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ON THE REDUCTIONS AND SCATTERING DATA FOR THE CBC SYSTEM

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Abstract. The reductions for the first order linear systems of the type:

$$L\psi(x,\lambda) \equiv \left(\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x} + q(x) - \lambda J\right)\psi(x,\lambda) = 0\,, \quad J \in \mathfrak{h}\,, \quad q(x) \in \mathfrak{g}_J$$

are studied. This system generalizes the Zakharov–Shabat system and the systems studied by Caudrey, Beals and Coifman (CBC systems). Here J is a regular complex constant element of the Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$ of the simple Lie algebra $\mathfrak{g}$ and the potential $q(x)$ takes values in the image $\mathfrak{g}_J$ of ad_J. Special attention is paid to the scattering data of CBC systems and their behaviour under the Weyl group reductions. The analytical properties of the generating functional of the integrals of motion and their reduced analogs are studied. These results are demonstrated on an example of N-wave type equations.

1. Introduction

The idea that the inverse scattering method (ISM) is a generalized Fourier transform has appeared as early as 1974 in [1]. In the class of nonlinear evolution equations (NLEE) related to the Zakharov–Shabat (ZS) system [26, 24] related to $sl(2)$ algebra was studied. This class of NLEE contains such physically important equations as the nonlinear Schrödinger equation (NLS), the sin-Gordon and modified Korteweg–de Vries (mKdV) equations.

The multicomponent ZS system leads to such important systems like the multicomponent NLS, the N-wave type equations, etc.

The classical results of [26, 24] have been generalized in several directions.