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CHAOTIC SCATTERING ON NONCOMPACT SURFACES OF CONSTANT NEGATIVE CURVATURE

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Abstract. The quantization of the chaotic geodesic motion on Riemann surfaces $\Sigma_{g,\kappa}$ of constant negative curvature with genus g and a finite number of points κ infinitely far away (cusps) describing scattering channels is investigated. It is shown that terms in Selberg's trace formula describing scattering states can be expressed in terms of a regularized time delay. Poles in these quantities give rise to resonances reflecting the chaos of the underlying classical dynamics. Illustrative examples for a class of $\Sigma_{g,2}$ surfaces are given.

1. Introduction

Let us consider the two dimensional sphere S^2 with three points removed. This is a two dimensional surface with three holes. A *classical* charged particle confined to move on the inner part of this "box" can enter and leave the box on any one of the holes. Taking these exceptional points infinitely far away with respect to some metric on S^2 we obtain a pants-like leaky surface $\Sigma_{0,3}$. This surface is called by mathematicians a noncompact Riemann-surface with three cusps and genus zero. It can serve as a model of a three channel scattering problem, where the channels are realized topologically. Taking instead of the sphere ($g = 0$), a torus ($g = 1$) or any higher genus multiply connected surface and moving κ points infinitely far away we obtain a wide variety of multichannel scattering systems. These systems describe the *classical* motion of a charged particle inside a noncompact box, modelled by a Riemann surface of type $\Sigma_{g,\kappa}$.

How can we obtain a unified description of such surfaces? According to Riemann uniformization, except for the sphere $\Sigma_{0,0}$, and the torus $\Sigma_{1,0}$ all