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$SU(3)$ GENERALIZATIONS OF THE CASSON INVARIANT FROM GAUGE THEORY

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Abstract. This paper is a survey of some recent joint work of Hans Boden, Paul Kirk and the author, as well as work by Cappell, Lee, and Miller, on generalizing the Casson invariant to the group $SU(3)$. The main challenge here is that in this setting there are nontrivial reducible representations. Because of this, the irreducible stratum is not compact, and as a consequence an algebraic count of points does not provide a topological invariant (independent of perturbation).

1. Introduction

In the late 1980's, Andrew Casson defined a new invariant of closed, oriented 3-manifolds X which have the same homology as S^3. Roughly speaking, the Casson invariant $\lambda_{SU(2)}(X)$ is an algebraic count of the conjugacy classes of irreducible $SU(2)$ representations $\rho\colon \pi_1 X \to SU(2)$. The reason for restricting to $SU(2)$ and imposing this homology restriction on the 3-manifold X is that reducible representations $\rho\colon \pi_1 X \to SU(2)$ are necessarily abelian and hence factor through the homology $H_1(X)$. For a homology 3-sphere, this homology group is trivial and so the only reducible representation is the trivial one. For cohomological reasons, the trivial representation is isolated, which implies that the irreducible portion of the set of representations modulo conjugation is compact.

We shall describe Casson's method for defining the invariant in more detail in the next section. For the moment, suffice it to say that Casson devised a means of perturbing to achieve a 0-dimensional set of points to count, in case the representation variety was not already a 0-dimensional manifold. The