Fourth International Conference on
Geometry, Integrability and Quantization
June 6–15, 2002, Varna, Bulgaria
Ivaïlo M. Mladenov and Gregory L. Naber, Editors
Coral Press, Sofia 2003, pp 206–234

UNDULOIDS AND THEIR CLOSED GEODESICS

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Abstract. We construct explicit parametrizations in terms of elliptic functions for unduloids depending on a parameter. We then use these parametrizations to study geodesics on unduloids. In particular, we use *Maple* to find interesting closed geodesics on unduloids.

1. Introduction

Finding closed geodesics on manifolds is an important and quite difficult problem in geometry (e. g. see [10]). For instance, it was a major advance when, around 1930, Lusternik and Schnirelmann showed that S^2 with any Riemannian metric possesses at least 3 closed geodesics. Recent acclaimed work of Franks [4] shows that, in fact, an infinite number of distinct closed geodesics exist. Lusternik and Fet proved that every closed manifold possesses at least one closed geodesic and this type of result can be extended for, say, surfaces not homeomorphic to the plane or cylinder [16]. In general, the focus has been on proving existence theorems rather than finding explicit examples of closed geodesics. In this paper, we will consider a non-compact cylinder-like surface called the *unduloid* and study some aspects of its geodesics. In particular, we will see that *Maple* may be used to find (within some tolerance) closed geodesics of an interesting type. We will *not* simply draw pictures and make claims, however, but rather, prove that our computer methods determine closed geodesics.[(1)] [(2)] For this, we bring classical differential geometry and

[(1)] We wish to thank David Singer for several conversations which contributed mightily to the development of the method for finding closed geodesics presented here.

[(2)] A preliminary version of this paper has appeared in the *Proceedings of Maple Workshop* 2002.