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STAR PRODUCT AND STAR EXPONENTIAL*

TOSHIO TOMIHISA and AKIRA YOSHIOKA

*Department of Mathematics, Faculty of Science, Tokyo University of Science
162-8601, Kagurazaka 1-3, Shinjyuku-ku, Tokyo, Japan*

Abstract. Here we extend the star products by means of complex symmetric matrices. In this way we obtain a family of star products. Next we consider the star exponentials with respect to these star products, and finally we obtain several interesting identities.

1. Introduction

In order to express the elements in Weyl algebra, we need to fix the ordering of the generators in monomials because of their non-commutativity. The ordering yields a linear isomorphism between the Weyl algebra and the space of all complex polynomials and the isomorphism naturally induces an associative product in the space of polynomials. This product is called a *star product*. For example, the normal ordering induces normal product, anti-normal ordering induces the anti-normal product and the Weyl ordering yields the Moyal product, respectively.

The so obtained star product algebra is isomorphic to the Weyl algebra, and then these are mutually isomorphic (see for example Omori-Maeda-Miyazaki-Yoshioka [1]). As an extension of these star products, Omori-Maeda-Miyazaki-Yoshioka [2] introduced a family of star products parameterized by the space of all complex symmetric matrices. Then a geometric picture is given for the family parameterized by the space of complex matrices. The family forms an algebraic bundle over the space of all complex symmetric matrices.

When one has to exponentiate elements in the star product algebra, one needs to deal with the infinite sum of the power series with respect to the Plank constant. Then, in order to discuss the convergence of these series it is necessary to introduce a topology and to take the completion of the star product algebra. A typical

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