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SCHRÖDINGER MINIMUM UNCERTAINTY STATES OF EM-FIELD IN NONSTATIONARY MEDIA WITH NEGATIVE DIFFERENTIAL CONDUCTIVITY

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Abstract. Quantization of the electromagnetic field in non-stationary media (linear with respect to $\boldsymbol{E}$, with negative differential conductivity) is investigated. The dynamical invariants and statistical properties of the field are found in such media. It is shown that in the eigenstates of linear dynamical invariant, the Schrodinger uncertainty relation is minimized. The time evolution of the tree independent second-order statistical moments (quantum fluctuations: covariance cov(q,p), var(q) and var(p)) are found out.

1. Introduction

The increasing use of energy as a result of the Industrial Revolution has brought a number of serious problems. Understanding the process of photosynthesis will play key role to solve these problems and to develop alternative energy sources in two asspects

- producing alternative fuels (such as H_2, biofuel etc.)
- producing directly Electricity using artificial photosynthesis in Dye-sensitized solar cells.

In the first processes (photosynthesis) the solar light is transformed into chemical energy, saved in molecule **adenosine triphosphate** (ATP). Thus a universal accumulator of energy is formed for widdely distributed biological processes [22].

During the process of photosynthesis [23], the ATP is formed from **adenosine diphosphate** (ADP) and inorganic phosphate. Besides light this conversion requires a donor of electrons as well as protons received from water.

The chemical energy stored in this "biological batery" (ATP) is used by plants to synthesize carbohydrates from CO_2 and H_2O.

Similar and more simplified processes are observed at artificial photosynthesis in **Dye-Sensitized Solar Cells** (DSSC), where the sun light energy absorbed by Ruthenium-polypyridine dye and is transform in the electrical energy, like the chlorophyll in green leaves. Absorbed photons create an excited state of the dye, from which an electron is transported directly into the conduction band of the porous TiO2 in the artificial cell (see also [7]).

2. Motivation for Our Approach

It is well known that electron transport in photosynthesis has Quantum Nature [15]. It is due to the tunneling effect of electron through the barrier, with the action of light quanta. The electron is being tunneled from one carrier to another (starting from chlorophyll molecule to acceptor molecule) with probability depending on the width and height of the barrier. The probability decreases exponentially with increasing the barrier's size.

We are going to aproach only part of the photosynthesis problem, concerning the creation of electrons and charge transport in photosensitive dyes (e.g., in chlorophyll and ruthenium-polypyridine dyes, at photosynthesis and artificial photosynthesis respectively), using tools from quantum mechanics.

For this purpose we consider electron transport chain in thylakoid membrane (or dye solution in DSSC) as a linear media with **Negative Differential Conductivity** (NDC). The light interacts with dye molecule, excites electrons, which overcomes the subsequent quantum potential barrier. This transfers electron from one molecule carrier to an other carrier in the whole chain of electron transport.

3. Schrödinger Uncertainty Relation and Dynamical Invariants in QM

The description of quantum systems is fundamental for understanding many problems in physics and particulary in chemistry. One of the most revolutionary consequences that quantum mechanics bequeathed as a fundamental principle in physics is the refusal of strong determinism. That is why the uncertainty relation plays fundamental role in this science. In 1930, a few years after Heisenberg, Schrödinger had generalized the famous **Uncertainty Relation** (UR) in quantum mechanics (QM) [2, 16, 18]

$$(\Delta q)^2(\Delta p)^2 \geq \frac{\hbar^2}{4} + \mathrm{Cov}^2(q,p). \tag{1}$$

The above inequality shows the general connection between all three independent statistical moments of second order of two quantum variables q and p - the covariance $\mathrm{Cov}(q,p)$

$$\mathrm{Cov}(q,p) = \frac{1}{2}\langle qp + pq\rangle - \langle q\rangle\langle p\rangle \tag{2}$$

and the variances $(\Delta q)^2$ and $(\Delta p)^2$ defined as particular case of covariance

$$(\Delta q)^2 = \mathrm{Cov}(q,q) \quad \text{respectively} \quad (\Delta p)^2 = \mathrm{Cov}(p,p). \tag{3}$$

Canonical variables $\hat{q}$ and $\hat{p}$ satisfy the canonical commutation relations

$$[\hat{q},\hat{p}] = \mathrm{i}\hbar\,\hat{1}. \tag{4}$$

In terms of the covariance matrix $\sigma_M(q,p)$ [6], [10] the uncertainty relation (1) takes the form

$$\det[\sigma_M(q,p)] \geq \frac{\hbar^2}{4}. \tag{5}$$

Other important notions of QM are the dynamical invariants (integrals of motion) $\hat{I}$. These are operators which do no depend on the time t. Using the definition of total derivative in QM of certain quantum system with Hamiltonian $\hat{H}$, the dynamical invariants $\hat{I}$ are defined as solutions to the equation [12]

$$\frac{\mathrm{d}\hat{I}}{\mathrm{d}t} = \frac{\partial\hat{I}}{\partial t} - \frac{\mathrm{i}}{\hbar}[\hat{I},\hat{H}] = 0. \tag{6}$$

The canonical commutation relations (4) show that quadratic in $\hat{q}$ and $\hat{p}$ Hamiltonians admit linear in $\hat{q}$ and $\hat{p}$ dynamical invariants. In [14] a family of (non-Hermitian) invariants $\hat{A}$ for the general nonstationary quadratic Hamiltonian

$$\hat{H} = a(t)\hat{p}^2 + b(t)(\hat{p}\hat{q}+\hat{q}\hat{p}) + c(t)\hat{q}^2 + d(t)\hat{p} + e(t)\hat{q} + f(t) \tag{7}$$

have been constructed in the form

$$\hat{A}(t) = \sqrt{\frac{a}{\hbar}}\left[\epsilon\hat{p} + \frac{1}{a}\left(\epsilon b - \dot{\epsilon} - \frac{\dot{a}}{2a}\epsilon\right)\hat{q}\right] \tag{8}$$

where ϵ is any solution of the second order equation (classical oscillator equation)

$$\ddot{\epsilon} + \Omega^2(t)\epsilon = 0. \tag{9}$$

Actually $\hat{A}^\dagger(t)$ and $\hat{A}(t)$ are generalization of boson creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ of the stationary oscillator (with Ω = constant). The time-dependent coefficients $a(t), b(t)$ and $c(t)$ in (7) establish the connection between the Hamiltonian $\hat{H}$ and the frequency $\Omega(t)$ of classical non-stationary harmonic oscillator

$$\Omega^2 = 4ac + 2b\frac{\dot{a}}{a} + \frac{\ddot{a}}{2a} - \frac{3\dot{a}^2}{4a^2} - 4b^2 - 2\dot{b}. \tag{10}$$

The linear part in the Hamiltonian (7) is not essential for the classical non-stationary harmonic oscillator, so it is assumed that $d(t) = e(t) = f(t) = 0$.

The commutator $[\hat{A}, \hat{A}^\dagger]$ is presented by Wronsky determinant W

$$[\hat{A}, \hat{A}^\dagger] = \frac{\mathrm{i}}{2}(\epsilon\dot{\epsilon}^* - \epsilon^*\dot{\epsilon}) \equiv \frac{\mathrm{i}}{2}W \tag{11}$$

so that $[\hat{A}, \hat{A}^\dagger] = \hat{1}$ iff

$$\epsilon = |\epsilon|\, \mathrm{e}^{\mathrm{i}\int_0^t \frac{\mathrm{d}t'}{|\epsilon(t')|^2}}. \tag{12}$$

4. Quantization of EM Field in Linear Media with Negative Differential Conductivity

The Maxwell equations in non-stationary linear media have the form

$$\boldsymbol{B}(\boldsymbol{r}, t) = \mu(t)\boldsymbol{H}(\boldsymbol{r}, t), \qquad \boldsymbol{D}(\boldsymbol{r}, t) = \varepsilon(t)\boldsymbol{E}(\boldsymbol{r}, t), \qquad \boldsymbol{j} = \sigma(\boldsymbol{t})\boldsymbol{E} \tag{13}$$

$$\begin{aligned} &\mathrm{div}\boldsymbol{D} = 0, &&\mathrm{rot}\boldsymbol{H} = \frac{\partial}{\partial t}\boldsymbol{D} + \sigma(\boldsymbol{t})\boldsymbol{E} \\ &\mathrm{div}\boldsymbol{B} = 0, &&\mathrm{rot}\boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}. \end{aligned} \tag{14}$$

Note that $\varepsilon(t) = \varepsilon_r(t)\varepsilon_0$ is the dielectric permittivity, and differs from the solution $\epsilon(t)$ of classical oscillator equation (9).

A scheme was proposed for quantizing the damped light in conducting media [3] (see references therein). We are going to apply the quantization not only for non-stationary media ($\epsilon(t), \mu(t)$ and $\sigma(t)$), but for a case of negative differential conductivity. For convenience we will consider one dimension case (in x direction). A linear and homogenous media could have some resistivity R (which is a positive constant). There are some special cases, when the resistivity (respectively - the conductivity) vary with the applied voltage. For example, this is the case with tunnel diodes, which are represented as over-dopped semiconductors with very narrow $p-n$-junction, playing a role of quantum mechanical potential barrier [4, 5] (see Fig. 1). One analytical expression [5] for such I-V characteristic is shown here

$$I = \frac{U}{R_0}\exp\left[-(\frac{U}{U_0})^m\right] + I_s \exp\left[\frac{U}{\eta U_{th}} - 1\right] \tag{15}$$

where R_0, U_0, I_s, η and U_{th} are appropriate constants.

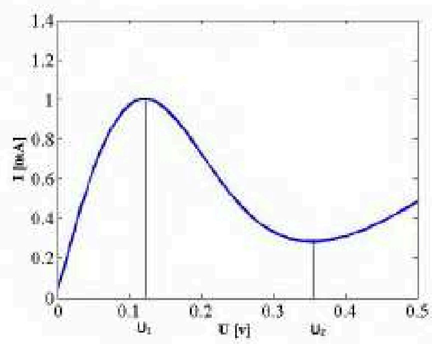

Fig. 1. I-U characteristic of device with negative differential conductivity.

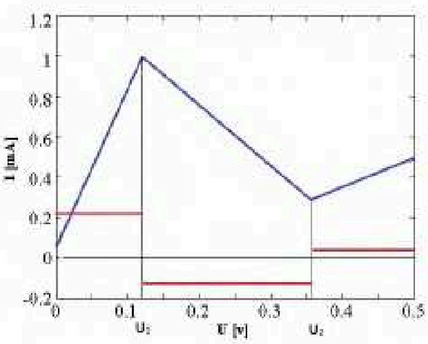

Fig. 2. Idealized I-U characteristic and the first derivative of σ, $\sigma_{\rm diff} < 0$ for $U_1 < U < U_2$.

To escape the problems with quantization in such media, we consider a hypothetic one, consisted of three different voltage domains, presenting the $I = I(U)$ with strait lines, as is shown on Fig. 2. This point of view is reasonable, because as is seen from Fig. 1, in all three domains there exists smaller sections, where the currents $I = I(U)$ could be presented approximately as strait lines. It is obvious that in the first and in the third domains, the secondary quantization could be solved in standard way (see for example [8,9,21]). Here we focus our attention mainly on the interesting second domain, which is called the regime with negative differential resistance (we do not take into account the transitions between domains, and leave this problem for future investigations). For the domain with negative differential conductivity around inflex point we always could apply linear approach. So, for our simplified model we have

$$\sigma = \text{const} > 0 \quad \text{but} \quad \sigma_{\rm diff} = \frac{{\rm d}\sigma(U)}{{\rm d}U} < 0 \qquad \text{for} \quad U_1 < U < U_2. \tag{16}$$

For this domain, where the differential conductivity is negative, we are going to find analytical solution for this quantum problem, when the condition (16) is satisfied also. Applying the Coulomb gauge, one can define vectors fields as

$$\boldsymbol{B} = \text{rot}\boldsymbol{A}, \qquad \boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} \tag{17}$$

and from Maxwell equations (14) we obtain the equation for $\boldsymbol{A}(\boldsymbol{r}, t)$

$$\nabla^2 \boldsymbol{A} - \mu(\sigma + \dot{\varepsilon})\frac{\partial \boldsymbol{A}}{\partial t} - \varepsilon\mu\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = 0 \tag{18}$$

As usual (see e.g. the books [11, 17, 24]) we expand vector potential $\boldsymbol{A}(\boldsymbol{r},t)$ in terms of mode functions $\boldsymbol{u}_l(\boldsymbol{r}) = \boldsymbol{e}_{l,\xi} u_{l,\xi}(\boldsymbol{r})$

$$\boldsymbol{A}(\boldsymbol{r},t) = \sum_{l,\xi} \boldsymbol{e}_{l,\xi} u_{l,\xi}(\boldsymbol{r}) q_{l,\xi}(t) \tag{19}$$

which satisfy the Helmholtz equation

$$\left(\nabla^2 + \frac{\omega_{0,l}^2}{c^2}\right) u_{l,\xi}(\boldsymbol{r}) = 0. \tag{20}$$

From Maxwell equations (14) it follows that (in case of linear media (13)) the time-dependent factors q_l are to obey the following linear equation (furthermore, unless otherwise stated, we suppress the polarization index ξ)

$$\frac{\partial^2 q_l}{\partial t^2} + \frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)} \frac{\partial q_l}{\partial t} + \omega_l^2(t) q_l = 0, \qquad \omega_l^2(t) = \frac{\omega_{0,l}^2}{c^2 \varepsilon(t) \mu(t)}. \tag{21}$$

One can see that the equation (21) could be obtained from the classical Hamilton equation with Hamilton function

$$H_l = \frac{1}{2}\left[\frac{1}{\varepsilon_0} \mathrm{e}^{-\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} p_l^2 + \varepsilon_0 \omega_l^2(t) \mathrm{e}^{\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} q_l^2\right] \tag{22}$$

Introducing the canonical operators $q- > \hat{q}_l$ and $p- > \hat{p}_l$, which obey the commutation relation (4) we receive for the total Hamiltonian of the EMF as a sum over all modes [3]

$$\hat{H} = \sum_l \left[\frac{1}{2\varepsilon_0} \mathrm{e}^{-\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} \hat{p}_l^2 + \frac{\varepsilon_0 \omega_l^2(t)}{2} \mathrm{e}^{\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} \hat{q}_l^2\right] \equiv \sum_l \hat{H}_l. \tag{23}$$

It can be seeing that time-dependent coefficients in equation (23) are

$$a(t) = \frac{1}{2\varepsilon_0} \mathrm{e}^{-\int_0^t \frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)} \mathrm{d}t}, \qquad b(t) = 0, \qquad c(t) = \frac{\varepsilon_0 \omega_l^2(t)}{2} \mathrm{e}^{\int_0^t \frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)} \mathrm{d}t} \tag{24}$$

and for the frequency $\Omega_l(t)$ of the non-stationary harmonic oscillator (10) we get

$$\Omega_l^2(t) = \omega_l^2(t) - \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)}\right) - \frac{1}{4}\left(\frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)}\right)^2. \tag{25}$$

In this case the invariants that satisfy the canonical boson relation $[\hat{A}, \hat{A}^\dagger] = \hat{\mathbf{1}}$ are

$$\hat{A}_l = \frac{1}{\sqrt{2\hbar\varepsilon_0}} \mathrm{e}^{-\frac{1}{2}\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} \times \left[\epsilon_l \hat{p}_l - \varepsilon_0 \mathrm{e}^{\int_0^t \frac{\sigma(t') + \dot{\varepsilon}(t')}{\varepsilon(t')} \mathrm{d}t'} \left(\dot{\epsilon}_l - \frac{1}{2} \frac{\sigma(t) + \dot{\varepsilon}(t)}{\varepsilon(t)} \epsilon_l\right) \hat{q}_l\right]. \tag{26}$$

The linear invariants $\hat{A}_l$ and the quadratic ones $\hat{A}_l^\dagger \hat{A}_l$ have the following eigenfunctions [14] ($\psi_{\alpha_l}(q_l,t) = \langle q_l|\alpha_l;t\rangle$, $\psi_{n_l}(q_l,t) = \langle q_l|n_l;t\rangle$

$$\psi_{\alpha_l}(q_l,t) = \psi_0(q_l,t)\exp\left[\sqrt{\frac{2}{a\hbar}}\frac{\alpha_l}{\epsilon_l}q_l - \frac{\epsilon_l^*}{2\epsilon_l}\alpha_l^2 - \frac{1}{2}|\alpha_l|^2\right] \tag{27}$$

$$\psi_{n_l}(q_l,t) = \psi_0(q_l,t)\frac{(\epsilon_l^*/2\epsilon_l)^{n_l/2}}{\sqrt{n_l!}}H_{n_l}(x_l), \qquad x_l = \frac{q_l}{|\epsilon_l|\sqrt{a}} \tag{28}$$

with eigenvalues α_l and n_l respectively. Here $H_n(x)$ are Hermite polynomials and $\psi_0(q_l,t)$ are the ground state wave functions ($\hat{A}_l\psi_0 = 0$)

$$\psi_0(q_l,t) = \left(\epsilon_l(\pi a\hbar)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\exp\left[\frac{\mathrm{i}}{2a\hbar}\left(\frac{\dot{\epsilon}_l}{\epsilon_l} + \frac{\dot{a}}{2a}\right)q_l^2\right]. \tag{29}$$

These time-dependent wave functions are normalized solutions to the Schrödinger equation with Hamiltonian $\hat{H}_l$, equation (23). Since $\hat{A}(t)$ and $\hat{A}_l^\dagger(t)\hat{A}_l(t)$ are dynamical invariant, the eigenvalues α_l and n_l are constant in time.

The system of $|\alpha_l;t\rangle$ is overcomplete in the one mode Hilbert space H_l (the set of $|n_l;t\rangle$ being complete)

$$\frac{1}{\pi}\int|\alpha_l;t\rangle\langle t;\alpha_l|d^2\alpha_l = \sum_{n_l}|n_l;t\rangle\langle t;n_l| = \hat{1}_l. \tag{30}$$

These states $|\alpha_l;t\rangle$ minimize the general uncertainty relation of Schrödinger

$$(\Delta q)^2(\Delta p)^2 = \frac{\hbar^2}{4} + \mathrm{Cov}^2(q,p). \tag{31}$$

According to the terminology of references [13,14] the states $|\alpha_l;t\rangle$ may be called generalized **Coherent States** (CS) of nonstationary system with Hamiltonian $\hat{H}_l$, equation (23). For the purpose of this paper and to make it more readable for physicist-experimentalists, biologist etc. we will call the states minimizing relation (31) **Schrödinger Minimum Uncertainty States** (SMUS) as it is done in [19]. Because the Hamiltonian $\hat{H}$, equation (23), is a sum over l, the SMUS for EM field with finite number of modes are product over l of one mode SMUS $|\alpha_l;t\rangle$.

The vector potential operator takes the form

$$\hat{\boldsymbol{A}}(\boldsymbol{r},t) = \sum_l \boldsymbol{u}_l(\boldsymbol{r})\hat{q}_l. \tag{32}$$

Note that it differs from linear invariants $\hat{A}(t)$, and it is denoted here in bold face. Replacing it in (17) one obtains the quantized electric and magnetic fields

$$\hat{\boldsymbol{E}}(\boldsymbol{r},t) = -\frac{1}{\epsilon_0}\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}\sum_l \boldsymbol{u}_l(\boldsymbol{r})\hat{p}_l \quad (33)$$

$$\hat{\boldsymbol{B}}(\boldsymbol{r},t) = \mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}\sum_l \nabla\times\boldsymbol{u}_l(\boldsymbol{r})\hat{q}_l. \quad (34)$$

Using the time derivatives of operators $\hat{q}_l$, $\hat{p}_l$ of the form

$$\frac{\mathrm{d}\hat{o}_l}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}[\hat{o}_l, \hat{H}] \quad (35)$$

we check that all Maxwell equations (14) are satisfied by operator fields $\hat{\boldsymbol{E}}$, $\hat{\boldsymbol{D}} = \varepsilon\hat{\boldsymbol{E}}$, $\hat{\boldsymbol{H}}$ and $\hat{\boldsymbol{B}} = \mu\hat{\boldsymbol{H}}$.

Evolution of second order statistical moments in SMUS. All three quantum-mechanical statistical moments for canonical operators $\hat{q}_l$ and $\hat{p}_l$ are defined in the evolved SMUS $|\alpha_l;t\rangle$. Using the general formulae [20] we find the variances

$$\begin{aligned}(\Delta q_l)^2_{\alpha_l} &= \frac{\hbar\varepsilon_0^{-1}\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}}{2}\rho_l^2, \qquad \rho_l = |\epsilon_l(t)| \\ (\Delta p_l)^2_{\alpha_l} &= \frac{\hbar}{\varepsilon_0^{-1}\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}}\left[\frac{1}{2\rho_l^2} + \left(\dot{\rho}_l(t) - \frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\rho_l\right)^2\right].\end{aligned} \quad (36)$$

From the general formula derived in [1], we obtain the covariance $\mathrm{Cov}(q,p)$ in terms of the negative differential conductivity

$$\mathrm{Cov}(q_l,p_l)_{\alpha_l} = -\frac{\hbar}{2}\rho_l\left(\dot{\rho}_l - \frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\rho_l\right). \quad (37)$$

Thus, we find the three statistical moments ($(\Delta q_l)^2$, $(\Delta p_l)^2$ and $\mathrm{Cov}(q,p)$) in the case of media with negative differential conductivity.

5. Vector Operators for EM Field and Their Statistical Properties

To express the statistical properties of vector operators $\hat{\boldsymbol{E}}(\boldsymbol{r},t)$ and $\hat{\boldsymbol{B}}(\boldsymbol{r},t)$ for EM field, it is convenient to present the Hermitian operators $\hat{q}$, $\hat{p}$ in terms of the invariants $\hat{A}$, $\hat{A}\dagger$. Taking into account (26) we get the following expressions

$$\hat{q}_l = \left(\frac{\hbar}{2\varepsilon_0}\right)^{1/2}\mathrm{e}^{-\frac{1}{2}\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}\left(-\mathrm{i}\epsilon_l(t)\hat{A}_l^\dagger(t) + \mathrm{i}\epsilon_l^*(t)\hat{A}_l(t)\right) \quad (38)$$

$$\hat{p}_l = \left(\frac{\hbar\varepsilon_0}{2}\right)^{1/2} \mathrm{e}^{\frac{1}{2}\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'} \times \left(-\mathrm{i}(\dot{\epsilon}_l - \frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\epsilon_l)\hat{A}_l^\dagger + \mathrm{i}(\dot{\epsilon}_l^* - \frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\epsilon_l^*)\hat{A}_l\right). \quad (39)$$

We shall consider the case of periodic boundary conditions with *complex mode functions* $\boldsymbol{u}_{l,\xi}^{(\pm)}(\boldsymbol{r}) = V^{-1/2}\boldsymbol{e}_{l,\xi}\exp(\pm\mathrm{i}\boldsymbol{k}_l\cdot\boldsymbol{r})$ [24], where $\boldsymbol{e}_{l,\xi}$ is the polarization vector of mode l, with wavevector $\boldsymbol{k}_l$. With these modes the vector potential operator, which obeys the equation (18) takes the following form

$$\hat{\boldsymbol{A}}(\boldsymbol{r},t) = \sqrt{\frac{\hbar}{2\varepsilon_0}}\mathrm{e}^{-\frac{1}{2}\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'} \sum_{l,\xi} \boldsymbol{e}_{l,\xi}\left[u_{l,\xi}^*(\boldsymbol{r})\epsilon_l\hat{A}_{l,\xi}(t) + h.c.\right]. \quad (40)$$

Replacing the vector potential operator $\hat{\boldsymbol{A}}(\boldsymbol{r},t)$ in the relations (17) we receive vector operator for EM field in the form

$$\hat{\boldsymbol{E}}(\boldsymbol{r},t) = \sqrt{\frac{\hbar}{2\varepsilon_0}}\mathrm{e}^{-\frac{1}{2}\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'} \times \sum_{l,\xi}\boldsymbol{e}_{l,\xi}\left[\left(\frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\epsilon_l - \dot{\epsilon}_l\right)u_{l,\xi}^*(\boldsymbol{r})\hat{A}_{l,\xi}(t) + h.c.\right] \quad (41)$$

$$\hat{\boldsymbol{B}}(\boldsymbol{r},t) = \mathrm{i}\sqrt{\frac{\hbar}{2\varepsilon_0}}\mathrm{e}^{-\frac{1}{2}\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'} \sum_{l,\xi}\boldsymbol{k}_l\times\boldsymbol{e}_{l,\xi}\left[u_{l,\xi}^*(\boldsymbol{r})\epsilon_l\hat{A}_{l,\xi}(t) - h.c.\right]. \quad (42)$$

The commutators between the j and m components of $\hat{\boldsymbol{E}}_l(\boldsymbol{r},t)$ and $\hat{\boldsymbol{B}}_l(\boldsymbol{r},t)$ are C-numbers, vanishing for $j = m$

$$[\hat{E}_{l,j}(\boldsymbol{r},t), \hat{B}_{l,m}(\boldsymbol{r},t)] = \mathrm{i}\frac{\hbar}{\varepsilon_0 V}\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'} \times \sum_{\xi} e_{l,\xi,j}(\boldsymbol{k}_l\times\boldsymbol{e}_{l,\xi})_j \,\mathrm{Re}\left(\dot{\epsilon}_l\epsilon_l^* - |\epsilon_l|^2\frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}\right)\delta_{jm}. \quad (43)$$

The three second moments are found as

$$\begin{aligned}
(\Delta E_l)_\alpha^2 &= \frac{\hbar}{2\varepsilon_0 V}\,\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}\left|\frac{1}{2}\frac{\sigma(t)+\dot{\varepsilon}(t)}{\varepsilon(t)}|\epsilon_l|^2 - \dot{\epsilon}_l\right|^2 \\
(\Delta B_l)_\alpha^2 &= k_l^2\frac{\hbar}{2\varepsilon_0 V}\,\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}|\epsilon_l|^2 \\
\mathrm{Cov}(E_l,B_l)_\alpha &= -k_l\frac{\hbar}{2\varepsilon_0 V}\,\mathrm{e}^{-\int_0^t \frac{\sigma(t')+\dot{\varepsilon}(t')}{\varepsilon(t')}\mathrm{d}t'}\mathrm{Im}\left(\frac{\sigma(t)+\dot{\varepsilon}(t)}{2\varepsilon(t)}|\epsilon_l|^2 - \dot{\epsilon}_l\epsilon_l^*\right)
\end{aligned} \quad (44)$$

(note also the presence of conductivity $\sigma(t)$ (with $\sigma_{\text{diff}} < 0$, which may vary in time also) in the expressions of all the above averages).

Schrödinger Uncertainty Relation for the j and m components of $\hat{\boldsymbol{E}}_l(\boldsymbol{r}, t)$ and $\hat{\boldsymbol{B}}_l(\boldsymbol{r}, t)$ in SMUS take the form

$$(\Delta E_l)^2_\alpha \, (\Delta B_l)^2_\alpha - \operatorname{Cov}^2(E_l, B_l)_\alpha = \frac{\hbar^2}{4} \, |\langle [E_l, B_l] \rangle_\alpha|^2 \,. \tag{45}$$

Thus the time-evolved SMUS $|\alpha_l; t\rangle$ in nonstationary and/or conductive media are minimizing uncertainty states with respect to the photon ladder operator quadratures $\hat{q}_l$, $\hat{p}_l$, and with respect to the electric and magnetic field components as well. The time evolution of these states can exhibit q_l-p_l and E_l-B_l covariance and squeezing.

Conclusion

Quantization of the electromagnetic field in non-stationary media (linear with respect to $\boldsymbol{E}$ and arbitrary with respect to time t) is investigated. The model presented here allow to be investigated and to perform secondary quantization in media with negative differential conductivity also. The dynamical invariants and statistical properties of the field are found in such media. It is shown that in the eigenstates of linear dynamical invariant, the Schrodinger uncertainty relation is minimized. The time evolution of the tree independent second-order statistical moments (quantum fluctuations: covariance cov(q,p), var(q) and var(p)) are found out. The model developed here, could be involved in quantum-mechanical explanation of electrons transport, when the electron jumps from one dye molecule to an other, overcoming the potential barrier between them. Thus, the tunnel effect, leading to negative differential conductivity, play essential role not only in chlorophyll, but in electron transport in ruthenium-polypyridine dyes at artificial photosynthesis, also.

References

[1] Angelow A., *Light Propagation in Nonlinear Waveguide and Classical Two-dimensional Oscillator*, Physica A **256** (1998) 485–498.

[2] Angelow A. and Batoni M., *Translation with Annotation of the Original Paper of E. Schrödinger (1930) in English*, Bulg. J. Phys. **26** (1999) 193–203 http://arxiv.org/abs/quant-ph/9903100.

[3] Choi J., *Coherent and Squeezed States of Light in Linear Media with Time-dependent Parameters by Lewis-Riesenfeld Invariant Operator Method*, J. Phys. A **39** (2006) 669–684.

[4] Esaki L., *New Phenomenon in Narrow Germanium p-n Junction*, Phys. Rev. **109** (1958) 603-604.

[5] Simin G., *Tunnel Diodes*, Elctronics & Comp. Tech. Course ELCT, **563** (2009) 1-30, http://www.ee.sc.edu/personal/faculty/simin/ELCT563/08 Tunnel Diodes.pdf.

[6] Gardiner C., *Handbook on Stochastic Methods*, Springer, Heidelberg 1983.

[7] Lubner C., Applegate A., Knorzer P., Ganago A., Donald A., Bryant D., Happe T. and Golbeck J., *Solar Hydrogen-producing Bionanodevice Outperforms Natural Photosynthesis*, PNAS (2011) **108** 20988–20991.

[8] Gosson M., *The Principles of Newtonian and Quantum Mechanics: The Need for Planck's Constant h*, Imperial College Press, London 2001.

[9] Gosson M., *Symplectic Geometry and Quantum Mechanics*, Operator Theory: Advances and Applications **166**, I. Gohberg (Ed), Birkhäuser, Basel 2006.

[10] Korn G. and Korn T., *Mathematical Handbook*, McGraw-Hill, New York 1961.

[11] Lousell W., *Quantum Statistical Properties of Radiation*, Wiley, New York 1973.

[12] Malkin I. and Man'ko V., *Dynamicheskie Sistemyi i Kogerentnyie Sostoianija Kvantovyih Sistem* (in Russian), Nauka, Moscow 1979.

[13] Malkin I., Man'ko V. and Trifonov D., *Coherent States and Transition Probabilities in a Time-Dependent Electromagnetic Field*, Phys. Rev. D **2** (1970) 1371–1385.

[14] Malkin I., Man'ko V. and Trifonov D., *Linear Adiabatic Invariants and Coherent States*, J. Math. Phys. **14** (1973) 576–582.

[15] Rubin A., *Primary Processes of Photosynthesis* (in Russian), Sorosovskij Obrazovatelnyj Journal **10** (1997) 79–84.

[16] Schrödinger E., *Zum Heisenbergschen Unschärfeprinzip*, Ber. Kgl. Akad. Wiss. Berlin **19** (1930) 296–303.

[17] Scully M. and Zubairy M., *Quantum Optics*, Cambridge University Press, Cambridge 1997.

[18] Trifonov D., Nikolov B. and Mladenov I., *On the Uncertainty Relations in Stochastic Mechanics*, J. Geom. Symmetry Phys. **16** (2009) 57-75.

[19] Trifonov D., *Completeness and Geometry of Schrödinger Minimum Uncertainty States*, J. Math. Phys. **34** (1993) 100–110.

[20] Trifonov D., *Coherent States and Uncertainty Relations*, Phys. Lett. A **48** (1974), 165; Trifonov D., *Coherent States and Evolution of Uncertainty Products*, ICTP Internal Report **IC/75/2** (1975) 1-6; Bulg. J. Phys. **2** (1975) 303–311 (Here expressions for the variances of $\hat{q}$ and $\hat{p}$ should be interchanged).

[21] Trifonov D. and Angelow A., *Dynamical Invariants and Robertson-Schrödinger Correlated States of Electromagnetic Field in Nonstationary Linear Media*, AIP Conf. Proc. **1340** (2011) 221–233.

[22] Volkeinstein M., *Molekulqrnaq Biofizika* (in Russian), Nauka, Moscow 1975.

[23] Volkeinstein M., *Biofizika* (in Russian), Nauka, Moscow 1981.

[24] Walls D. and Milburn G., *Quantum Optics*, $2^{\rm nd}$ Edn, Springer, Heidelberg 2008.