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THE SPINNING GAS CLOUDS WITH PRECESSION: THE SYMMETRY GENERATORS

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Abstract. The Dyson model of a spinning ellipsoidal gas cloud expanding into a vacuum has been found to be Liouville integrable under certain additional assumptions, such as the absence of either vorticity or of angular momentum. Here we present a new formulation in the form of a 4x4 matrix equation, which generalizes a similar result obtained in rotationless cases. This implies to consider an extended affine space of seven dimensions, in which the seven coordinates of the point-mass representative of the cloud obey differential equations of the same general form as those defining the elliptic functions. This leads very directly to the linearization of the system in the so-called degenerate cases. We obtain also explicit expressions for the symmetry generators, a prerequisite in the task of constructing a Backlund transformation.

1. Introduction

Here we consider the model of a spinning cloud of gas of ellipsoidal shape, expanding into a vacuum, proposed by Dyson [5]. This belongs to a more general class of self-gravitating models, studied in particular by Dirichlet [4], Riemann [19], Chandrasekhar [3] in the case of incompressible fluids, and by Ovsiannikov [18], Dyson and by Fujimoto [6] in the compressible case.

The Dyson model becomes completely integrable by quadratures (Gaffet [8], hereafter Paper I) in the absence of either vorticity or of angular momentum, when the cloud is constituted of an ideal gas with the adiabatic index $\gamma = 5/3$ characteristic of monatomic gases.

Under the restriction of rotation about a fixed principal axis, it has been found (Gaffet [7, 9]) that the equations of motion are amenable to the puzzling form

$$M_{ij}x'^{j} = 0 \tag{1}$$