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INTEGRABILITY OF CONTACT SCHWARZIAN DERIVATIVES AND ITS LINEARIZATION

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Abstract. We define the contact Schwarzian derivatives $s_{[ij,k]}(\phi)$ for a contact transformation $\phi : \mathbb{K}^3 \to \mathbb{K}^3$. Using the contact Schwarzian derivatives as coefficients, we give a system of linear differential equations such that the solutions give the contact transformation.

1. Introduction

For a contact transformation $\phi : (x, y, z) \mapsto (X, Y, Z)$, we define the contact Schwarzian derivatives $s_{[ij,k]}(\phi)$. A system of non-linear differential equations for a quadruple of functions is given as the condition that the quadruple is the Schwarzian derivatives of a contact transformation. Using a quadruple of functions on $\mathbb{K}^3$ as coefficients, we give a system of linear differential equations. The integrable condition of the linear system is just equal to the non-linear system. We call the linear system a linearization of the integrability condition of the contact Schwarzian derivatives. If the linear system is integrable, the solutions give the contact transformation whose contact Schwarzian derivatives are the coefficient functions. Details will appear in a joint paper with Tetsuya Ozawa [2].

2. Contact Schwarzian Derivative

On the affine 3-space $\mathbb{K}^3$ ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) with the usual coordinate (x, y, z), we give the contact form $\alpha = dy - zdx$.

$$v_1 = \frac{\partial}{\partial x} + z\frac{\partial}{\partial y}, \quad v_2 = \frac{\partial}{\partial z}, \quad v_3 = \frac{\partial}{\partial y}, \quad v_4 = v_2 v_1 + v_1 v_2 .$$