Geometry,
Integrability
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GEOMETRY OF THE RECURSION OPERATORS FOR CAUDREY-BEALS-COIFMAN SYSTEM IN THE PRESENCE OF MIKHAILOV TYPE $\mathbb{Z}_p$ REDUCTIONS*

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Abstract. We give geometric picture for the Recursion Operators related to the Caudrey-Beals-Coifman linear problem and the Nonlinear Evolution Equations associated to it in the presence of $\mathbb{Z}_p$ reductions of Mikhailov type.

1. Introduction

It is well known that the characteristic property of the nonlinear evolution equations (NLEEs) of soliton type is that they admit the so called Lax representation $[L, A] = 0$. In the last expression L, A are linear operators on ∂_x, ∂_t depending also on a family of functions $q_\alpha(x, t)$, $1 \le \alpha \le s$ (called 'potentials') and a spectral parameter λ. Since the Lax equation $[L, A] = 0$ must be satisfied identically in λ it is equivalent to a system (in case A depends linearly on ∂_t) of the type

$$(q_\alpha)_t = F_\alpha(q, q_x, ...), \quad \text{where} \quad q = (q_\alpha)_{1 \le \alpha \le s} \tag{1}$$

which is the NLEE (soliton equation) associated with A and L. Usually one fixes the linear problem $L\psi = 0$ (auxiliary linear problem) and considers all the NLEEs (of certain form) that can be obtained changing the operator A. There are various schemes used to find exact solutions to a given soliton equation but the essential is that the Lax representation permits to pass from the original evolution defined by the equations (1) to the evolution of some spectral data related to the problem $L\psi = 0$. Since finding the evolution on the spectral data is easy, the principal difficulty is then to recover from the spectral data the potentials, a process called Inverse Scattering Method, see the monograph books [4, 8].