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# GENERALIZED ADHM-CONSTRUCTION ON WOLF SPACES

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## 1. Introduction

On $4n$-dimensional quaternion-Kähler manifolds, self-dual (SD) connections can be defined, which is the same as self-dual connections in 4-dimensional Riemannian geometry in the case $n = 1$.

However the situation in higher dimensionl case is quite different. For example, there exists "rank 2" vector bundles with an ASD connection on 4-dimensional sphere $S^4 \cong \mathbb{H}P^1$, which is one of conclusions from ADHM-construction. On the contrary, algebraic geometers believe that there do *not* exist any indecomposable "rank 2" holomorphic vector bundles on 5-dimensional complex projective sapce $\mathbb{C}P^5$ which is the Salamon twistor space of $\mathbb{H}P^2$. Hence it is conjectured that we have no "rank 2" ASD bundle on $\mathbb{H}P^2$. In general, the lack of low-rank holomorphic vector bundles on higher dimensional Kähler manifolds prevented us from finding concrete examples of ASD bundles on higher dimensional quaternion-Kähler manifolds via the twistor theory.

It is natural that we try to generalize ADHM-construction, when intending to construct some ASD bundles on higher-dimensional quaternionic projective spaces, because so called ADHM-data is comprised of finite dimensional vector spaces and linear maps between them with some conditions [4, p.97]. Indeed, this approach is adopted by Mamone Capria and Salamon [10]. (These "$k$-instantons" in higher dimensional case can be *classified* via vanishing theorems ([11] and [9].)) As examples independent of ASD bundles on 4-dimensional manifolds, Mamone Capria and Salamon found that the well known Horrocks bundle (rank "3"!) on $\mathbb{C}P^5$ can be obtained as the pull-back bundle of an anti-self-dual bundle on $\mathbb{H}P^2$ [10]. They also showed that there exists a rank 3 ASD homogeneous vector bundle on $G_2/SO(4)$. These were the only known concrete examples of anti-self-dual bundles on higher-dimensional quaternion-Kähler manifolds until 1990.

In my talk, these ASD bundles are reinterpreted from **representation theory** of compact Lie groups (and complexified Lie groups of them). The method of **monad** (or ADHM-construction) is generalized to the Wolf spaces from the viewpoint of representation theory. The purpose of my talk is to give classification of irreducible homogeneous bundles with ASD canonical connections and construct non-homogeneous ASD connections on the Wolf spaces. We will obtain many examples of ASD bundles systematically, which include all the examples provided by Mamone Capria and Salamon. Secondly, the moduli spaces of such connections are described via the theory of monad and the Bott-Borel-Weil theorem. Finally, we focus attention on the boundary of the moduli spaces. Such a boundary point represents an ASD-connection "with a singular set". The relation between such a singular set and a vector bundle on which a singular ASD connection is defined will be understood through the Poincaré duality.

## 2. Preliminaries

**• The Wolf spaces and the Salamon twistor spaces**

*Theorem* 2.1. [19] For every complex simple Lie group, there exists only one compact quaternion symmetric space. (These compact quaternion symmetric sapces are called the Wolf spaces.)

*Example.* type $A_{n+1} = Gr_2(\mathbb{C}^{n+2})$, type $C_{n+1} = \mathbb{H}P^n$.
In particular, we have only two **4-dimensional** compact quaternion symmetric space :
type $A_2 = Gr_2(\mathbb{C}^3) = \mathbb{C}P^2$, type $C_2 = \mathbb{H}P^1 = S^4$

*Theorem* 2.2. [7] A compact 4-dimensional manifold of which the twistor space admits a Kähler metric is conformally equivalent to $\mathbb{C}P^2$ or $S^4$ with a standard metric.

Let $Z$ be the Salamon twistor space. (In the 4-dimensional case, $Z$ is the Penrose twistor space.)

*Theorem* 2.3. [18] The total space of the twistor space $Z$ has a natural complex structure and so, $Z$ is a complex manifold whose dimension is $2n+1$. The fibre of $Z$ is a complex submanifold and is holomorphically isomorphic to $\mathbb{C}P^1$.

*Example.*

$$\begin{array}{ccc} \mathbb{C}P^{2n+1} & & F^{2n+1} \\ \downarrow{\scriptstyle \mathbb{C}P^1} & & \downarrow{\scriptstyle \mathbb{C}P^1} \\ \mathbb{H}P^n & & Gr_2(\mathbb{C}^{n+2}) \end{array} \qquad F^{2n+1} = \frac{SU(n+2)}{S(U(1)\times U(n)\times U(1))}$$

**• ASD-connection**

We shall treat metric connections on a complex vector bundle $E$ equipped with a Hermitian metric $h$ over a quaternion-Kähler manifold $M$.

*Definition.* [6, 10, 16] A connection $\nabla$ is called **anti-self-dual(ASD)**

$$\overset{\text{def}}{\Longleftrightarrow} R^{\nabla}(IX, IY) = R^{\nabla}(JX, JY) = R^{\nabla}(KX, KY) = R^{\nabla}(X, Y), \quad \text{for all } x \in M \text{ and all } X, Y \in T_x M,$$

where $R^{\nabla}$ is the curvature of $\nabla$, which is regarded as $\operatorname{End} E$ valued 2-form on $M$.

A vector bundle with an ASD connection is called **ASD bundle** or **instanton** (bundle).

*Theorem* 2.4. [6, 10, 16] Any ASD connection is a Yang-Mills connection.

*Remark.* Moreover, if $M$ is compact, then ASD connection minimizes the Yang-Mills functional [6, 10].

*Example.* <u>4-dimensional case</u>
• **ADHM**-construction (Atiyah-Drinfeld-Hitchin-Manin)
All instanton bundles on $S^4$ are classified by the twistor method (for example, see [1, 5]).
• All instanton bundles on $\mathbb{C}P^2$ are also classified in a similar way by Buchdahl [3].

*Remark.* Before 1990, in the **higher-dimensional** case, concrete examples of vector bundles with ASD connections had not known except examples presented by Mamone Capria and Salamon [10].

The twistor method in the examples is explained in the next theorem, originated with Atiyah, Hitchin and Singer.

*Theorem* 2.5. [2, 10, 16] The pull-back connection of an ASD connection induces a holomorphic structure on the pull-back bundle on the twistor space $Z$, and so the pull-back bundle is a holomorphic vector bundle on $Z$.

## 3. Homogeneous ASD bundles

By ADHM-construction [1, 5] and Buchdahl [3], the standard ASD bundles with $c_2 = 1$ on $S^4$ and $\mathbb{C}P^2$ are homogeneous bundles with canonical connections. In this section, we determine irreducible homogeneous vector bundles with ASD canonical connections in terms of weights.

*Definition.*

| | |
|---|---|
| $\mathfrak{g}^{\mathbb{C}}$ : complex simple Lie algebra | $B$ : the Killing form of $\mathfrak{g}^{\mathbb{C}}$ |
| $\theta$ : maximal root | $I$ : the set of integral weights |

*Definition.* $f : I \to \mathbb{Z}$

$$\underline{f(\lambda) = B(\lambda, \theta^{\vee}) \qquad (\lambda \in I)}$$

where, $\theta^{\vee}$ is the co-root of $\theta$. ($\theta^{\vee} = 2\theta / B(\theta, \theta)$.)

**Notation**

$$E(\lambda) := \text{the irreducible representation space of } \mathfrak{g}^{\mathbb{C}} \text{ has } (-\lambda) \text{ as an extremal weight}$$
$$E_{\mathfrak{p}}(\lambda) := \text{the irreducible representation space of } \mathfrak{p} \text{ has } (-\lambda) \text{ as an extremal weight}$$

where, $\mathfrak{p} \subset \mathfrak{g}^{\mathbb{C}}$: parabolic subalgebra.

$G^{\mathbb{C}}$ : simply connected Lie group whose Lie algebra is $\mathfrak{g}^{\mathbb{C}}$

$\mathfrak{g}$ : a compact real form of $\mathfrak{g}^{\mathbb{C}}$

$G$ : the corresponding compact simply connected Lie group to $\mathfrak{g}$

$G/K_4$ : compact quaternion symmetric space

$G/K_Z$ : the twistor space

*Remark.* Since the twistor space $G/K_Z$ is a compact simply connected homogeneous Kähler manifold, we can also express the twistor space using a complex simply connected Lie group $G^{\mathbb{C}}$. Then the twistor space is denoted by $G^{\mathbb{C}}/P$, where $P$ is the corresponding parabolic subgroup of $G^{\mathbb{C}}$.

*Definition.* $\mathcal{O}_{\mathfrak{p}}(\lambda) = G^{\mathbb{C}} \times_P E_{\mathfrak{p}}(\lambda)$ : irreducible homogeneous holomorphic vector bundle on the twistor space $G^{\mathbb{C}}/P$.

We have the classification theorem for ASD irreducible homogeneous vector bundles.

*Theorem* 3.1. [12] Let $E$ be an irreducible homogeneous bundle over $G/K_4$ of which the canonical connection is ASD. Then, there exists an integral weight $\lambda$ with $f(\lambda) = 0$ such that $\mathcal{O}_{\mathfrak{p}}(\lambda)$ is the pull-back bundle of $E$ on the twistor space $G/K_Z$. Conversely, if an integral weight $\lambda$ satisfies $f(\lambda) = 0$, an irreducible homogeneous holomorphic bundle $\mathcal{O}_{\mathfrak{p}}(\lambda)$ on $G/K_Z$ is the pull-back of an ASD homogeneous bundle on $G/K_4$.

## 4. Monad and Representation Theory

In this section, we show that a dominant integral weight induces a monad of vector bundles on the twistor space of which the cohomology bundle is the pull-back of an ASD bundle.

*Definition.* (cf.[17]) A "**monad**" is a complex of vector bundles

$$A \xrightarrow{a} B \xrightarrow{b} C,$$

with homomorphisms $a$ and $b$ between them, such that $a$ is injective and $b$ is surjective. The quotient bundle

$$E = \operatorname{Ker} b / \operatorname{Im} a$$

is called the **cohomology of the monad**.

Let $W$ be the Weyl group of $\mathfrak{g}^{\mathbb{C}}$ and $w^0$ be the longest element of $W$.

☐ **The unified construction of monad**

1. We take an irreducible representation space $E(\lambda)$ of $G^{\mathbb{C}}$, where $\lambda$ is a dominant integral weight.
2. Restrict the homomorphism $G^{\mathbb{C}} \longrightarrow \operatorname{End} E(\lambda)$ to $P$, then we have a complex of representation spaces:
$$E_{\mathfrak{p}}(w^0\lambda) \xrightarrow{\ i\ } E(\lambda) \xrightarrow{\ \pi\ } E_{\mathfrak{p}}(\lambda)$$
where, $i$:**injection**, $\pi$:**surjection**, $\pi \circ i = 0$ and
$i$, $\pi$: $P$-**equivariant homomorphism**.
We call this complex *a monad of representation*.
3. A complex of representation spaces yields a monad of vector bundles.
$$\mathcal{O}_{\mathfrak{p}}(w^0\lambda) \xrightarrow{\ \alpha\ } G^{\mathbb{C}}/P \times E(\lambda) \xrightarrow{\ \beta\ } \mathcal{O}_{\mathfrak{p}}(\lambda),$$
where,
$$\alpha\big([g,e]\big) = \big([g], gi(e)\big) \quad \text{and} \quad \beta\big([g],u\big) = \big[g, \pi(g^{-1}u)\big],$$
$$g \in G^{\mathbb{C}}, e \in E_{\mathfrak{p}}(w^0\lambda) \text{and} u \in E(\lambda).$$

*Example.* • If we take the dominant integral weight $\varpi_1$ of $C_{n+1}$ ($\mathbb{H}P^n$), then we have
$$\mathcal{O}_{\mathfrak{p}}(w^0\varpi_1) \xrightarrow{\ \alpha\ } \underline{E(\varpi_1)} \xrightarrow{\ \beta\ } \mathcal{O}_{\mathfrak{p}}(\varpi_1),$$
where, $\underline{E(\varpi_1)} = G^{\mathbb{C}}/P \times E(\varpi_1)$. In the case $n = 1$, this is nothing but a monad for 1-instanton bundle which is presented by ADHM-construction [1]: (In higher dimensional case, see [10] and [9].)
$$\mathcal{O}(-1) \xrightarrow{\ \alpha\ } \underline{E(\varpi_1)} \xrightarrow{\ \beta\ } \mathcal{O}(1).$$

• If we take two dominant integral weights $\varpi_1$ and $\varpi_n$ of $A_{n+1}$ ($Gr_2(\mathbb{C}^{n+2})$), then we have
$$\mathcal{O}_{\mathfrak{p}}(w^0\varpi_1) \oplus \mathcal{O}_{\mathfrak{p}}(w^0\varpi_n) \xrightarrow{\ \alpha\ } \underline{E(\varpi_1) \oplus E(\varpi_n)} \xrightarrow{\ \beta\ } \mathcal{O}_{\mathfrak{p}}(\varpi_1) \oplus \mathcal{O}_{\mathfrak{p}}(\varpi_n).$$
In the case $n = 1$, this is nothing but a monad for 1-instanton bundle which is presented by Buchdahl [3]. (In higher dimensional case, see [15].)
$$\mathcal{O}(0,-1) \oplus \mathcal{O}(-1,0) \xrightarrow{\ \alpha\ } \underline{E(\varpi_1) \oplus E(\varpi_n)} \xrightarrow{\ \beta\ } \mathcal{O}(1,0) \oplus \mathcal{O}(0,1),$$

*Definition.* A monad of vector bundles on $G^{\mathbb{C}}/P$ obtained in the above way is called *the standard monad induced by* $\lambda$.

*Theorem* 4.1. For an integral dominant weight $\lambda$, the following two conditions are equivalent:

1. $f(\lambda) = 1$.

2. The cohomology bundle of the standard monad induced by $\lambda$ is the pull-back of an ASD bundle on $G/K_4$.

In the next section, we apply this method on the Wolf spaces of type $B, D, E, F$ and $G$.

## 5. Moduli Spaces

¿From now on, we pick up the dominant integral weights (or the corresponding irreducible representation spaces) of $G^{\mathbb{C}}$:

$$\begin{aligned} A_n :& E(\varpi_1) \oplus E(\varpi_n), \quad B_n : E(\varpi_n), \quad C_n : E(\varpi_1), \\ D_n :& E(\varpi_{n-1}) \oplus E(\varpi_n), \quad E(\varpi_1) \oplus E(\varpi_{n-1}), \quad E(\varpi_1) \oplus E(\varpi_n), \\ E_6 :& E(\varpi_1) \oplus E(\varpi_6), \quad F_4 : E(\varpi_4), \quad G_2 : E(\varpi_1). \end{aligned}$$

These dominant integral weights (say, $\lambda$) which we choose satisfy $f(\lambda) = 1$. As in the previous section, we obtain the standard monad of vector bundles. We describe moduli spaces of ASD bundles, which are obtained by deforming vector bundle homomorphisms $\alpha$ and $\beta$ of the standard monad.

□ **Description of moduli**

For simplicity, we restrict ourselves to the case that the weight which we pick up is $\varpi_1$ of type $C_n$.

$$\mathcal{O}_{\mathfrak{p}}(w^0\varpi_1) \xrightarrow{\alpha} \underline{E(\varpi_1)} \xrightarrow{\beta} \mathcal{O}_{\mathfrak{p}}(\varpi_1),$$

1. Applying the Bott-Borel-Weil theorem:
   we have the identification as the $G$-representation spaces such that

$$\begin{aligned} &H^0(\mathrm{Hom}(\mathcal{O}_{\mathfrak{p}}(w^0\varpi_1),\, E(\varpi_1))) \cong \mathrm{End}\, E(\varpi_1), \\ &H^0(\mathrm{Hom}(E(\varpi_1),\, \mathcal{O}_{\mathfrak{p}}(\varpi_1),\, )) \cong \mathrm{End}\, E(\varpi_1). \end{aligned}$$

   Hence $\alpha$ and $\beta$ are identified with $A \in \mathrm{End}\, E(\varpi_1)$ and $B \in \mathrm{End}\, E(\varpi_1)$, respectively.
2. $\beta \circ \alpha = 0 \Leftrightarrow BA \in \mathbb{C} \oplus E(\varpi_2) \subset \mathrm{End}\, E(\varpi_1)$
3. $\alpha$:injection, $\beta$:surjection(non-degeneracy condition) $\Leftrightarrow \det BA \neq 0$
4. the cohomology bundles are the pull-back of some ASD bundles (reality condition) $\Leftrightarrow B = A^*$

As a result, we obtain the following.

*Theorem* 5.1. [12] The moduli spaces are identified with the following spaces, respectively.

• Table 5.1(The moduli spaces of "1-instanton bundles")

| base spaces | rep. spaces | moduli space |
|---|---|---|
| $A_n$ | $E(\varpi_1) \oplus E(\varpi_n)$ | an open cone over $\mathbb{P}(E(\varpi_2))$ |
| $B_n$ | $E(\varpi_n)$ | an open ball in $E(\varpi_1)^{\mathbb{R}}$ |
| $C_n$ | $E(\varpi_1)$ | an open ball in $E(\varpi_2)^{\mathbb{R}}$ |
| $D_n$ | $E(\varpi_{n-1}) \oplus E(\varpi_n)$ | an open cone over $\mathbb{P}(E(\varpi_1))$ |
| | $E(\varpi_1) \oplus E(\varpi_{n-1})$ | an open cone over $\mathbb{P}(E(\varpi_{n-1}))$ |
| | $E(\varpi_1) \oplus E(\varpi_n)$ | an open cone over $\mathbb{P}(E(\varpi_n))$ |
| $E_6$ | $E(\varpi_1) \oplus E(\varpi_6)$ | an open cone over $\mathbb{P}(E(\varpi_1))$ |
| $F_4$ | $E(\varpi_4)$ | an open ball in $E(\varpi_4)^{\mathbb{R}}$ |
| $G_2$ | $E(\varpi_1)$ | an open ball in $E(\varpi_1)^{\mathbb{R}}$ |

where, for example, $E(\varpi_1)^{\mathbb{R}}$ denotes the real representation space of $G$ in $E(\varpi_1)$.

*Remark.* In the case of $A_2\,(Gr_2(\mathbb{C}^3) \cong \mathbb{C}P^2)$, the moduli space is an open cone over $\mathbb{P}(E(\varpi_2)) \cong \mathbb{P}(\mathbb{C}^3) \cong \mathbb{C}P^2$. In the case of $C_2\,(\mathbb{H}P^1 \cong S^4)$, the moduli space is an open ball in $E(\varpi_2)^{\mathbb{R}} \cong \mathbb{R}^5$. These are well known moduli spaces of 1-instantons.

*Remark.* In the case of type $G_2$ $(G_2/SO(4))$, "the center" of the moduli space represents the canonical ASD connection which is found by Mamone Capria and Salamon [10].

On the complex Grassmannian manifold $Gr_2(\mathbb{C}^{n+2})$ (the Wolf space of type $A_{n+1}$), we obtain another type of ASD bundles in a slightly different way. However these ASD bundles are also 1-instantons in the case $n = 1\,(\mathbb{C}P^2)$.

*Theorem* 5.2. [13] The moduli space is identified with an open cone over $\mathbb{P}(E(\varpi_1)) \cong \mathbb{C}P^{n+1}$.

Finally, we introduce generalized Horrocks bundles on odd-dimensional complex projective spaces.

*Theorem* 5.3. [12] On $\mathbb{C}P^{2n+1}$ $(n \geqq 2)$, we have a monad of the following type:

$$\mathcal{O}(-1) \longrightarrow \mathcal{O}_{\mathfrak{p}}(-\varpi_1 + \varpi_{n+1}) \longrightarrow \mathcal{O}(1),$$

and the cohomology bundle of this monad is the pull-back of an anti-self-dual bundle on $\mathbb{H}P^n$. In particular, in the case of $n = 2$, this cohomology bundle is the well known Horrocks bundle on $\mathbb{C}P^5$ [8, 10].

## 6. Singular sets

In our geometric description in §5, obvious compactifications are suggested, (though we do not explicitly refer to the topology of the moduli spaces.)

For simplicity, we explain our theorem in the case of 1-instanton bundle $E$ on the Wolf space of type $B_3$ $(Gr_4(\mathbb{R}^7)^{\sim})$. In our description of moduli by monad, the boundary point of the moduli space represents bundle homomorphisms $a : \mathcal{O}_{\mathfrak{p}}(w_0\varpi_3) \to \underline{E(\varpi_3)}$ and $b : \underline{E(\varpi_3)} \to \mathcal{O}_{\mathfrak{p}}(\varpi_3)$ which are *not* injective and surjective, respectively. We fix $G$-invariant Hermitian metrics on the homogeneous bundles $\mathcal{O}_{\mathfrak{p}}(w_0\varpi_3)$ and $\mathcal{O}_{\mathfrak{p}}(\varpi_3)$. Using Hermitian metrics and bundle homomorphisms $a$ and $b$, we obtain a bundle homomorphism

$$B := a^* \oplus b : \underline{E(\varpi_3)} \to \mathcal{O}_{\mathfrak{p}}(w_0\varpi_3) \oplus \mathcal{O}_{\mathfrak{p}}(\varpi_3).$$

Because of the reality condition ($B = A^*$) in §5, a bundle homomorphism $B$ pushed down to $Gr_4(\mathbb{R}^7)^{\sim}$. The subset $S$ in $Gr_4(\mathbb{R}^7)^{\sim}$ is defined:

$$S := \left\{ x \in Gr_4(\mathbb{R}^7)^{\sim} \middle| B_x : \underline{E(\varpi_3)}_x \to \mathcal{O}_{\mathfrak{p}}(w_0\varpi_3)_x \oplus \mathcal{O}_{\mathfrak{p}}(\varpi_3)_x \text{ is not surjective} \right\}.$$

The subset $S$ is called singular set.

*Theorem* 6.1. • The restricted bundle Ker$B$ to $Gr_4(\mathbb{R}^7)^{\sim} \setminus S$ is still an ASD bundle.
• The singular set $S$ is a quaternion submanifold $Gr_2(\mathbb{C}^4) \subset Gr_4(\mathbb{R}^7)^{\sim}$.
• The Poincaré dual of $S$ is the second Chern class $c_2(E)$.
• In some sense, on the singular set $S$, $E|_S$ is identified with the standard 1-instanton bundle on $Gr_2(\mathbb{C}^4)$ which corresponds to the vertex of the moduli space (see Table 5.1).

• Table 6.1 (Singular set)

| **base spaces** | **singular set** | **Poincaré dual** |
|---|---|---|
| $(1)Gr_2(\mathbb{C}^{n+2})$ | 1point, $\mathbb{H}P^1, \cdots,$ $\mathbb{H}P^{[\frac{n}{2}]}$ | $c_{2n}(E)$, $c_{2n-2}(E)$, $\cdots$, $c_n(E)$($n$:even), $c_{n+1}(E)$($n$:odd) |
| $(2)Gr_2(\mathbb{C}^{n+2})$ | $Gr_2(\mathbb{C}^{n+1})$ | $c_2(E)$ |
| $Gr_4(\mathbb{R}^7)^{\sim}$ | $Gr_2(\mathbb{C}^4)$ | $c_2(E)$ |
| $\mathbb{H}P^n$ | 1point, $\mathbb{H}P^1, \cdots, \mathbb{H}P^{n-1}$ | $c_{2n}(E)$, $c_{2n-2}(E)$, $\cdots$, $c_2(E)$ |
| $G_2/SO(4)$ | $\mathbb{C}P^2$ | $c_2(E)$ |

where $[m]$ is the greatest integer not greater than $m$.

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