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RIEMANNIAN CURVATURES OF THE FOUR BASIC CLASSES OF REAL HYPERSURFACES OF A COMPLEX SPACE FORM

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Abstract. Any real hypersurface of a Kähler manifold carries a natural almost contact metric structure. There are four basic classes of real hypersurfaces of a Kähler manifold with respect to the induced almost contact metric structure. In this paper we study the basic classes of real hypersurfaces of a complex space form in terms of their Riemannian curvatures.

1. Introduction

Let $\bar{M}^{2n+2}(J,G)$ be an almost Hermitian manifold with almost complex structure J and Riemmannian metric G: $J^2 = -\operatorname{Id}$, $G(J\bar{X}, J\bar{Y}) = G(\bar{X},\bar{Y})$, $\bar{X},\bar{Y} \in \mathfrak{X}\bar{M}^{2n+2}$.

If M^{2n+1} is a hypersurface in $\bar{M}^{2n+2}$ with a unit normal vector field N, then there arises naturally an almost contact metric structure (φ,ξ,η,g) on M^{2n+1} in the following way [3, 10, 12]:

$$\xi = -JN, \quad g = G_{|M}, \quad \varphi = J - \eta\otimes N\,,$$
$$\eta(X) = g(\xi, X), \quad X \in \mathfrak{X}M^{2n+1}\,.$$

Let ∇ and ∇' be the Levi-Civita connections on M^{2n+1} and $\bar{M}^{2n+2}$, respectively. We denote by Φ the fundamental 2-form of the structure (φ,ξ,η,g)

$$\Phi(X,Y) = g(X,\varphi Y)\,, \qquad X,Y \in \mathfrak{X}M^{2n+1}$$

and by $F' = -\nabla'\Phi, F = \nabla\Phi$. If A is the shape operator and $h(X,Y) = g(AX,Y), \quad X,Y \in \mathfrak{X}M^{2n+1}$ is the second fundamental tensor of M^{2n+1},