CONFORMAL SCHWARZIAN DERIVATIVES AND DIFFERENTIAL EQUATIONS

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Abstract. We investigate the fundamental system of equations in the theory of conformal geometry, whose coefficients are considered as the conformal Schwarzian derivative. The integrability condition of the system is obtained in a simple method, which allow us to find a natural geometric structure on the solution space. From the solution spaces, using this geometric structure, we get a transformation whose Schwarzian derivative is equal to the given coefficients of the equation.

1. Introduction

Some years ago Sasaki and Yoshida [10] gave the fundamental system of linear equations, which is the key system connecting the theory of conformal connections and the uniformizing differential equations in the geometry of symmetric domains of type IV. It is a system of equations with n variables such that the maximal dimension of the solution space is $n + 2$. The solutions naturally provide a map into the projective space whose image is contained in the hyperquadric, and accepts the conformal transformation group as its symmetry.

Sasaki and Yoshida considered the equations as a higher dimensional analogue of Gauss–Schwarz equation. In projective geometry of higher dimension, they defined Schwarzian derivatives as a difference of normal Cartan connections moved by a diffeomorphism. Using the Schwarzian derivatives, they got the system of linear equations such that the maximal dimension of the solution space is $n + 1$ on n variables [9, 12]. In conformal geometry of higher dimension, the problem is much harder. As the Schwarzian derivatives we need more