Eleventh International Conference on
Geometry, Integrability and Quantization
June 5–10, 2009, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2010, pp 157–169

NEW RESULTS ON THE GEOMETRY OF TRANSLATION SURFACES*

MARIAN I. MUNTEANU and ANA I. NISTOR

Department of Mathematics, University Al. I Cuza of Iaşi, Bd. Carol I, n. 11
Iaşi 700506, Romania

Abstract. In this paper we study the second mean curvature for different hypersurfaces in space forms. We furnish some examples and we remind some connections between II-minimality and biharmonicity. The main result consists in proving that there are no II-minimal translation surfaces in the Euclidean three-space.

1. Introduction

The study of the second fundamental form II was initiated in the early papers of Weingarten [16], Darboux [5] and Cartan [3] where appeared for the first time notions like connection or curvature associated to II. Later on, Erard [7] introduced the second fundamental form as metric on the surface. This is possible only when II is non-degenerate and hence it can be regarded as a (pseudo)-Riemannian metric on the surface. At this point one can consider a connected smooth surface M endowed with II as metric in order to study new characteristics associated to (M, II). In the classical case when the metric on the surface is given by the first fundamental form I, i.e., for (M, I), there are well known formulae to compute the **Gaussian curvature** K and the **mean curvature** H in order to analyze the properties of M that arise from this *"measures"*. In a similar manner, the **second Gaussian curvature** denoted by K_{II} and the **second mean curvature**, denoted by H_{II}, were considered. In [3], K_{II} was introduced for the first time by Cartan, as the analogous of the Gaussian curvature. Concerning H_{II}, it was defined by Glässner in [8]. An overview over the literature dedicated to the second fundamental form and the associated curvatures for different type of submanifolds in different ambient spaces can be found in [15] and its references. Regarding the second

*Reprinted from JGSP **18** (2010) 49–62.