From G.H.H. and Littlewood to XML and Maple: Changing Needs and Expectations in Mathematical Knowledge Management

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Abstract: This paper concerns changing needs and expectations in the way mathematics is practiced and communicated. The time frame is mainly the early twentieth century to the present and the scope is all activity that can be considered to fall under the purview of the mathematical community. Unavoidably, the idea of a mathematical community is confronted; what it means to claim ownership in this community and how knowledge management practices effect the community. Finally, a description of an extendable mathematical text-based database which can be used to manage user defined forms of mathematical knowledge is presented.

Hardy and Littlewood: a Study in Collaboration

The anecdote is related by C.P. Snow in his introduction to *A Mathematician's Apology.* It is a pleasant May evening some time in the 1930's and Hardy is in his fifties. He and Snow are walking at Fenner's cricket ground when the 6 o'clock chimes ring out from the nearby Catholic chapel. "It is rather unfortunate", Hardy remarks, "that some of the happiest hours of my life should have been spent within sound of a Roman Catholic church".[1]

One of the preeminent mathematicians of his day and indeed, of the century, it is perhaps not entirely surprising that, living as he did, in the intellectual communities of Cambridge and Oxford, Gottfried Harold Hardy was an atheist. While our immediate concern is not his views on religion, it is germane only that Hardy was, in this fundamental domain, a non-conformist in an age that put a great deal of stock in conformity. Our concern *is* with his mathematics and, in particular, with his lively and productive collaboration with John Edensor Littlewood. Commencing in 1912, spanning some 36 years[2], and resulting in enough papers to fill several thick volumes of Hardy's seven volume *Collected Papers*[3] the Hardy-Littlewood collaboration stands as one of the most celebrated and productive academic partnerships of the twentieth century. It also provides an excellent starting point for an examination of mathematical knowledge management; but first, a little background is in order.

Hardy was already established in his career when at 33 years old he first met the 25 year-old John Edensor Littlewood at Cambridge in 1910. Prior to their meeting, each man had distinguished himself as a first rate mathematician. Falling into the broader definitions of the fields of Number Theory and Analysis, the work that they did together contributed immensely to the reputation of each and helped to bring about a renewal in English mathematics which had, for some time, been overshadowed by work from schools on the continent. There are several reasons why the Hardy-Littlewood collaboration provides a good backdrop to a discussion about mathematical knowledge management. One of the most important is the very fact that it was a collaboration between peers and thus entailed sharing of mathematical knowledge. The fact that their efforts were prolific and well-documented[4] means that resources are relatively easy to obtain. Equally important is that both men were very much part of the mathematical community of their day and they conducted themselves, at least in the way they practised mathematics, according to community standards. But there is another reason that their collaboration bears examination in the context of mathematical knowledge management and that is that they lived and worked in the proverbial *interesting times* and it is to the intellectual climate of those times that we first turn our attention.

[1] Hardy, G.H., *A Mathematician's Apology. (London: Cambridge University Press, 1967), 21.*
[2] Their first co-authored paper, was published in 1912 and their last co-authored paper was published in 1948.
[3] Bosanquet, L.S. et al (editors), *Collected Papers of G.H. Hardy*, (Oxford: Clarendon Press, 1967).
[4] The collected letters of Hardy are in the archives of Kings College, Cambridge.

Towards Foundational Pluralism...

> Hardy 'asked 'What's your father doing these days. How about that esthetic measure of his?' I replied that my father's book was out. He said, 'Good, now he can get back to real mathematics'.[5] Garret Birkhoff

The date is 1910 and the location is Cambridge; the time and place of the first encounter between Hardy and Littlewood. The first world war was still four years away but tremors were already being felt in old orders both inside and outside the socio-political domain. In art and literature the modernist perspective had informed such works as Picasso's early cubist piece, *Les Desmoiselles d'Avignon* and Santayana's *The Life of Reason*. In physics, Einstein had published *The Special Theory of Relativity*, challenging the determinism of Newtonian mechanics. Mathematics did not emerge unscathed. In his book, *What is Mathematics Really?*, Reuben Hersh describes the fractures that arose in the philosophy of mathematics after the widely accepted idea that all of mathematics could be ultimately derived from the principles of Euclidean geometry fell victim first to logically consistent non-euclidean geometries and second to geometrically counter-intuitive concepts such as space filling curves and such unavoidable consequences of analysis as continuous everywhere but nowhere differentiable curves. In Hersh's words:

> The situation was intolerable. Geometry served from the time of Plato as proof that certainty is possible in human knowledge - including religious certainty. Descartes and Spinoza followed the geometrical style in in establishing the existence of God. Loss of certainty in geometry threatened loss of all certainty.[6]

The response of mathematicians concerned with the philosophy of their subject was an attempt to replace geometry at the foundation of mathematical knowledge with arithmetic and set theory; thus giving rise to the field of Mathematical Logic. It is reasonable to state that the culmination of these efforts was the enunciation by David Hilbert of what came to be known as *Hilbert's Program*. In an address entitled *The Foundations of Mathematics* given in July of 1927 at the Hamburg Mathematical Seminar, he stated:

> ...I pursue a significant goal, for I should like to eliminate once and for all the questions regarding the foundations of mathematics, in the form that they are now posed, by turning every mathematical proposition into a formula that can be concretely exhibited and strictly derived, thus recasting mathematical definitions and inferences in such a way that they are unshakable and yet provide an adequate picture of the whole science.[7]

Hilbert's objective and the objectives of others, such as Brouwer, von Neumann, and Weyl, whose goals were similar, albeit while starting from slightly different sets of assumptions, were famously shown to be impossible to achieve by the 1931 incompleteness result of the Austrian mathematician, Kurt Gödel. The result is that by the mid 1930's, all hope of a unified perspective regarding the foundations of mathematics is lost. In the main, at least initially, the competing perspectives on the foundations of mathematics are Hilbert's *formalism* and Brouwer's *intuitionism*. Later, *constructivism* articulated by among others, the American mathematician Erret Bishop, would emerge as a radical extension of some of the ideas of intuitionism. From the point of view of mathematical knowledge management, the specifics of these ideologies are not important; what is important is that, in the end, there is a plurality of perspectives regarding the foundations of mathematics. Which brings the discussion back to Hardy and Littlewood and the question of how all of the turmoil in the foundation of their science affected the lives of the "working mathematician".

[5] Zitarelli, David E. (quoting Garret Birkhoff), *Towering Figures, 1890-1950*, MAA Monthly Aug-Sept, Vol 108, (2001), 618.

[6] Hersh, Reuben *What is Mathematics Really*, (Oxford: Oxford University Press, 1997) 137.

[7] Hilbert, David *The Foundations of Mathematics* translation by Stefan Bauer-Mengelberg and Dagfinn Føllesdal, in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879-1931*, Jean van Heijenoort (editor), (Boston: Harvard University Press, 1967) 464.

If we could have asked Hardy and Littlewood about how the shifting philosophical ground affected the way they think, talk, and write about mathematics as individual mathematicians and, in particular, how the philosophical upheaval affected the nature of their collaboration, it is quite likely that their reply would have been that the philosophical debate was of little or no consequence. Like most practicing mathematicians of the time and today, when pressed, they seemed to adhere to a Neo-Platonist perspective of mathematical investigation. Here's Hardy from the *Apology*:

> I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our "creations", are simply the notes of our observations.[8]

This was all the philosophy that any practicing mathematician needed and all the philosophy that any mathematician continues to need. It explains mathematics as a process of discerning truths.

So, what about these two whose intent it was to simply get on with the business of doing mathematics? What influenced their day to day experience as mathematicians and what influenced the nature of their collaboration? The answer lies in the structures of the broader mathematical community in which they existed and in which, to be certain, both individually and in collaboration, they played prominent roles.

The Mathematical Community

> A man is necessarily talking error unless his words can claim membership in a collective body of thought.[9] Kenneth Burke

The notion of community is loosely defined and can be used to refer to a lot of quite different types of social groupings; it is important to spend some time with the definition of "community". Intentionally, we will broadly define the mathematics community to include those involved with advancing the understanding of mathematics; either at its frontiers, the primary occupation of researchers, or within the existing body of mathematical knowledge such as teachers and students. The boundary is a porous one and relatively few would claim full time membership. Many others are interlopers, jumping in and out as the need arises or circumstances dictate. This idea of community will need to be passed through a prism, allowing us to consider separately four inter-related factors that help to bind the community: the *language* of the community, the *purposes* of the community, the *methods* of the community, and the *meeting places* of the community.

The Language of the Community

> A precisian professor had the habit of saying:'...quartic polynomial $ax^4 + bx^3 + cx^2 + dx + e$, where e need not be the base of natural logarithms.'[10]
> J.E. Littlewood

It is tempting, but tautological, to state that the language of the community is the language of mathematics and it only extends the tautomerism to state that anyone who claims membership in the community knows what this statement means. In reality, it may be argued that the paradigm for mathematical discourse is the language of the published research paper. All other discourse approximates the paradigm by degree according to what level of rigor is appropriate to the situation and audience.

The special symbols of mathematics present a particular challenge to expressing mathematics in mechanically type set or digital forms. An individual claiming membership in the mathematical community can generally be assumed to have some understanding of how to overcome those challenges.

[8]Hardy, 123.

[9]Burke, Kenneth cited in Booth, Wayne C. *Modern Dogma and the Rhetoric of Assent* (Chicago: University of Chicago Press, 1974) 86.

[10]Littlewood, J.E. *Littlewood's Miscellany*, edited by Béla Bollobás, (London: Cambridge University Press, 1986), 60.

The Purposes of the Community

> If intellectual curiosity, professional pride, and ambition are the dominant incentives to research, then assuredly, no one has a fairer chance of gratifying them than a mathematician.[11]
> G. H. Hardy

In the sense it is used here, 'purpose' does not refer to the overriding *raison d'être* of the community; that has already been defined to be an interest in the advancement of mathematics. Rather, purpose here refers to what motivates an individual to seek membership in the community; and there are many. There is a professional motive which expresses itself by the simple statement that "I am involved with mathematics because this is how I earn my living". There is an egotistical motive which is expressed in the statement that "I am involved with mathematics because I take pleasure from proving to myself and others that I can overcome the challenges that the field affords". There is a social motive which is evident in the statement that "I am involved with mathematics because I benefit from the company of others who are involved with mathematics". And, finally, there is an aesthetic motive that is reflected in statements like "I am involved with mathematics because I wish to help unlock the beauty of mathematics".

With respect to mathematical knowledge management, an individual's reasons for being involved with mathematics strongly affects the individual's role in the community. This sense of purpose in the community in turn helps to determine the individual's information management needs.

The Methods of the Community

Next, we look at the *methods of the community.* Under this rubric, we examine the question of how do mathematicians do what they do and what tools do they use. Traditionally, and certainly through the period encompassed by the Hardy-Littlewood collaboration, mathematics has always been one of the most purely cerebral of the sciences, depending, for its practice, on little more than pencil and paper. This austerity is tightly associated with underlying philosophical assumptions about the nature of mathematics. The foundational shifts of the last century and developments in computer technology paved the way to the situation we find at present, with mathematicians lining up with theoretical physicists, molecular biologists, and others to claim time on the world's most powerful super computers. An important consideration regarding the question of how mathematicians do mathematics is the question of how and to whom to mathematicians *express* their mathematics. This warrants separate treatment.

The Meeting Places of the Community

> J.J. Sylvester sent a paper to the London Mathematical Society. His covering letter explained, as usual, that this was the most important result in the subject for 20 years. The secretary replied that he agreed entirely with Sylvester's opinion of the paper; but Sylvester had actually published the results in the L.M.S. five years before.[12]
> J. E. Littlewood

Tightly associated with the methods of the community, is the notion of the *meeting places of the community.* These are the venues in which mathematics is presented and discussed. Not only the offices, classrooms, seminar rooms, labs, and conference halls, but also the notes, postcards, letters, journals, and, in our electronic age, their digital equivalents.

The factors that bind the mathematical community help to explain how Hardy and Littlewood, and, indeed, all mathematicians, could continue their particular practice of mathematics despite the state of disarray in the underlying ideas which attempted to bind their subject on an intellectual level. Their relationship to the community and their role in the community prevented questions about the intellectual foundations of mathematics from getting in the way of *their* mathematics. On a formal level, these roles

[11] Hardy, 80.
[12] Littlewood, 148.

and relationships were defined by the unwritten rules of community membership. In Hardy and Littlewood's case both were, after all, professors at very established universities and as such, were expected to mix teaching responsibilities with research and to submit papers in the accepted form to acceptable journals. On an informal level however, they were completely free, as were all community members, to define their own rules of engagement. And they did. In his foreword to A Mathematician's Miscellany, Bélla Bollobás quotes a letter in which Harald Bohr describes Hardy and Littlewood's four "axioms" for successful collaboration:

> The first [axiom] said that when one wrote to the other (they often preferred to exchange thoughts in writing instead of orally), it was completely indifferent whether what they said was right or wrong. As Hardy put it, otherwise they could not write completely as they pleased, but would have to feel a certain responsibility thereby. The second axiom was to the effect that, when one received a letter from the other, he was under no obligation whatsoever to read it, let alone answer it, - because, as they said, it might be that the recipient of the letter would prefer not to work at that particular time, or perhaps that he was just then interested in other problems....The third axiom was to the effect that, although it did not really matter if they both thought about the same detail, still, it was preferable that they should not do so. And, finally, the fourth, and perhaps most important axiom, stated that it was quite indifferent if one of them had not contributed the least bit to the contents of a paper under their common name; otherwise there would constantly arise quarrels and difficulties in that now one, and now the other, would oppose being named co-author.[13]

This example of informal mathematical knowledge management on the micro scale provides an excellent point of departure for an accelerated trip through the remainder of the twentieth century with the goal of examining the impact of digital technology on the exchange and management of knowledge within the mathematical community. However, before traveling forwards in time, it will be useful to travel backwards; back to the interface between scribal and typographic culture in the fifteenth and sixteenth centuries.

From Scribal Culture to Typographic Culture

> The difference between the man of print and the man of scribal culture is nearly as great as between the non-literate and the literate. The components of Gutenberg technology were not new. But when brought together in the fifteenth century there was an acceleration of social and personal action tantamount to "take off" in the sense that W.W. Rostow develops this concept in *The Stages of Economic Growth* "that decisive interval in the history of a society in which growth becomes its normal condition."[14] Marshall McLuhan

In *The Gutenberg Galaxy*, Marshall McLuhan describes the change in cultural orientations, expectations, and assumptions that occurred with the wide spread adoption of typography in the fifteenth and early sixteenth centuries and is occurring today with the adoption of electronic media. Typical of McLuhan's style, his main ideas are developed from a number of different perspectives in a non-sequential fashion. Examining the impact of the printing press, McLuhan argues that, while pre-typographic culture was characterized by localized production and limited distribution of production - most abbeys would have at least one scribe but a single scribe can only produce so many manuscripts - typographic culture would come to be characterized by centralized production and mass distribution; a limited number of publishing houses producing and distributing many copies of individual texts. McLuhan suggests that, with printing, came notions of authority, authorship, and intellectual property that were completely unknown in scribal culture. He cites E.P. Goldschmidt, a scholar in medieval studies:

[13]Littlewood, J.E. *Littlewood's Miscellany*, edited by B. Bollobás (editor), 10-11.
[14]McLuhan, Marshall *The Gutenberg Galaxy*, (Toronto: University of Toronto Press, 1962), 90.

> One thing is immediately obvious: before 1500 or thereabouts, people did not attach the same importance to ascertaining the precise identity of the author of a book they were reading or quoting as we do now. We very rarely find them discussing these points...Not only were users of manuscripts, writes Goldschmidt, mostly indifferent to the chronology of authorship and to the "identity and personality of the author of the book he was reading, or in the exact period at which this particular piece of information was written down, equally little, did he expect his future readers to be interested in himself."[15]

Despite the fact that Samuel Morse had brought in the age of electronic communication with his "What hath God wrought?" transmission of 1844, electronic technology had made little impact in the early part of the twentieth century. Phones were few and far between and calls were expensive. The telegraph had found its niche in the long distance communication of simple messages. With advancements in the technology of typography, the typographic age was at its apogee. It can reasonably be argued that in so far as their reputation was earned primarily through the reception by the mathematical community of their published works, the *public identities* of Hardy and Littlewood were creations of typographic culture. The question that confronts us today, in this new era of "take off", is what is the effect of electronic media on the factors that bind the mathematical community and, in particular, what is the effect of electronic media on the organization of mathematical knowledge.

From Typographic Culture to Electronic Culture

> Today, with the arrival of automation, the ultimate extension of the electro-magnetic form to the organization of production, we are trying to cope with such new organic production as if it were mechanical mass production.[16]
> Marshall McLuhan

While the transition from scribal culture to typographic culture represented a shift from loose notions of authorship with distributed loci of publication and limited distribution to firm notions of authorship with centralized loci of publication and mass distribution, the transition to electronic culture turns the equation inside out, presenting the possibility of distributed authorship via mass collaboration and multiple nodes of production with various forms of near instantaneous mass publication. The transformation that occured in the foundations of mathematics, from a unified perspective to a plurality of perspectives, finds resonance in the media environment in which the mathematical community exists and has the potential to affect the language, purposes, methods, and meeting places of the community. In a speech entitled *The Medieval Future of Intellectual Culture: Scholars and Librarians in the Age of the Electron*, professor Stanley Chodorow states:

> In the not-so-distant future, our own intellectual culture will begin to look something like the medieval one. Our scholarly and information environment will have territories dominated by content, rather than by distinct individual contributions. The current geography of information was the product of the seventeenth-century doctrine of copyright. We are all worrying about how the electronic medium is undermining that doctrine. In the long run, the problem of authorship in the new medium will be at least as important as the problem of ownership of information.
>
> ...
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> Works of scholarship produced in and through the electronic medium will have the same fluidity - the same seamless growth and alteration and the same de-emphasis of authorship - as medieval works had. The harbingers of this form of scholarship are the listservs and bulletin boards of the current electronic environment. In these forums, scholarly exchange is becoming

[15] McLuhan, 131 ff.
[16] McLuhan, .130

> instantaneous and acquiring a vigor that even the great scholarly battlers of old - the legendary footnote fulminators - would admire. Scholars don't just work side by side in the vineyard; they work together on common projects[17]

Applied to mathematics, Chodorow's ideas suggest the possibility that the community's elites, long having been composed of those individuals who demonstrate a particular "*individual* vision and brilliance", may undergo a process of reconstruction, resulting in elites whose members are those who have learned how to start with good ideas and develop them by using the internet to harness the intellectual power of the community. In an age of massively parallel mathematical computation, the potential exists for massively parallel mathematical collaboration. Perhaps the best idea of what a fully digital mathematical scholarship and teaching environment *might* look like can be gleaned from the "hacker culture" of the open source programming community. The meeting places of this community are primarily email, threaded bulletin boards, and implementations of the Concurrent Version System. Those who identify themselves as members, speak of the community's "gift culture" which rewards the most talented and generous of members with status in the community meritocracy. In the opening section of *The Cathedral and the Bazaar*, Eric S. Raymond describes hacker culture:

> Many people (especially those who politically distrust free markets) would expect a culture of self-directed egoists to be fragmented, territorial, wasteful, secretive, and hostile. But this expectation is clearly falsified by (to give just one example) the stunning variety, quality and depth of Linux documentation. It is a hallowed given that programmers hate documenting; how is it, then, that Linux hackers generate so much of it? Evidently Linux's free market in egoboo [coined by the author for 'ego boost'] works better to produce virtuous, other-directed behavior than the massively-funded documentation shops of commercial software producers.[18]

He goes on to invoke the idea of a "community of interest":

> I think the future of open-source software will increasingly belong to people who know how to play Linus's game, people who leave behind the cathedral and embrace the bazaar. This is not to say that individual vision and brilliance will no longer matter; rather, I think that the cutting edge of open-source software will belong to people who start from individual vision and brilliance, then amplify it through the effective construction of voluntary communities of interest.[19]

If, indeed, *doing mathematics* in the digital age were to develop in a similar fashion to the way that *doing software development* has in the open source community, then the mathematics community must prepare itself for the loss of fixed notions of authorship and ownership and the accountability and economic models that those notions sustain. There are good reasons to believe, however, that despite changes in patterns of collaboration, doing mathematics in twenty-first century, will not be too unlike doing mathematics in the twentieth century.

[17] Chodorow, Stanley *The Medieval Future of Intellectual Culture: Scholars and Librarians in the Age of the Electron* a speech before Association of Research Libraries Membership Meeting, October, 1996

[18] Raymond, Eric S., *The Cathedral and the Bazaar.*

[19] Raymond, *The Cathedral and the Bazaar.*

Implications and a Proposal

> Mathematics books and journals do not look as beautiful as they used to. It is not that their mathematical content is unsatisfactory, rather that the old and well-developed traditions of type-setting have become too expensive. Fortunately, it now appears that mathematics itself can be used to solve this problem.[20]
> Donald Knuth

In *Digital Typography*, Donald Knuth describes his efforts to capture the traditions of mathematical type-setting in a digital publishing environment; efforts which ultimately led to the development of TEX and METAFONT. It is noteworthy that, faced with a new technology for communicating and presenting mathematics, one of the first major projects related to mathematical publishing was a truly awesome effort to preserve the most valued qualities of traditional typography. The enormous success of Knuth's enterprise stands as a testament to the high value that is placed upon traditional methods of representing mathematical knowledge. It is an interesting aside that the TEX and METAFONT projects were among the first open source programming efforts, functioning without the support of the web, the code being released to collaborators via email.

Apart from the value that members of the mathematical community evidently place on good quality digital typography, there are a number of other reasons to believe that traditional methods of knowledge representation such as the refereed journal and the bound textbook together with the ideas of copyright and accountability that they encapsulate, will not completely fall victim to the distributed modes of digital technology. The educational and research institutions that support traditional forms of knowledge representation are well established and, as evidenced by the success of firms which offer support for open source software, there is every reason to believe that there will continue to be a market for mathematical content that comes with some form of explicit or implicit guarantee and accountablity.

This is not to imply that digital modes of expression can or should be left to develop unscrutinized. The complexity of modern mathematics and the volume of work produced has led to classification schema that are being adapted and extended to digital publication. The use of metatags to describe digital documents has an interesting antecedent in medieval scholarship. In his book *Medieval Theory of Authorship*, A.J. Minnis describes the use of formal prologues found at the beginning of manuscripts. Here, he describes the so-called 'type C' prologue:

> In the systematisation of knowledge which is characteristic of the twelfth century, the 'type C' prologue appeared at the beginning of commentaries on textbooks os all disciplines: the arts, medicine, Roman law, canon law, and theology. Its standard headings, refined by generations of scholars and to some extent modified through the influence of other types of prologue, may be outlined as follows: *Titulus*, the title of the work...*Nomen auctoris*, the name of the author...*Intentio Auctoris*, the intention of the author...*Materia Libri*, the subject matter of the work...*Modus agendi*, the method of didactic procedure...*Ordo libri*, the order of the book...*Utilitas*, utility...*Cui parti philosophiae supponitur*, the branch of learning to which the work belonged.[21]

Cast as medieval metadata, these prologues indicate some effort on the part of medieval scholars to protect the integrity of scholarly works in a distributed publishing environment. The question arises as to whether or not the unique modes of digital publishing that members of the mathematical community may create to express mathematical thought might be supceptible to some form of classification via metadata. For example, what might Hardy and Littlewood's correspondence look like in a digital environment? The

[20] Knuth, Donald *Digital Typography*, (Stanford: CSLI Publications, 1999), 19.
[21] Minnis, A.J. *Medieval Theory of Authorship: Scholastic literary attitudes in the later Middle Ages*, (London: Scolar Press, 1984) 19.

default answer is that it would *look* much the same as long as they were to access their work on devices that presented a viewing area that resembled the paper and postcards that they used. The qualified answer is that it would look much the same unless they were reading the information using a display with quite different geometry from note paper or postcards. If, for example, they were to attempt to read the data using a cell phone, then ideally the logic of the the digital environment would make the appropriate adaptations and do the best job possible to make the data readable. The correct answer is that the correspondence wouldn't look like anything at all because it would never be in digital form; Hardy would refuse to have anything to do with it. By all accounts, he was a true Luddite who mistrusted the telephone and would refuse even to use ball point pens. Hardy's likely reluctance aside, however, it is possible to imagine how the appropriate digital environment might have been tremendously useful to the Hardy-Littlewood collaboration. If their correspondence had been instantly stored in a database, then a minimal effort invested in specifying metadata would permit them efficient searches and the other digital data manipulations that we now take for granted such as cutting and pasting. Depending on what permissions they chose to grant to the data, a wider audience could be included as observers, or partial or full collaborators.

Emkara, the Extensible Mathematical Knowledge Archiving and Retrieval Agent, is the working title of a project at Simon Fraser University's *Centre for Experimental and Constructive Mathematics* that is designed to investigate how user defined mathematical knowledge construction might conform to metadata driven information management. Emkara is, in effect, a database management system which affords qualified users the ability to create their own data structures, while encouraging thoughtful use of metadata to describe what they are creating. The act of creating a data structure is a table creation operation on the database. All user-defined metadata is stored internally as well-formed XML and, at present, mathematical content fields store data as mathML embedded in xhtml. When a qualified user creates a new mathematical object, emkara generates a default 'edit mode' script, a default 'view mode' script, and the corresponding style sheet for each. These scripts and style sheets are accessible to the object creator for modification or replacement. The object creation interface requests certain metadata by default. These include:

- basic elements of the *Math-Net* metadata set[22]
- some elements of *eduML* mark-up[23]
- information concerning the *copyright* of the object data.

The research incentive of this project is two-fold; to gain insight into what forms of metadata "work" in electronic mathematics knowledge management and to develop a framework for describing the characteristics of electronic mathematics interfaces that meet their objectives.

Sequel...

In the theory of software development processes, *Conway's Law* is cited as a caveat regarding the tendency of a software project's logical design to take on the characteristics of the organizational structure of the work groups that create it. The full statement of the law describes a set of complementary forces in which software architecture informs organizational structure and vice versa. Ultimately, however, the two become aligned and it is therefore important in the early stages of a project to build as much flexibility as possible into both architecture and organizational structure[24]. While these statements about the software development process were never intended to apply to an undertaking such as the development of systems for mathematical knowledge management, the relationship between organizational structure and system design is worth considering.

[22] Math-Net
[23] eduML
[24] Coplien, James O *A Development Process Generative Pattern Language*, (AT&T, 1995), p.19

Earlier, a broad definition of *mathematical community* was adopted encompassing all "those involved with advancing the understanding of mathematics; either at its frontiers, the primary occupation of researchers, or within the existing body of mathematical knowledge such as teachers and students". This definition is at odds with the experience of most who might claim either full or part time membership in the community. If there is truth to the idea that our individual experience of "community" is formed by the group of people with whom we exchange ideas, then it can be argued that any undertaking that attempts to define systems of mathematical knowledge management is, perforce, also defining the structure of the mathematical community.

It is possible to argue that many of the factors that have determined the current divisions within the broad mathematics community, such as domain specialization in research and age group specialization in education, have their origin in the perspectives of typographic culture. It is not necessary here that these arguments be completed. Rather, it is important to point out that there is the possibility of defining management systems that can support the exchange of ideas between smaller communities within the broad mathematics community. These systems would need to create "meeting places" that bring the broader community together. The schism that presently exists between school level and university level mathematics provides a good example. There is no reason that those interested in exchanging ideas regarding a topic in a high school mathematics curriculum could not visit the same electronic mathematical community centre as members of a particular research community. While members of one group may never use nor even look at the resources that are designed for members of the other group, the fact that they pass through the same digital front doors and may even linger, looking at the notices posted in the digital entrance hall, offers hope that each group, even if only accidentally, might gain a better understanding of the priorities and concerns of the other group.

Conclusion

Men despise religion; they hate it, and they fear it is true.
Pascal, from *Pensées*, 1670.

The idea of organized religion representing a fixed, centralized view of the world has come up several times in this paper starting with the anecdote concerning Hardy's atheism and most recently with the citations from the metaphorically titled *The Cathedral and the Bazaar.* With some exceptions, it is probably inaccurate to imply that modern churches are unyielding and inflexible in their outlook and community structure. It would be even more inaccurate to suggest that the broad mathematical community has much in common with organized religion. If this were true, then in his day, Hardy would certainly have been one of the high priests; an idea that he surely would have found either very amusing or very annoying or both. What is true however is that over the course of this century and particularly with the accelerated spread of computer technology that has occured in the last fifteen years, the mathematical community has been faced with the challenge of adapting its language, methods, and meeting places to new technology. The affect that digital technology had on the methods of the community was clearly reflected in the types of knowledge considered valid by the community. For example, it is today not unusual to find mathematical papers with blocks of Maple code. Networking technology is having a more complicated effect on the forms of knowledge that are accepted by the community. Bearing in mind that, in 1969, the *Culler-Fried Interactive Mathematics Center* at the University of California at Santa Barbara became the third node on the arpanet, it is fair to say that mathematicians have seen the potential of the network and encountered its problems from its genesis. At this still early stage in the development of network technologies for mathematical knowledge management, it is important to consider the effect that those technologies can have on the meeting places of the community and, among other things, who is invited to those meeting places.