Eleventh International Conference on
Geometry, Integrability and Quantization
June 5–10, 2009, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2010, pp 71–84

PROPERTIES OF BIHARMONIC SUBMANIFOLDS IN SPHERES*

ADINA BALMUŞ, STEFANO MONTALDO† and CEZAR ONICIUC

Faculty of Mathematics, "Al.I. Cuza" University of Iasi
Blvd. Carol I No. 11, 700506 Iasi, Romania

†*Dipartimento di Matematica, Università degli Studi di Cagliari*
Via Ospedale 72, 09124 Cagliari, Italia

Abstract. In the present paper we survey the most recent classification results for proper biharmonic submanifolds in unit Euclidean spheres. We also obtain some new results concerning geometric properties of proper biharmonic constant mean curvature submanifolds in spheres.

1. Introduction

Biharmonic maps $\phi : (M, g) \to (N, h)$ between Riemannian manifolds are critical points of the **bienergy functional**

$$E_2(\phi) = \frac{1}{2}\int_M \|\tau(\phi)\|^2 \, v_g$$

where $\tau(\phi) = \operatorname{tr} \nabla \mathrm{d}\phi$ is the tension field of ϕ that vanishes for harmonic maps (see [17]). The Euler-Lagrange equation corresponding to E_2 is given by the vanishing of the **bitension field**

$$\tau_2(\phi) = -J^\phi(\tau(\phi)) = -\Delta\tau(\phi) - \operatorname{tr} R^N(\mathrm{d}\phi, \tau(\phi))\mathrm{d}\phi$$

where J^ϕ is formally the Jacobi operator of ϕ (see [24]). The operator J^ϕ is linear, thus any harmonic map is biharmonic. We call **proper biharmonic** the non-harmonic biharmonic maps. Geometric and analytic properties of proper biharmonic maps were studied, for example, in [2, 25, 27].

*Reprinted from JGSP **17** (2010) 87–102.