the Bognor Campus of the Chichester Institute of Higher education! That campus contains an excellent mathematics centre.) For greater verisimilitude, various humorists added features to this imaginary university, such as a coat of arms on christmas cards when David Wishart, a keen amateur printer, became conference chairman. They invented a ruthless Principal of poor judgement but skilled dialectic, who would force choices between buying computers and sacking unworldly staff; and exercises were formulated to explore and defend such choices. Practice of this kind might have been useful in 1997 or so, to faculty in the university of Rochester (USA), when their President wanted to close down its graduate school of mathematics. To them, reprieve did eventually come, but the defensive arguments reported in the *Monthly Notices of the American Mathematical Society* (AMS) would not have got by the Principal of "Bognor University", who was unmoved by cultural argument that was not underpinned by "realist" economics. In the 80's I was present when a university rector was indeed moved by cultural arguments containing no financial arithmetic; but that was in the old German tradition, and the rector and faculty had not been exposed to fashionable "management" doctrines. With the world-wide spread of "Anglo-Saxon" economic views based on assigning numerical cost and ignoring qualitative value, more and more university presidents of the future will be recruited as financial managers, demanding counter-argument from academics in very hard-nosed terms. Such argument requires skill and practice; so the conferences tried to provide appropriate Briefs, as well as those directly concerning the teaching and learning of mathematics.

**7. The book under review**

Coming now to the book itself, there is some introductory explanation before Chapters 2–9. Something of the scope of these is indicated by their titles:

Ch. 2: The process of teaching mathematics
Ch. 3: Content of A-Level and undergraduate mathematics
Ch. 4: Detailed Expositions (eg. Analysis, Number Theory, Friction)
Ch. 5: History of mathematics
Ch. 6: Needs of Society and the Professions
Ch. 7: Applications and Modelling
Ch. 8: Learning mathematics
Ch. 9: Assessment.

Nowadays, it seems strange not to find a Chapter on computers, but these were rather rare during the early years. However, each Chapter is subdivided into several sub-topics, and it is in these that we find the first substantial computing discussion via a Brief in 1980, on "the computer as a teaching aid" (in Ch. 2), and 1984 saw a Brief on "Microcomputers for Mathematicians" (in Ch. 4). Generally, the editors have composed the text by choosing suitable extracts from the conference proceedings, and linking these by brief commentary. Most of the extracts are from the reports of working groups, and often contain interesting ideas for teaching mathematics (both in content and style of teaching). These necessitate too much detail to allow useful quotation here, but I hope to give something of the flavour shortly. A few extracts are from plenary lectures, where we see that the rule excluding opinion was stretched a bit. One (factual) is by Atiyah, about the work of the Cockcroft Inquiry, which contains statistics concerning such matters as the decline of the afore-mentioned double mathematics. Another, by Adrian Oldknow on "Using Derive to approach the frontiers of knowledge" contains interesting mathematics, with a warning (on p. 67), about an early piece of software that used high-resolution graphics, but was intended to teach the use of the Vernier gauge – at a time when the same digital electronics were being used to make the Vernier gauge obsolete. As he says: "The parallel with mathematics needs to be considered". In 1977, Tony Gardiner was allowed a plenary called "The Art of Coarse Teaching" (in Ch. 1) to give a hard-hitting critique of a common feature of the reports of most working groups. He picks out several common assertions, which seem at first sight to be reasonable, but which he considers to be false or meaningless; and he bewails the lack of rigour. In their defence, one should say that the groups were short of time, and lacked the experience of rigorous argument (outside mathematics) that we hoped

would emerge from the imposition of the Briefs. Transfer to other areas, of fluency in mathematical rigour, seems to need a lot of practice!

Most of the assertions that Gardiner questions are of the form: "Students (or lecturers) should acquire a particular skill", – stated with the implication that such acquisition can safely be left to happen on its own, yet deplorably seems to be lacking. But that is just the type of statement that some working group would eventually get to grips with, by

(1) leaving the generalities,
(2) analysing the terms,
(3) suggesting ways in which particular skills might be cultivated, and
(4) acknowledging that conventional teaching in universities leaves us with too little practical experience with the suggested ways. (Such acknowledgement itself was progress, in view of the standard belief of the time that "British universities are the best in the world".)

Reports from some of the later conferences indicate that new types of teaching were beginning to appear, and the final Chapter concerns discussions of the new types of assessment that these required. The accounts of these new techniques indicate honestly how much effort (and hence expense) is unfortunately needed relative to the conventional (but unsatisfactory) ways; the improvements may now have to be abandoned in some universities because of managerial pressures to concentrate on "measurable parameters". Quite apart from Gardiner's criticisms, the extracts in this book are largely good, thoughtful expositions, or else are chosen to contradict each other in interesting ways.

Through no fault of the editors, the material on history of mathematics worries me most, although it contains many useful bibliographical references. Firstly, the boundaries (History of ideas? of individuals? Social history?) are never explicitly defined by the participants in the discussions. It is remarkable that Islam is never mentioned, especially in such a multicultural society as that of Britain, and in the context of a proposed education for students who are woefully ignorant of any history before their birth. Secondly, a plenary lecture is quoted (presumably in full) from 1986, which considers "the bearing of the history of mathematics upon mathematical education." I shall dwell on it here as an example of the "confident ignorance" mentioned in Sect. 2 that was so widespread among many academics of the time: one wonders how much matters have improved nowadays. The lecturer mentions education as if it has neither discipline nor literature. He lists only one paper on education – 2-pages by an engineer in 1979, containing (we are advised) "excellent pithy criticism of the state of mathematical education in Britain"; the mere attempt to summarise such a complex subject in 2 pages suggests that the engineer's is one of the many ignorant diatribes of the genre. Our lecturer goes on to say: "thus a regrettable history of ignorance of history has been generated in educational circles", without either defining such circles or even sketching a proof. Being an expert historian of mathematics, he is quite reasonable when suggesting how history can be blended with specific mathematics. But he goes off the rails in his Section 3 when he attacks "New Mathematics" (left undefined, and without mention of which country he is talking about). Many hard arguments, often by very good mathematicians in several countries, underlay the materials of various non-traditional projects of the 1960's and 70's; and some, but alas not all, contained input by experienced teachers. Yet our lecturer was able to predict global failure and absurdity after "only 3 weeks historical research". I suspect that his lecture occurred on a one-day visit to the conference, and that he knew nothing of the whole Nottingham enterprise. I guess that the groups who were to criticise reports on history of mathematics, were either too similar, or too inhibited by lack of expertise, to deal with these points.

The process of cultivating honest sophistication takes a long time. When the late Frank Adams was professor in Cambridge, he told me how sorry he was that his funding would allow him to send only two members of staff to each conference, so it would take 20 years to follow up his hope of exposing them all to the Nottingham treatment. That process can be shortened, if only people will read this book; but they still need practice in rigorous discussion with others. Times have changed, and the *Monthly Notices* of the AMS often now contain articles and letters about the teaching of mathematics, often by lone voices who have not been involved in critical dialectic. Underlying them we still find the same naïve attitudes that we heard in the Nottingham discussions, and though there are some claims of improvement within the US context, it is clear that there, as elsewhere, the basic problems of teaching mathematics to undergraduates are still unsolved.

**8. Reference**

Griffiths, H. B. (1999): Fudge and Fiddlesticks: A Century After. – In: C. Hoyles; C. Morgan; G. Woodhouse (Eds.), Rethinking the Mathematics Curriculum. London: Falmer Press, p. 174–183

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